cet The Center for Educational Technology

JERUSALEM COLLEGE OF TECHNOLOGY
LEV ACADEMIC CENTER

BUDAPEST METROPOLITAN UNIVERSITY

The Chair for Mathematics, Education and Judaism

Erasmus+
KA107 STEAM 2016

# Glimpses at Mathematics and Jewish Art

## (with a focus on symmetry)

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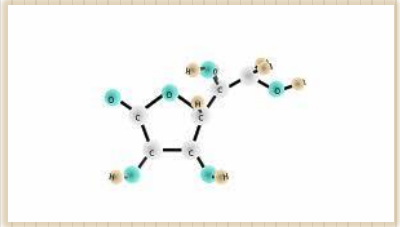
Budapest, December 6th , 2016



# Limitations: Short tidbits on a Jewish attitude towards Art

Whether there exists a form of art that can be described as "Jewish Art" has long been a matter for discussion. What is indisputable is that at every stage of their history the Jews and their ancestors of biblical times expressed themselves in various art forms which inevitably reflect contemporary styles and fashions and the environment in which they lived. For purposes of cult and of religious observance, as well as for household and personal adornment, Jews have constantly produced or made use of objects which appealed in some fashion to their aesthetic sense. In a famous passage (Shab. 133b), the rabbis, commenting on Exodus 15:2, prescribed that God should be "adorned" by the use of beautiful implements for the performance of religious observances. A problem exists, however, regarding the Jewish attitude toward figurative and representational art. The Pentateuchal code in many places (Ex. 20:4; Deut. 5:8 and in great detail 4:16–18) ostensibly prohibits, in the sternest terms, the making of any image or likeness of man or beast.

**Exodus 20,3 – Ten commandements**

"Thou shalt not make unto thee a graven image, nor any manner of likeness, of any thing that is in heaven above, or that is in the earth beneath, or that is in the water under the earth"





**Ref: the Jewish Virtual Library**

# Pros and Cons until 19th century

## May exist

- Torah scrolls ornaments
- Enluminated books
- Music
- Etc…

## May not exist

- Monuments (also because political decision of the Gentiles' governments)
- Sculptures (Figurative pictures of animals and human beings)
- Etc…

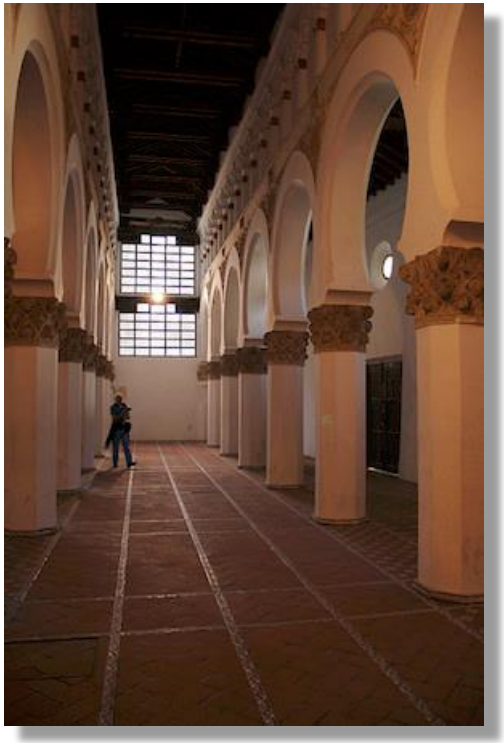
2D ≠ 3D

# Number Theory & Geometry

« It is important to study the theories of numbers and of conic sections as their study help man to get closer to G.d » (Maimonides, 14th century – free translation)

# Jewish Art: Ancient items

Panel from a Torah Shrine from the Ben Ezra Synagogue in Cairo, 11th century, wood (walnut) with traces of paint and gilt, 87.3 x 36.7 cm (The Walters Art Museum). The patterns of vine scrolls and lozenges shows the influence of Islamic art.



Santa Maria la Blanca, former synagogue in Toledo, Spain. Erected in 1180, it may be the oldest synagogue in Europe still standing. It is now owned and preserved by the Catholic Church as a museum

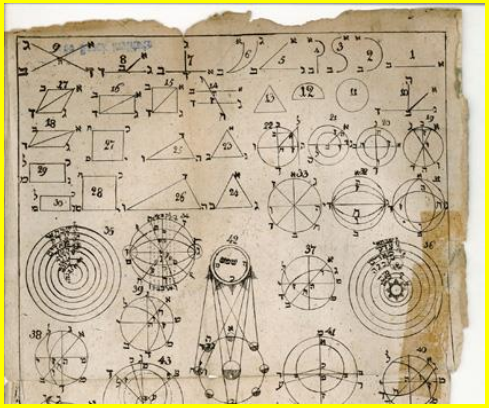
photo: Nik McPhee (CC BY-SA 2.0)

# Religious items (Judaica)

**Torah Crown**, 1698-99, Bolzano, Italy (The Jewish Museum, New York) "Originally dedicated to an Italian synagogue in 1698/99, this crown was later plundered during a Russian pogrom and then recovered. It became part of the collection of the Great Synagogue of Danzig in the early 20th century. In 1939, it was sent to the Jewish Theological seminary in New York for safekeeping when the Nazis' rise to power forced the Danzig Jewish community to disband."

**Torah Case**, Iraq, 19th-early 20th century, silver overlaid on wood, with coral set cresting (The Jewish Museum, London)

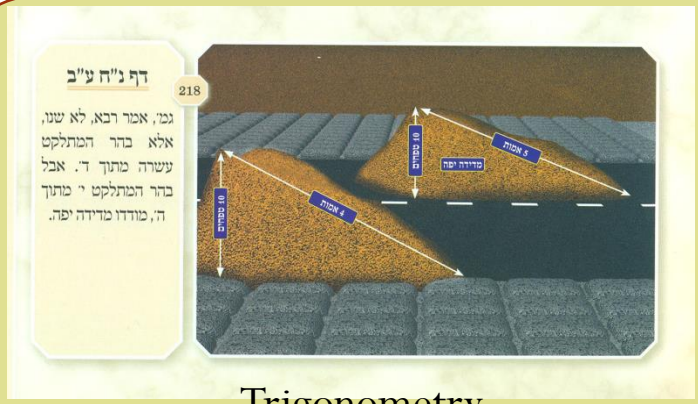
# Jewish Maths (Talmud)



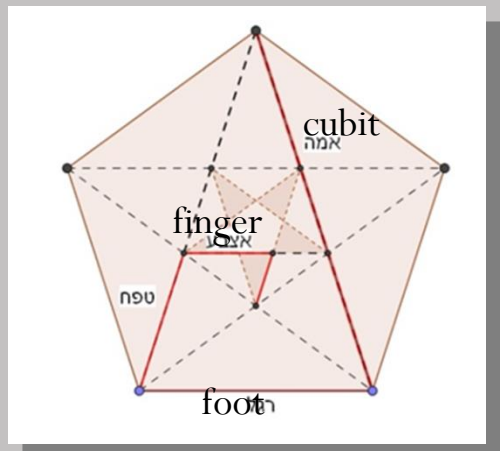
Computing the Jewish Calendar (lunar and solar)

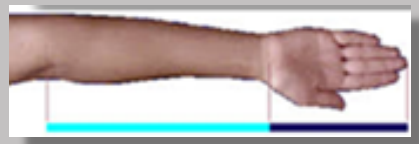
An approach looking as integral calculus






