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On the Effect of Granularity on Dynamic Range and Information Content of Photographic Recordings*

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The presence of graininess in a photographic emulsion provides a threshold below which a signal will not be detectable. Graininess thus acts as a limiting factor on dynamic range.

Granularity data of photographic emulsions are used to determine the minimum usable spot size when the dynamic range or the number of discernible grey steps of the recording is specified, or the resulting dynamic range, if the resolution element size is specified.

A brief analysis is made of the information content as affected by the element size, i.e., scanning spot size.

INTRODUCTION

RESOLVING power is generally defined by the limit of detectability of signal, such as the minimum separation between two lines in the object that enables an observer to identify them as two lines in the image, with no regard to deterioration of the contrast rendition. When such contrast rendition performance is specified, however, this will serve in the definition of resolving power.

The required contrast rendition, in turn, may be prescribed by psychological requirements, such as ease of recognition. Alternatively, it may be prescribed in more objective terms such as information content. We are here concerned with this latter type of specification.

Because photographic materials exhibit certain random fluctuations of density, usually referred to as "graininess," the exposure at any one point cannot be determined accurately from a density measurement. The magnitude of these random fluctuations is a measure of the probable error in an exposure determination and will, therefore, be a factor in specifying the required contrast rendition.

Conversely, if we desire a certain contrast performance in terms of discernible shades of density or in terms of dynamic range, the permissable fluctuations in density will be thereby determined.

It is the purpose of this paper to investigate how such specifications will affect the resolving power—or, better, the minimum usable element size—of grainy photographic material.

This type of investigation may be of practical importance in a recording instrument which uses a photographic process. In such an instrument the opacity of the recording would be used as a measure of the amplitude of the signal which produced it. The signal amplitude can thus be determined with no greater accuracy than the opacity.

If we now specify the accuracy to which opacity should be determinable, we limit ourselves as to the minimum element size, or in other words the tolerance on the opacity measurement determines the minimum element size. If the total number of elements across the recording is specified, the element size will, of course, determine the over-all dimensions of the recording.

For a general treatment of the information capability of grainy photographic materials, the reader is referred to a paper by Fellgett.[1]

A thorough investigation on liminal contrast was made by H. R. Blackwell.[2]

RESOLVING POWER

Once the permissible density fluctuation has been determined, the graininess of the photographic material will limit the resolution capability of material. This limitation will become apparent from the following considerations.

If we scan a uniformly exposed piece of photographic film after processing, we expect to measure a uniform density. If the emulsion is grainy, though, and the scanning aperture is not much larger than the grains, we shall measure large fluctuations of density, just as we should observe speckling with considerable variations in the density upon enlarging the exposed film. As this enlargement is increased, these visible fluctuations will become more and more pronounced. In the same fashion, the measured density fluctuations will become more and more pronounced as the scanning aperture is decreased. Thus the permissible density fluctuation will determine the minimum scanning aperture and consequently the resolution capability of the photographic material.

To arrive at a quantitative measure of resolution, a quantitative measure of the graininess of the photographic material is required. One such measure, granularity (G), was devised by E. W. H. Selwyn.[3] It states that the probability (dp) of measuring a density, between D and $D+dD$ is given by

$$dp=\left(\frac{a}{\pi G^2}\right)^{\frac{1}{2}} \exp[-(D-D_0)^2 