Trigonometry




**Biblical length units: Fibonacci numbers**

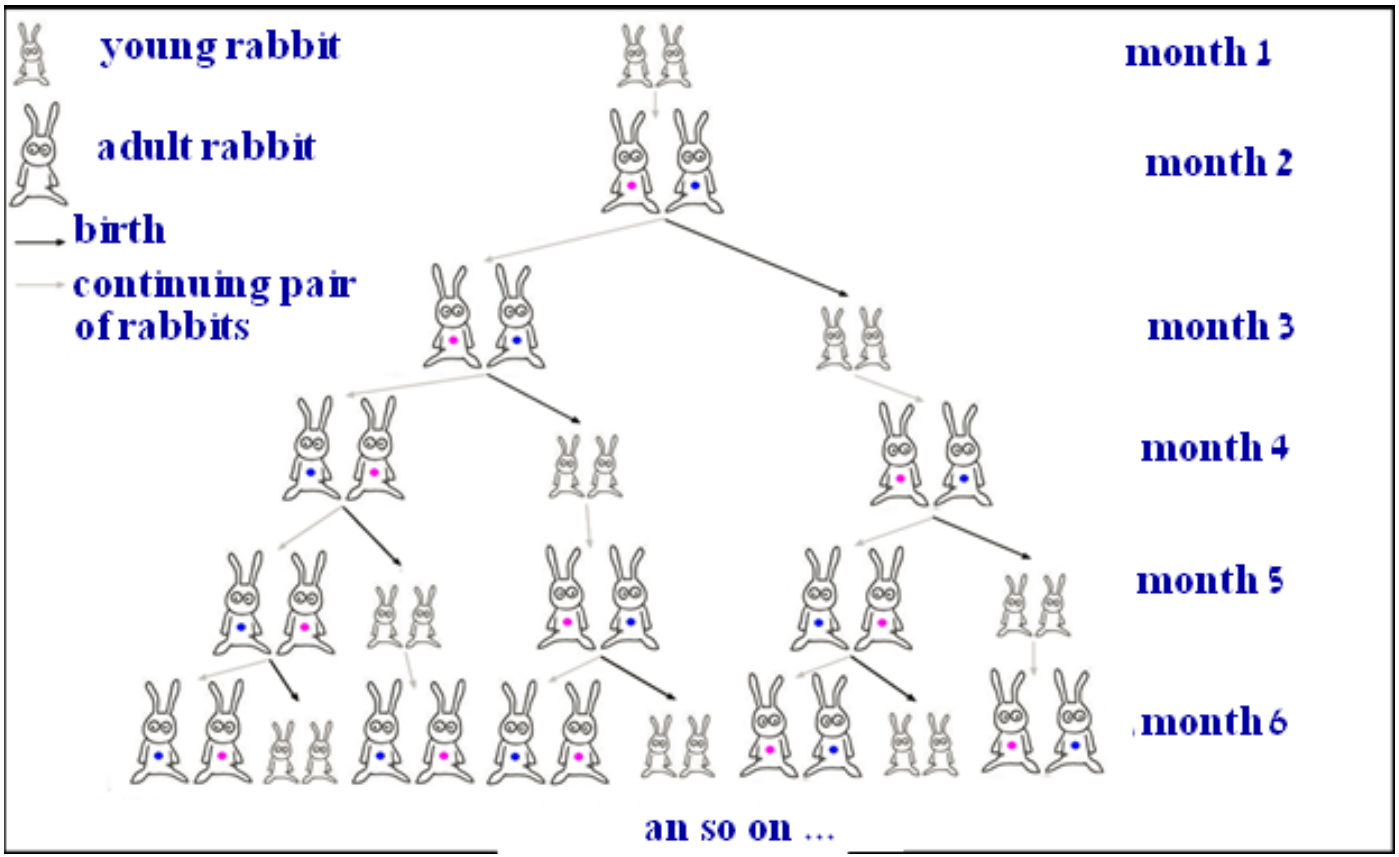




**Ref: DP (2014): JCRME2.**

# Fibonacci numbers

$$\begin{cases} F_1 = F_2 = 1 \\ F_{n+2} = F_{n+1} + F_n, n \in \mathrm{N} \end{cases}$$




1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, 233, 377, 610, 987, 1597, 2584,…

# Numerical values of letters in the Hebrew Alphabet (Gematria)

| | ה | ב | ה | א |
|---|---|---|---|---|
| 1 | | | | א |
| 2 | | ב | | |
| 3 | | ב | | א |
| 5 | | | ה | |
| 8 | | ב | ה | א |
| 13 | ה | ב | ה | א |

**Love = אהבה**

**Fibonacci numbers = numbers of love**





1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, 233, 377, 610, 987, 1597, 2584,…

# Numerical values of words

**One traditional way to interpret the Bible is based on numerical values of letters, whence of words and sentences.**

**The clue: if two expressions have the same numerical value, then they have some profound meaning in common**

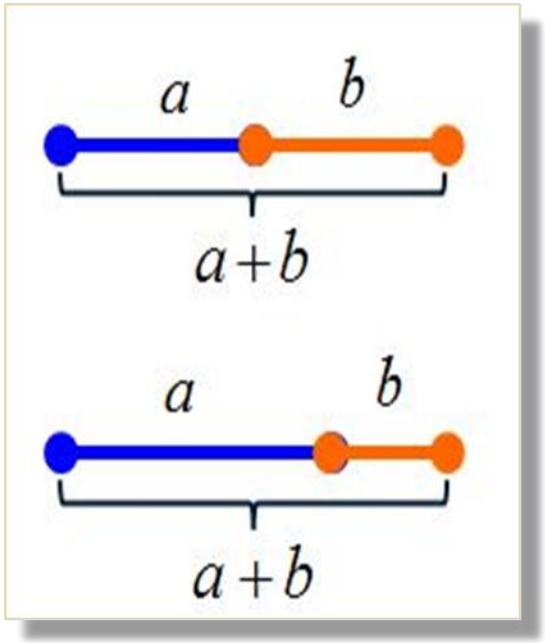
**One example (with Fibonacci numbers):**

- **Isaac = יצחק=208 = 8*26 =8*(2*13)**
- **Jacob = יעקב = 182 = 7*26**
- **Esau = הברכה אחת היא לך?**

**Don't you have one more blessing for me?**

# The Golden Section aka the Divine Proportion[1]

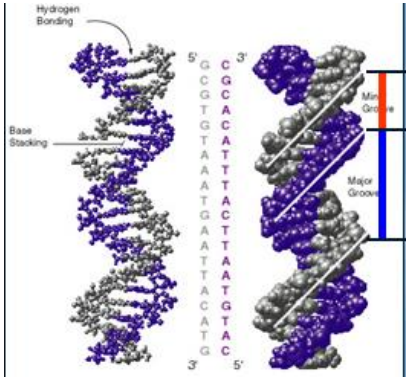
How to divide a quantity into two parts in a harmonious way?



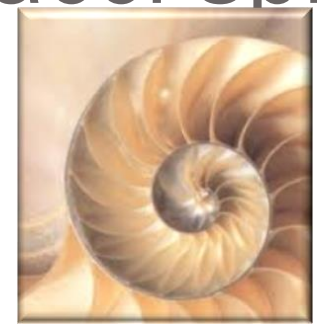

The Golden Section is realized when you take two numbers a et b such that the ratio of the sum a+b over the greatest one a is equal to the ratio of the greatest a over $\frac{a+b}{a} = \frac{a}{b}$

1 Luca Bartolomes Pacioli (Luca di Borgo): 1445 in Borgo Sansepolcro, Tuscany - 1517 in Rome.

# Fibonacci spiral in nature



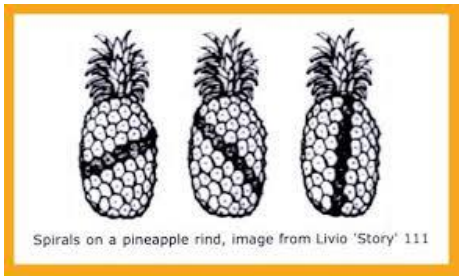

ADN

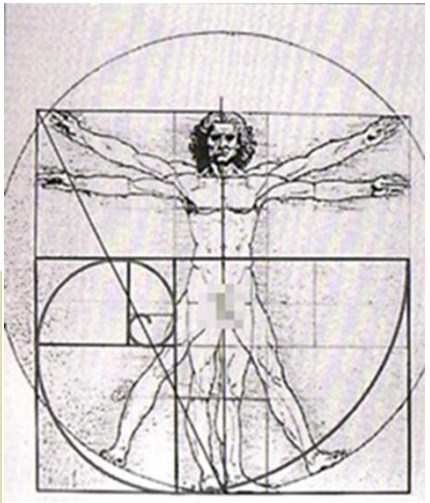


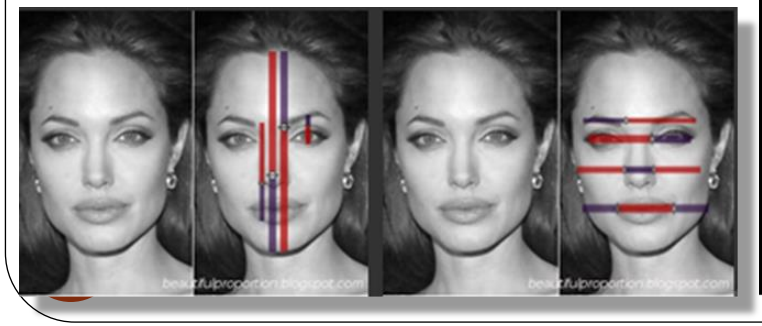
Nautile

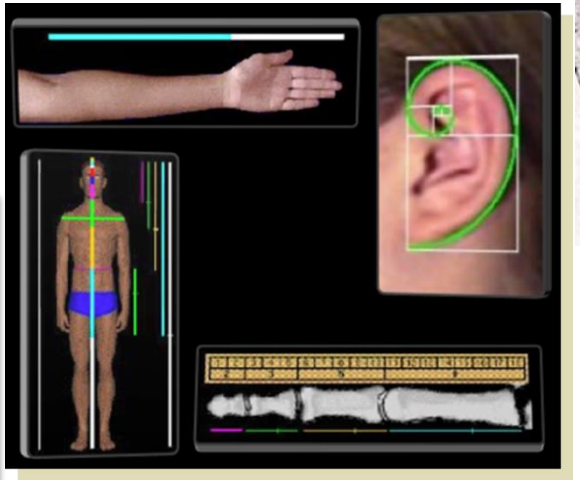
Sunflower

Pine

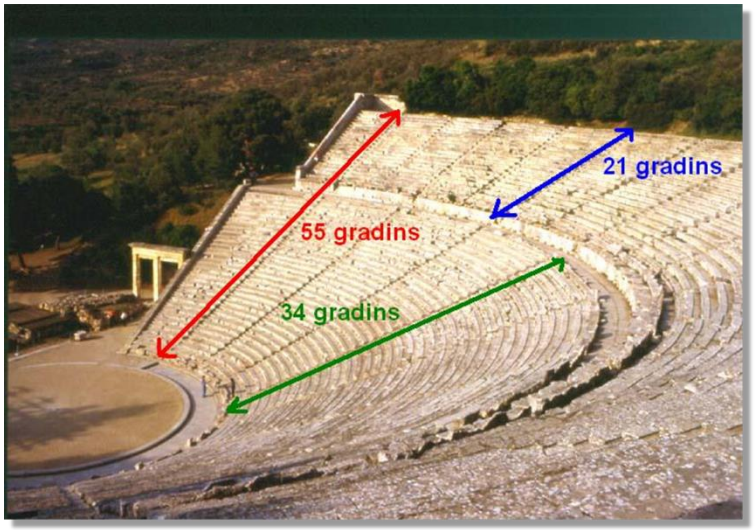


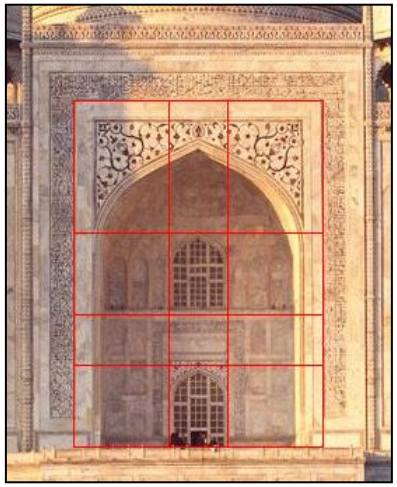


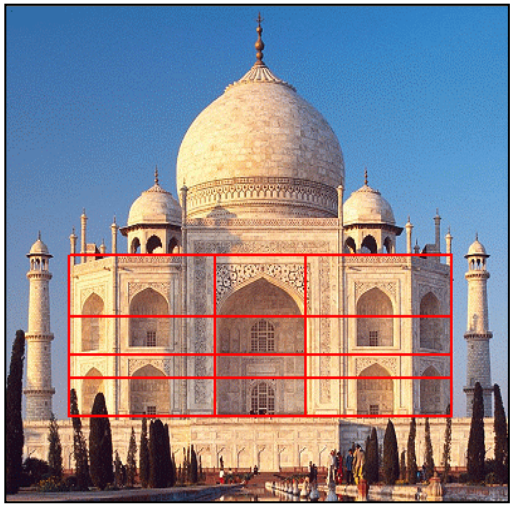


# Golden Section in Architecture

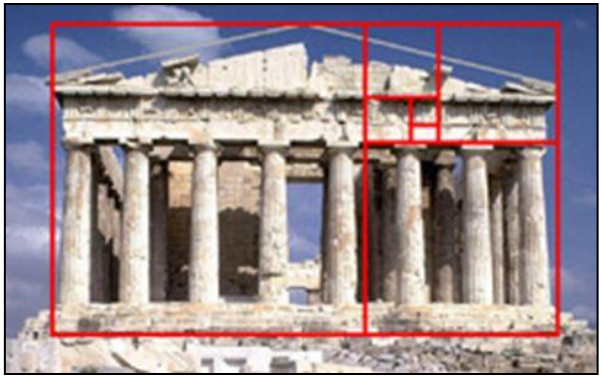



The theater in Epidauros

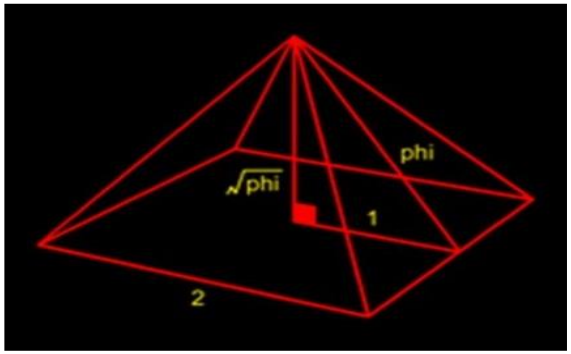




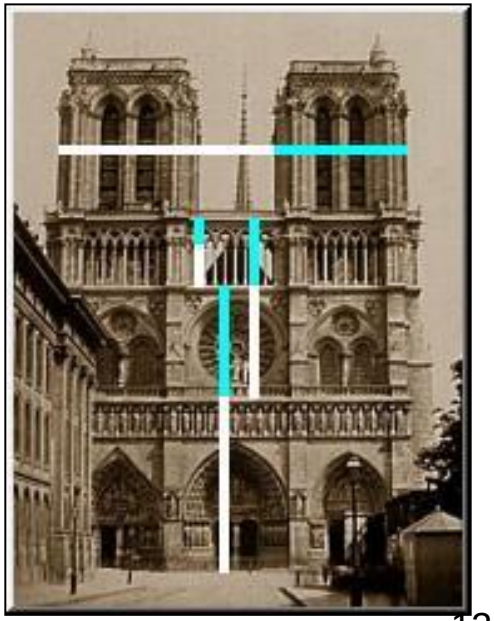
Taj Mahal



Parthenon




Cheops pyramid



# Counterexamples

## Power (Macht)

Versailles, France

Bucharest, Romania

## Anxiety vs aspiration to infinity

# And what in Jewish life?

- Biblical texts and Tradition
- Artefacts
- More modern buildings

Ref: DP and G. Morali, book to appear (in Hebrew), 2016.

# Noah's Ark – Genesis 6,15

וְזֶה אֲשֶׁר תַּעֲשֶׂה אֹתָהּ: שְׁלֹשׁ מֵאוֹת אַמָּה
אֹרֶךְ הַתֵּבָה חֲמִשִּׁים אַמָּה רָחְבָּהּ
וּשְׁלֹשִׁים אַמָּה קוֹמָתָהּ

And this is how thou shalt make it: the length of the ark three hundred cubits, the breadth of it fifty cubits, and the height of it thirty cubits.

$$\frac{50}{30} = \frac{5}{3} = \frac{2.5}{1.5} = 1.666$$

# The Arch of Covenant
# Exodus 25,10

$$\frac{2.5}{1.5} = \frac{5}{3} = 1\frac{2}{3} \approx 1.66$$

וְעָשׂוּ אֲרוֹן עֲצֵי שִׁטִּים
אַמָּתַיִם וָחֵצִי אָרְכּוֹ וְאַמָּה וָחֵצִי
רָחְבּוֹ וְאַמָּה וָחֵצִי קֹמָתוֹ.

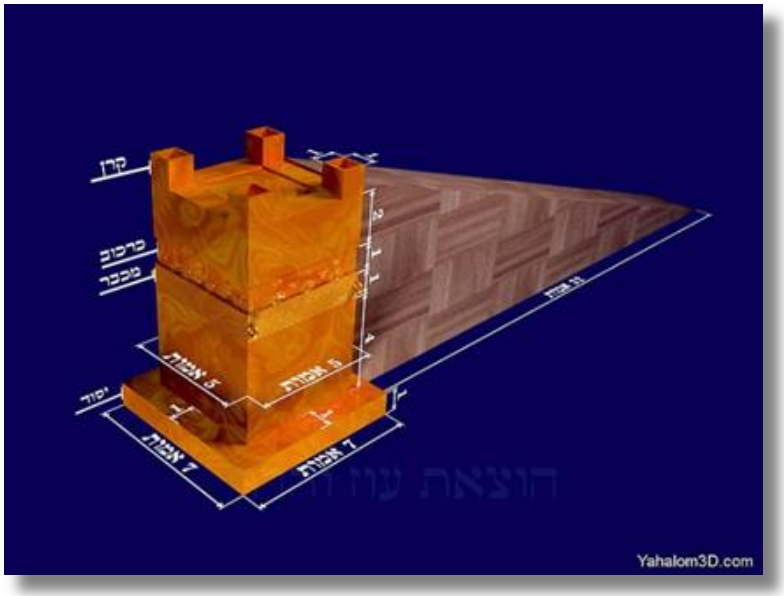
And they shall make an ark of acacia-wood: two cubits and a half shall be the length thereof, and a cubit and a half the breadth thereof, and a cubit and a half the height thereof.

# The exterior (copper) altar
# Exodus 27,1

וְעָשִׂיתָ אֶת מִזְבֵּחַ עֲצֵי שִׁטִּים חֲמֵשׁ אַמּוֹת אֹרֶךְ וְחָמֵשׁ אַמּוֹת רֹחַב
רָבוּעַ יִהְיֶה מִזְבֵּחַ וְשָׁלֹשׁ אַמּוֹת קֹמָתוֹ

And thou shalt make the altar of acacia-wood, five cubits long, and five cubits broad; the altar shall be four-square; and the height thereof shall be three cubits.

$$\frac{5}{3} = 1\frac{2}{3} \approx 1.66$$



# The fringes @ the 4 corners of a cloth – Numbers 15,38

**וְעָשׂוּ לָהֶם צִיצִת עַל כַּנְפֵי בִגְדֵיהֶם לְדֹרֹתָם וְנָתְנוּ עַל צִיצִת הַכָּנָף פְּתִיל תְּכֵלֶת**

… bid them that they make them throughout their generations **fringes** in the corners of their garments, and that they put with the fringe of each corner a **thread of blue**.

According to Maimonides:

- At least 7 groups of 3 knots

  7x3=21 = a Fibonacci number!

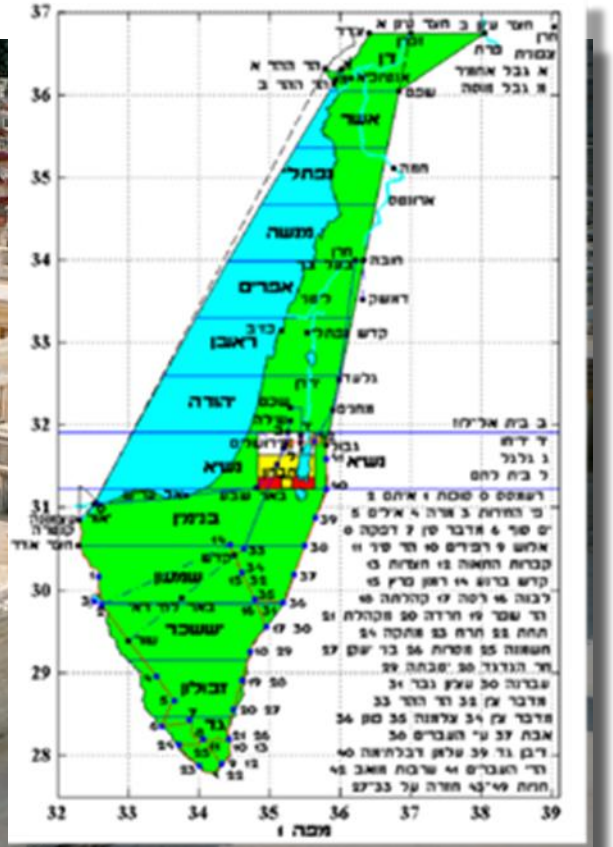
- At most 13 groups

  13 = a Fibonacci number!

# Architecture
# The Holy Temple of Jerusalem

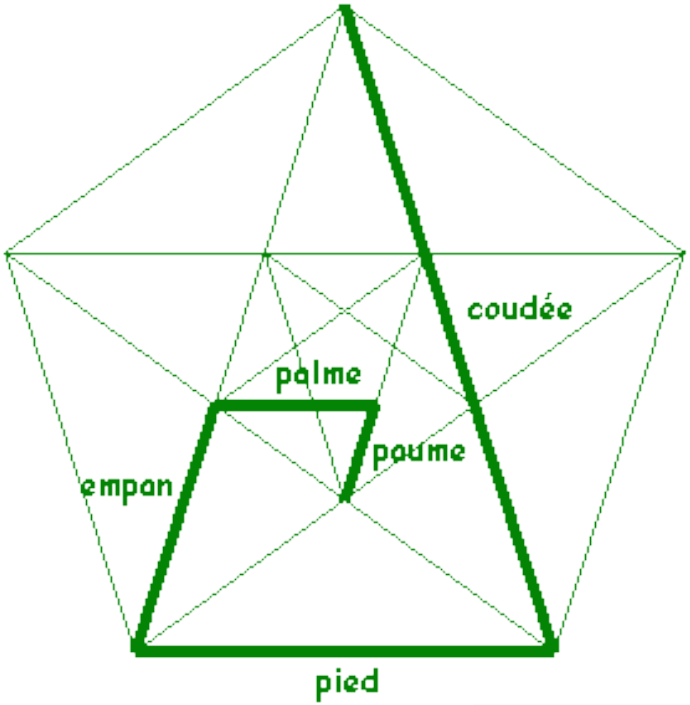
**Model of the Temple (Herod's period)**
**Jerusalem Museum**

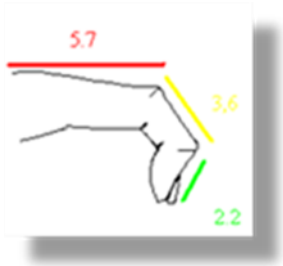
The Temple's dimensions and its surrounding area 's dimensions are related to Fibonacci numbers



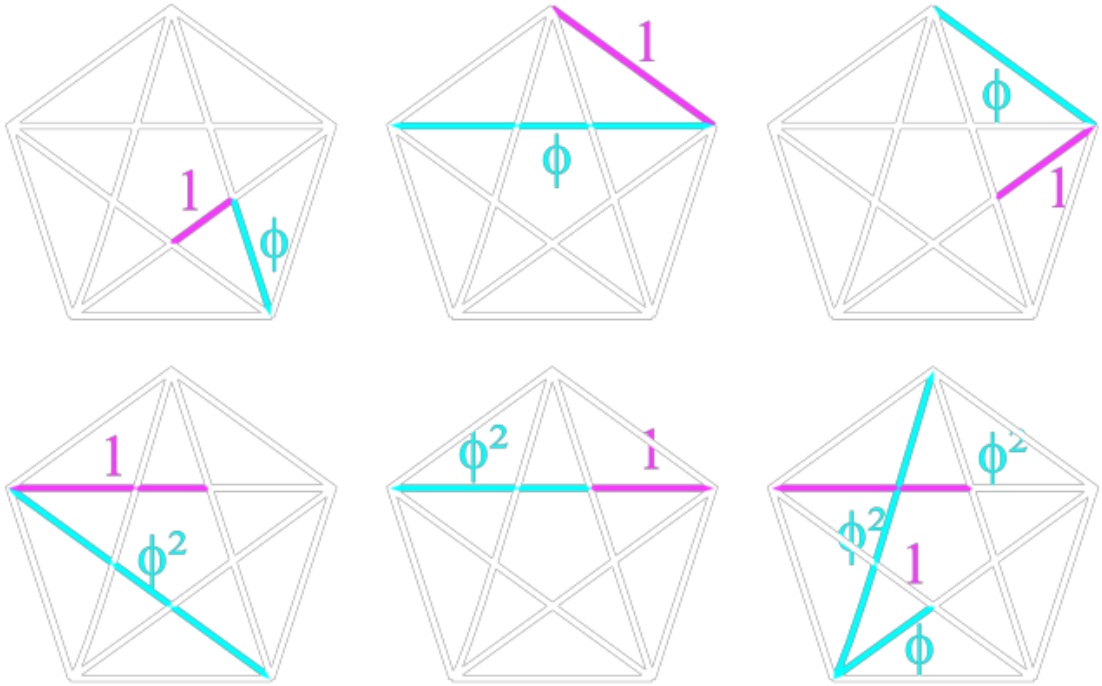

The ratio of the distances to the northern and the southern biblical boarder lines of the Land of Israel is an approximation of the Golden Ratio

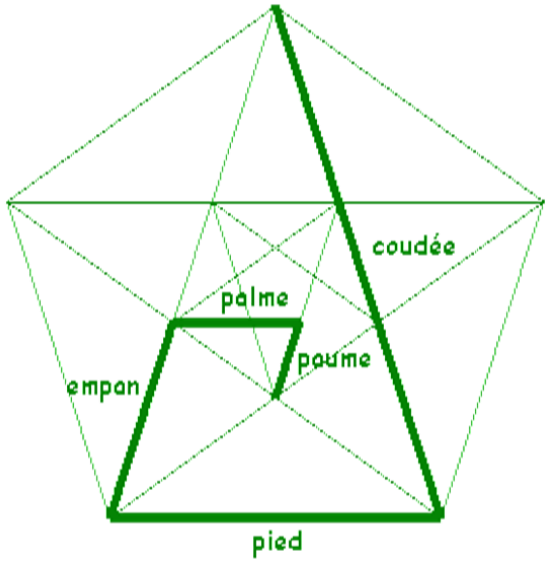
# The pentagram



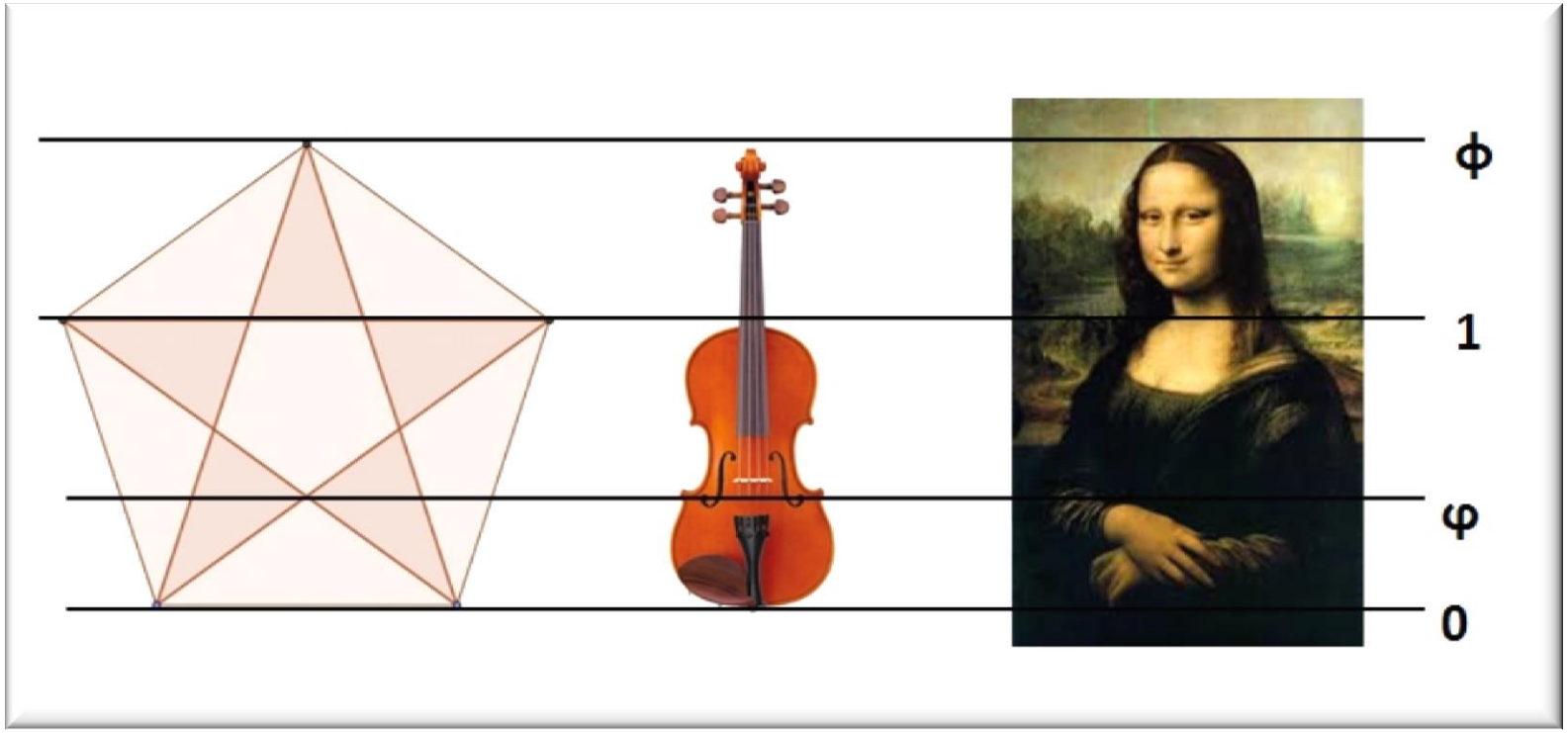

# The pentagram - biblical measures




**Cubit vs palm**

# A non official reason why Jews play so often the violin



# Symmetry and grace

**ונח מצא חן בעיני ה׳**

Noah was graceful in G.d's eyes

# Budapest Great Synagogue

**Symmetries**

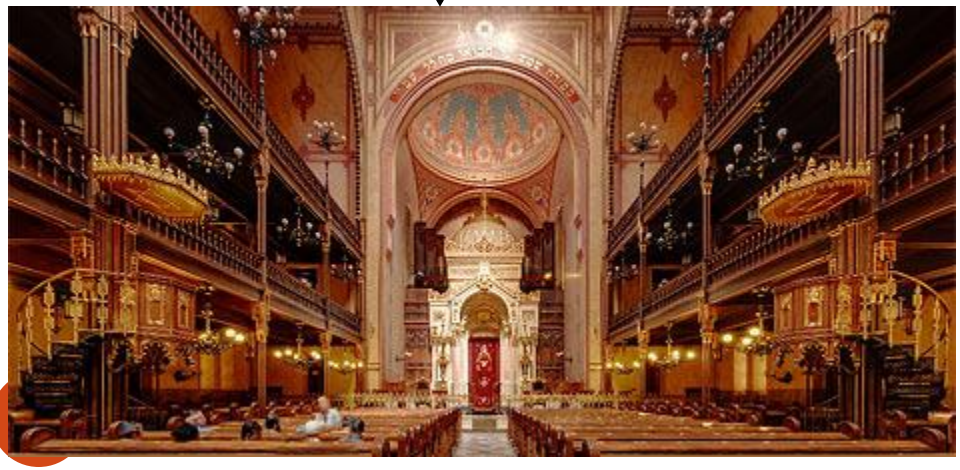
**Fibonacci numbers**

**Tesselations**

↑

**Rundbogenstil (Round-arch style)**

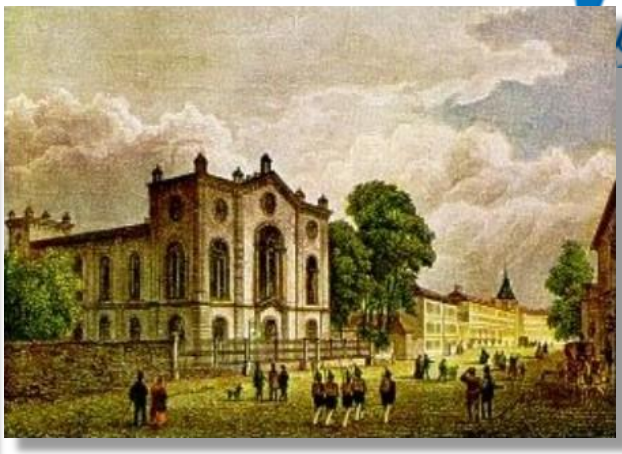
↓



# Hard vs soft symmetry

Bratislava

Liptovský Mikuláš
(Slovakia)

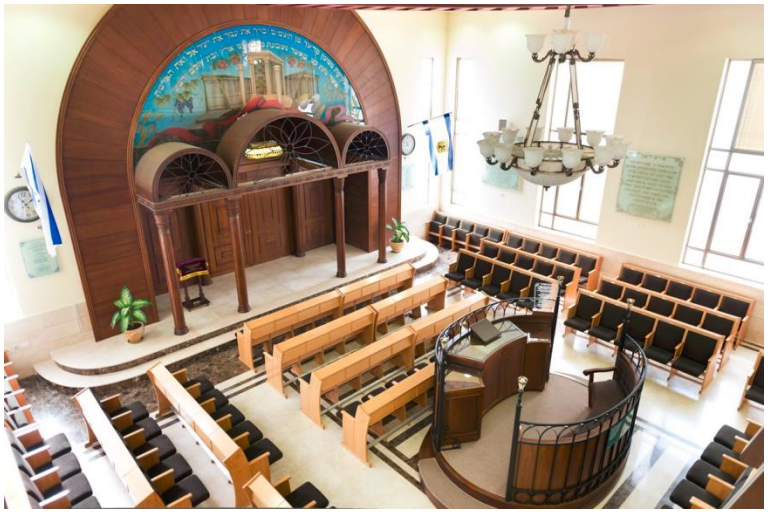


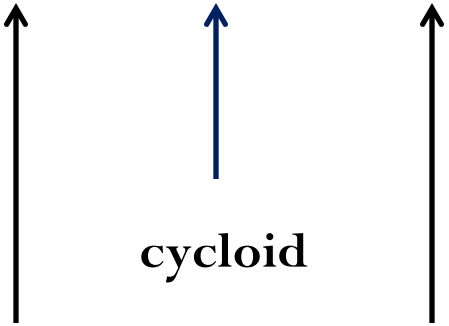
1839-1938
Kassel (Germany)
today

**Trenčín (Slovakia)**

# Softening symmetries

Mishkan Its'hak synagogue in Jerusalem






Bet Its'hak synagogue in Jerusalem

# The same motives in a more ancient synagogue

Synagogue «Yohanan ben Zakkai »
In the Old City of Jerusalem

The vaults‘ shapes are not circles

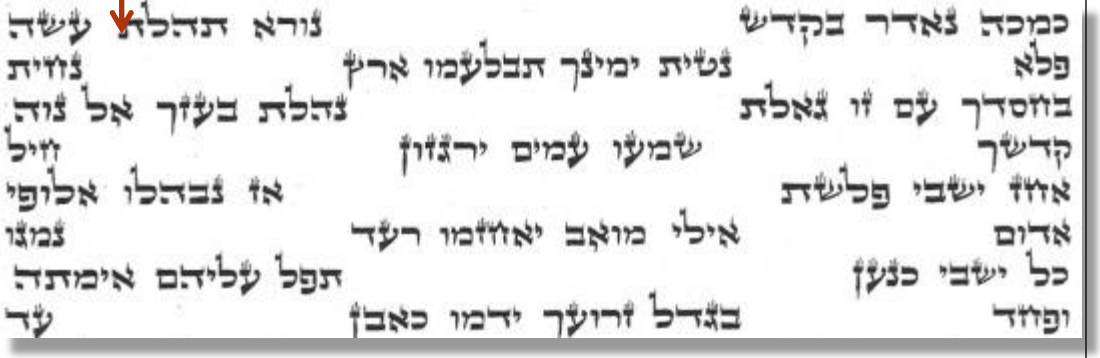
Numbers of windows

**Note the symmetries!**

# Special presentation of biblical text

Text with a very positive atmosphere (joy, happyness, gratitude after Exodus): the scripture expresses stability like the building of a stone wall

**Exodus 15,1**



Text with anxiety and lack of hope: the scripture expresses unstability

**Deuteronomy 32,1**

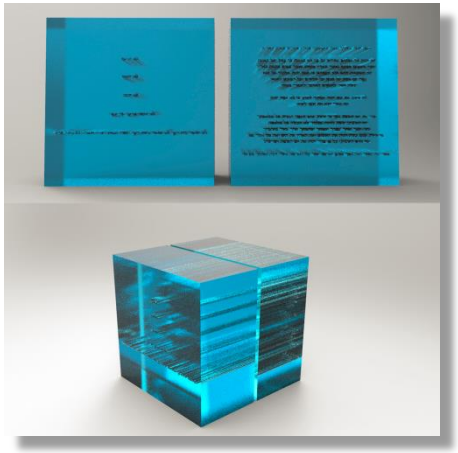
**Note the symmetry in numbers of words**

Symmetry exists but not « absolute »

# Symmetry in Tables of the Cov


Synagogue of Lausanne

Prague: the
Spanish Synagogue



The issue of the tables' symmetry has been addessed in details by **Rabbi Moshe ben Yossef di Trani** (1500-1580), one of the most important Talmudists from his time until today, in Safed (Galilee).

# Relevance to Math Education

In Israel:

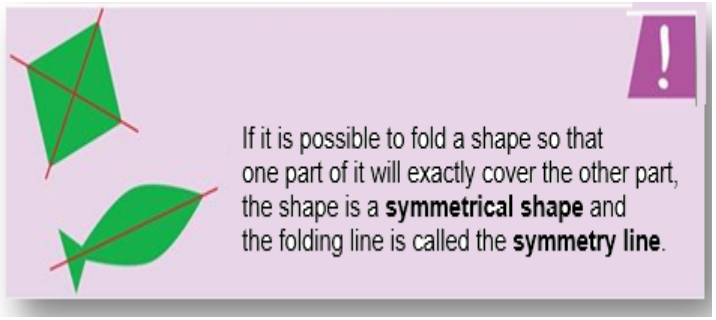
- Pupils learn reflection in 1st grade, translation in 2nd grade and rotation in 3rd grade.
- They are requested not only to learn the theory, but to look for examples in their surroundings

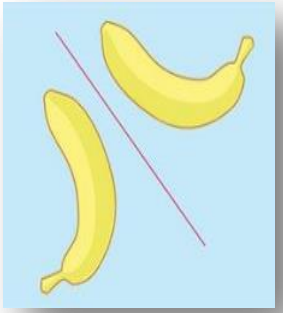
| grade | | To be learned: | By means of…… |
|---|---|---|---|
| 1 | Reflection | Copying a shape with respect to a line (the reflection line) so that every point on the shape is copied to its "mirror image" with respect to the line.<br>**Properties:**<br>• Distances are preserved (a point and its image are at equal distances from the reflection line).<br>• The reflection changes orientation.<br>• Points on the line are stationary points. | Mirrors<br>Cutting out<br>Folding and perforating |
| 2 | Translation | Each point in the shape is moved in the same direction and the same distance.<br>**Properties:**<br>• Distances are preserved (the distance between two points on the shape is the same after the translation).<br>• Each shape is translated to a congruent shape.<br>• The translation preserves orientation.<br>• No stationary points. | Drawing using movable stencils on square grids |

| grade | | To be learned: | By means of…… |
|---|---|---|---|
| 3 | Rotation | Rotating a shape around a point (the rotation point) through an angle (the rotation angle) so that every point on the shape is rotated round the same point through the same angle.<br>**Properties:**<br>• Distances are preserved (the distance between two points on the shape is the same after the rotation).<br>• The rotation preserves orientation.<br>• There is one stationary point (the rotation point). | Drawing using stencils |

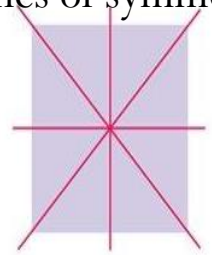
| grade | | To be learned: | By means of…… |
|---|---|---|---|
| 4 | Symmetry | A shape has reflectional symmetry if it is possible to copy the shape onto itself by means of reflection.<br>A shape has rotational symmetry if it is possible to copy the shape onto itself by means of rotation. | |
| 5 | Tessellation | Tessellation by polygons.<br>**Properties:**<br>• No spaces between the polygons.<br>• The polygons are joined to each other along the entire length of a whole side (each vertex coincides with another vertex).<br>• The is a repeating pattern in the tessellation, which can be continued in every direction. | |

# Reflection and symmetry

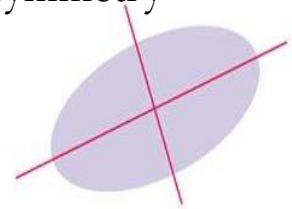


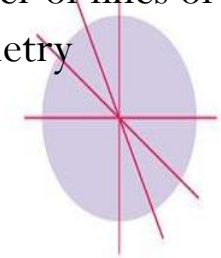
The square has 4 lines of symmetry

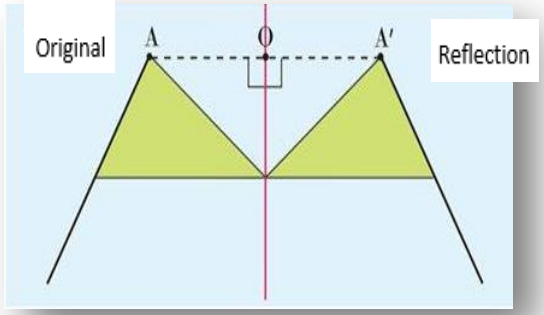


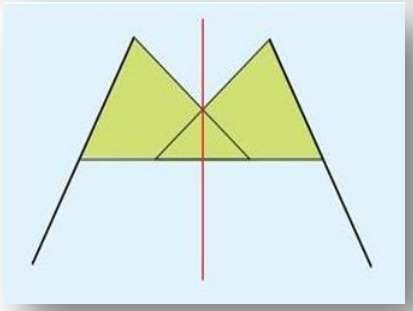
The ellipse has 2 lines of symmetry

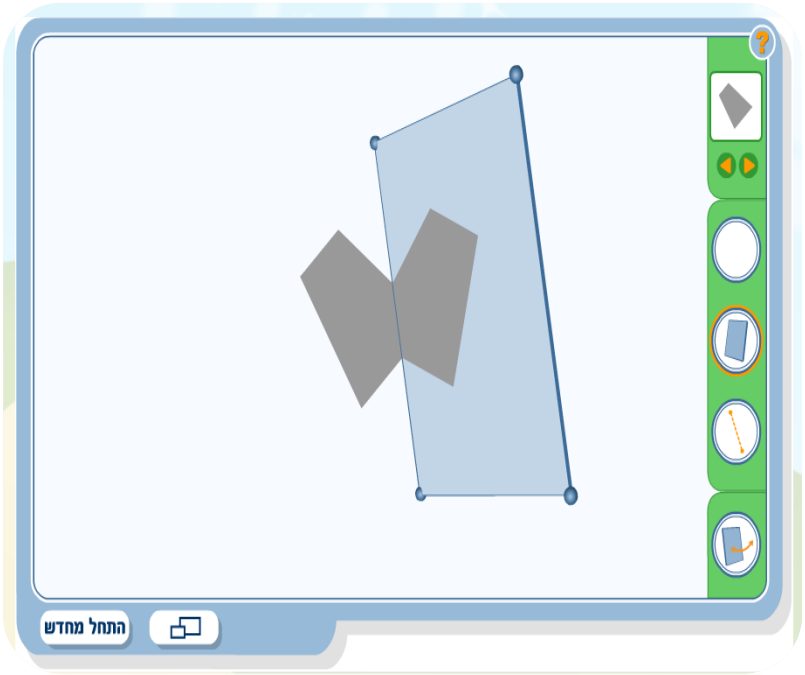


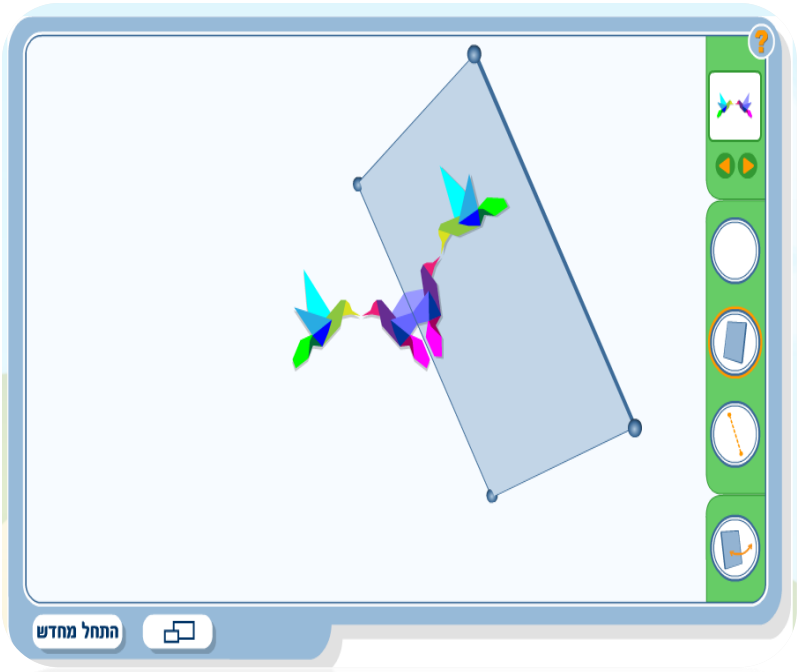
The circle has an infinite number of lines of symmetry



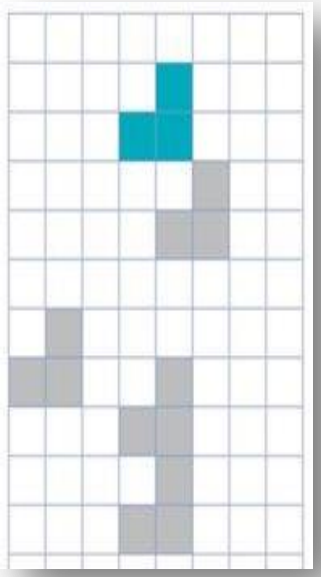


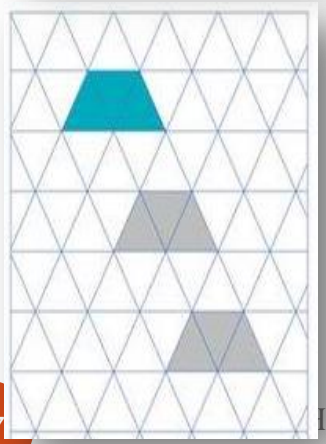



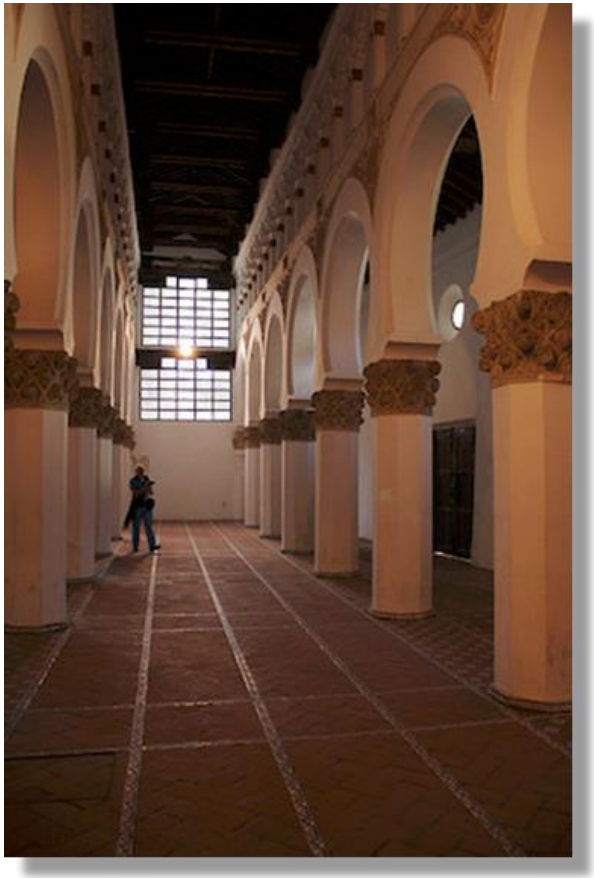

התחל מחדש

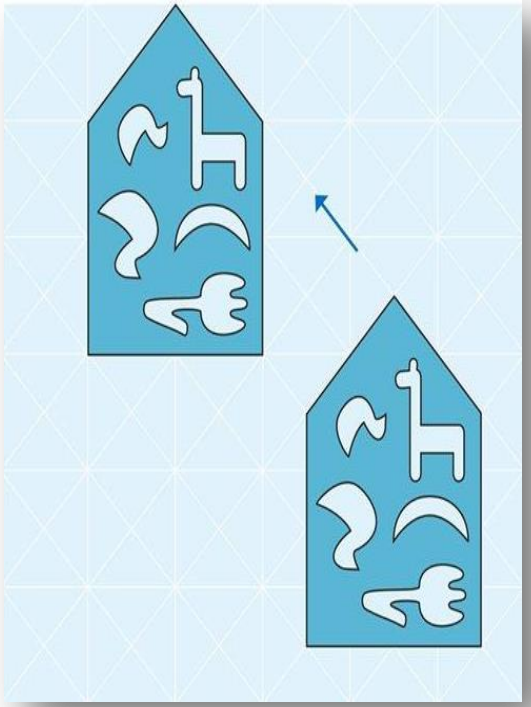

התחל מחדש

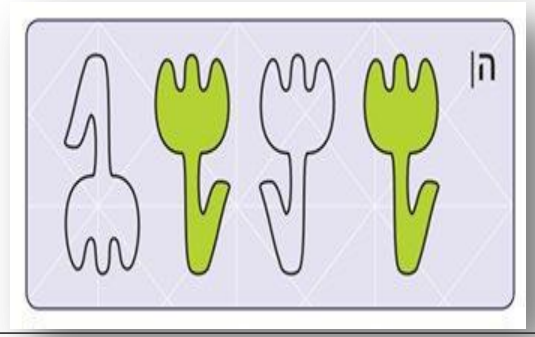
# Translation





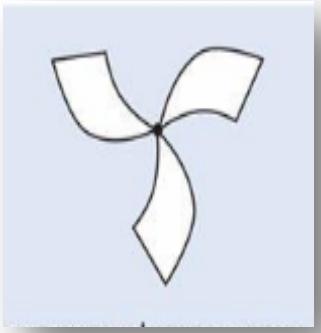


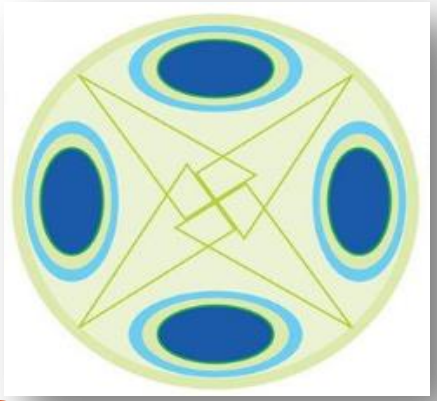


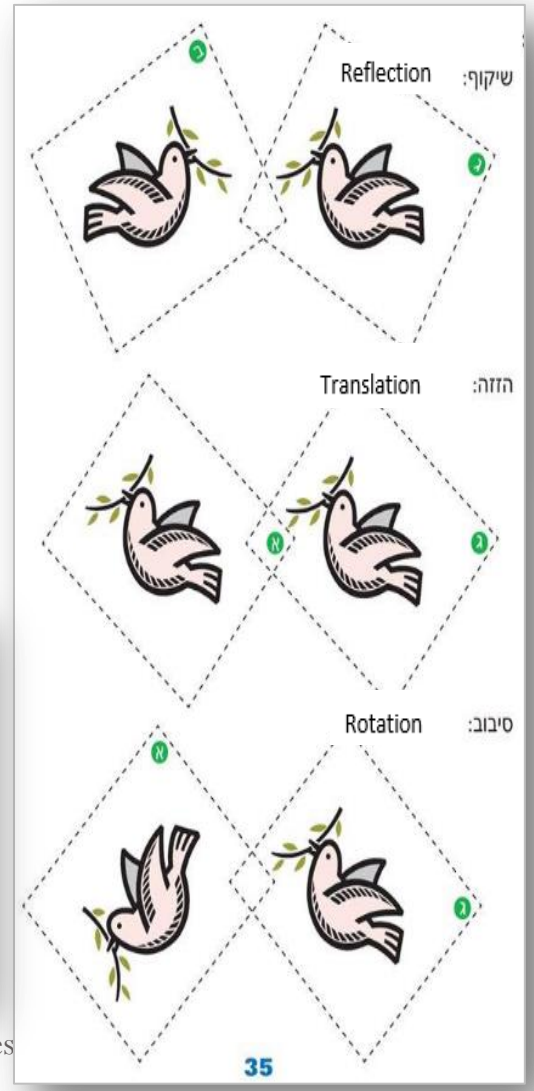


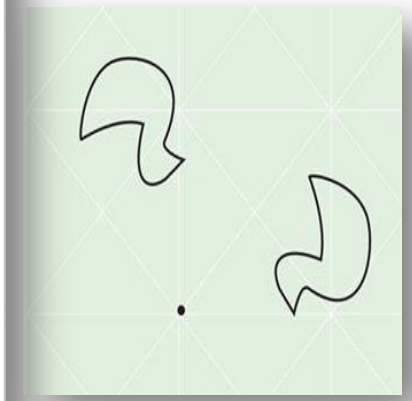
# Reflectional / rotational symmetry

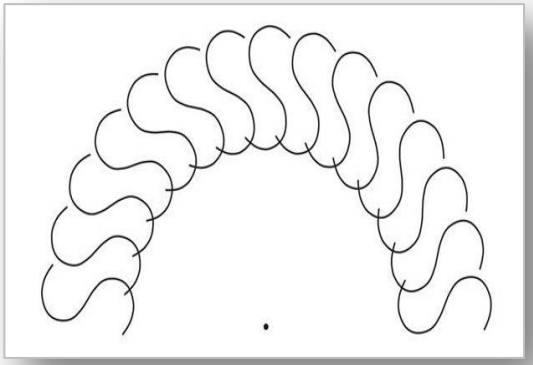
**Rotational symmetry**



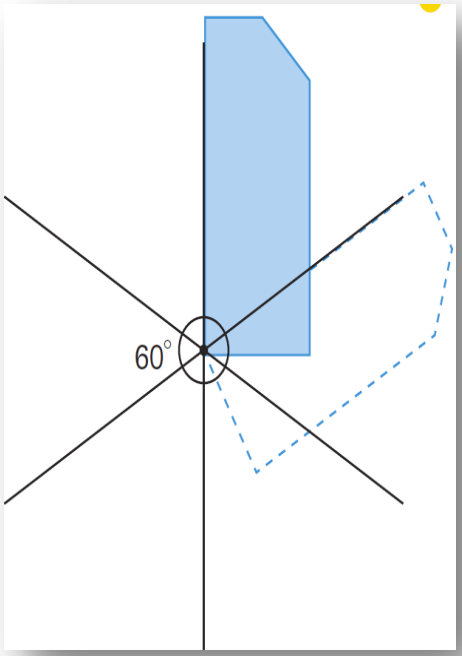


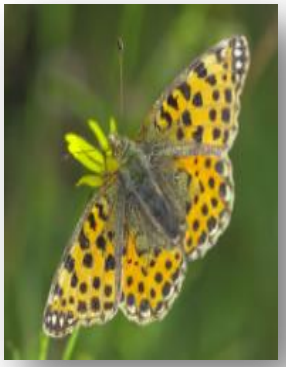


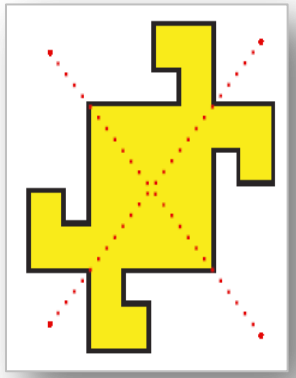





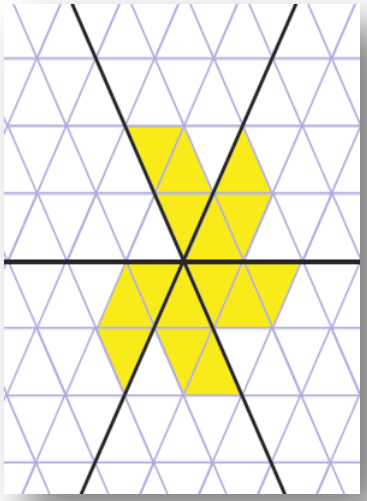
# Reflectional / rotational symmetry

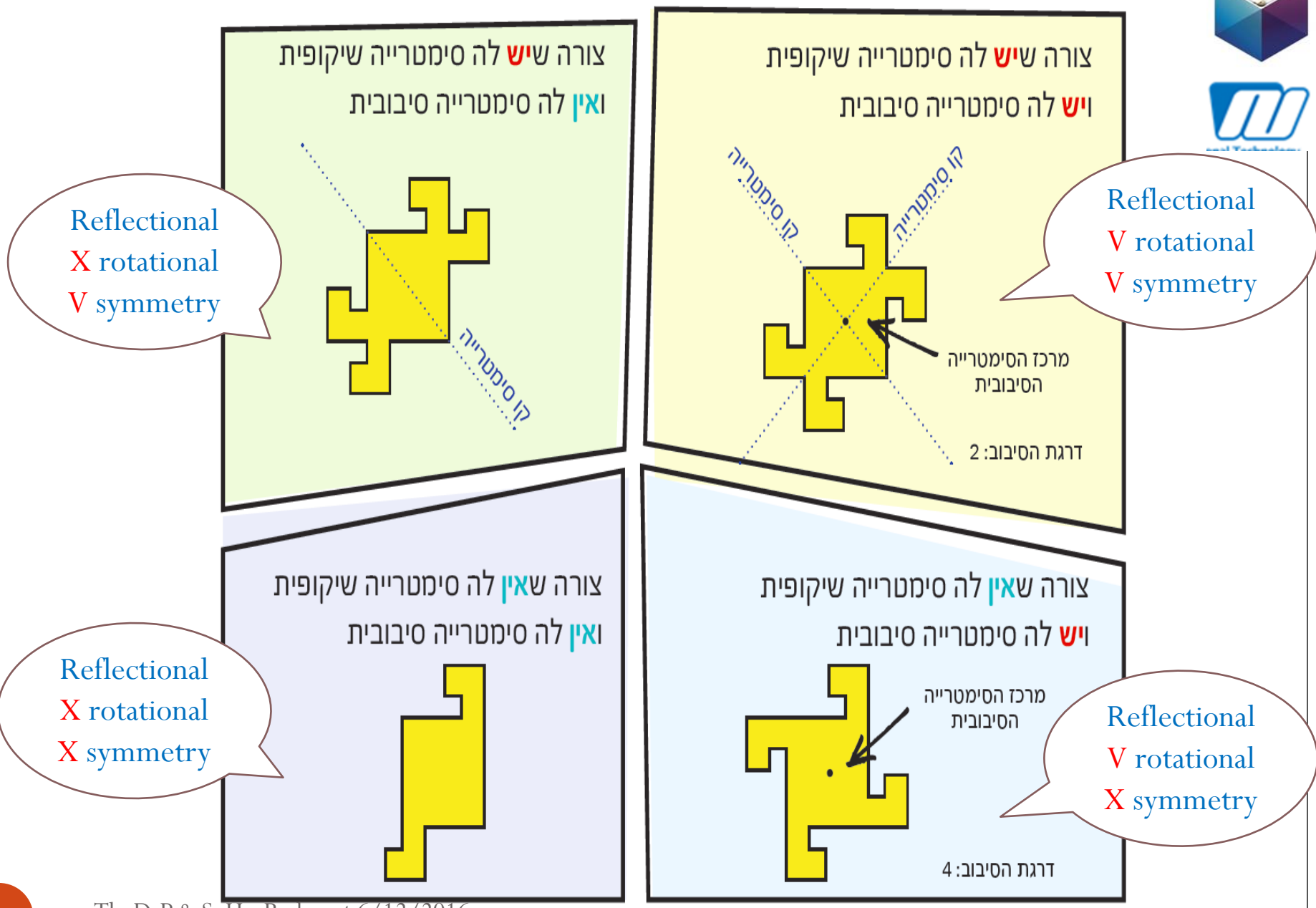


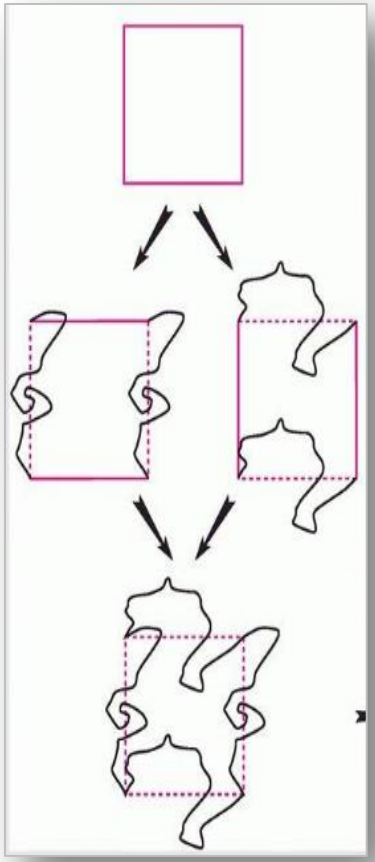


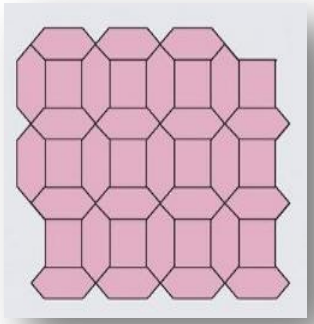
Synagogue in Pilsen

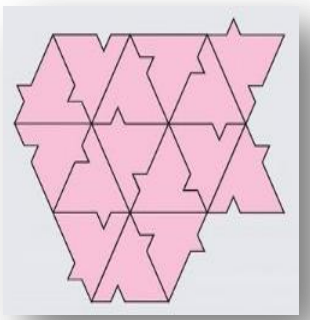


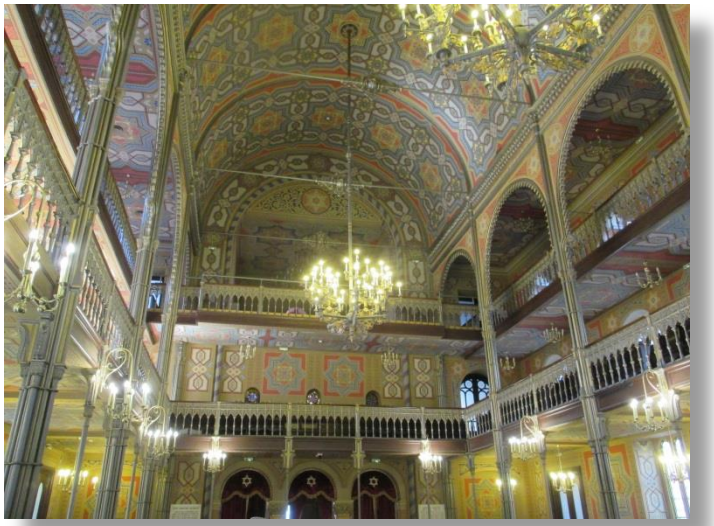

צורה שיש לה סימטרייה שיקופית
ואין לה סימטרייה סיבובית
קו סימטרייה
קו סימטרייה
Reflectional
X rotational
V symmetry
צורה שיש לה סימטרייה שיקופית
ויש לה סימטרייה סיבובית
קו סימטרייה
קו סימטרייה
מרכז הסימטרייה הסיבובית
דרגת הסיבוב: 2
Reflectional
V rotational
V symmetry
צורה שאין לה סימטרייה שיקופית
ואין לה סימטרייה סיבובית
Reflectional
X rotational
X symmetry
צורה שאין לה סימטרייה שיקופית
ויש לה סימטרייה סיבובית
מרכז הסימטרייה הסיבובית
דרגת הסיבוב: 4
Reflectional
V rotational
X symmetry

# Tessellation









**Coral Synagogue, Bucharest, Romania**

# Relevance to Math Education

## The example of the orthodox population

- A large fringe of the so-called orthodox population does not learn maths
- A move has been made during the last years:
  - Private initiative of certain institutions/associations
  - Governmental decisions afterwards
- In mathematics, students are requested, not only to learn the theory, but also to look for concrete example in their natural surroundings.
- Regarding orthodox children, this includes home, the synagogue where they go to pray together with their dad, etc.

# Relevance to Math Educ advantages

- Maths do not contradict  the orthodox way of life
- Maths do not contradict the orthodox way of thinking
- Maths can be conveyed using objects of students' everyday life as examples
- People who have learnt Talmud for years are accustomed to logical reasoning

# Relevance to Math Educ advantages

- The Talmud itself is full of mathematical content:
- Geometry
  - Number Theory
  - The bases of the infinitesimal reasoning
  - Taxicab geometry
  - Logic
  - Combinatorics
  - Statistics
- Last but not least: orthodox and non-orthodox students can learn together

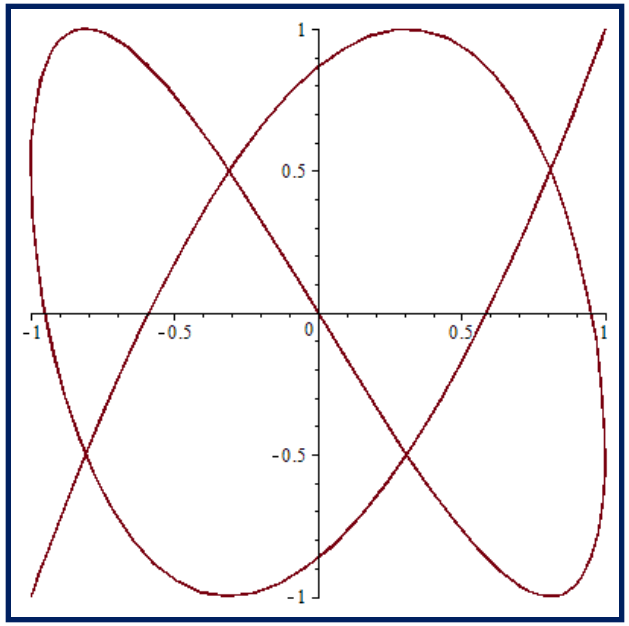
# Mixing symmetries

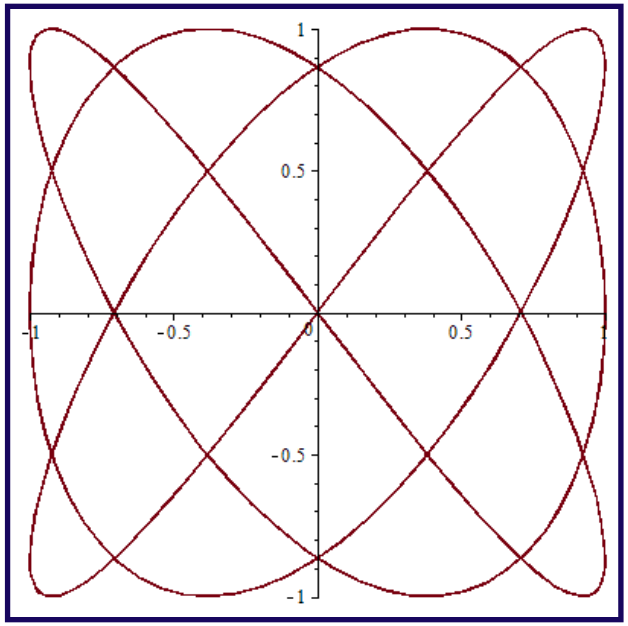
**Synagogue in Tirgu Mures/ Marosvásárhely/Neumarkt**


**Dohany Synagogue, Budapest**

# Mixing symmetries

- Examples in Physics/Electronics: Lissajous curves




$$\begin{cases} x(t) = \cos 3t \\ y(t) = \cos 5t \end{cases}, 0 \le t \le 2\pi$$

$$\begin{cases} x(t) = \cos 6t \\ y(t) = \sin 8t \end{cases}, 0 \le t \le 2\pi$$

# Mixing symmetries

**Ceiling of Rumbach synagogue Budapest**

# The Tree of Life: monument in the Raoul Wallenberg memorial garden, Budapest


**The tree is not symmetric: life implies a rupture of symmetry**

**Chirality at the macro scale**


# THANK YOU FOR YOUR ATTENTION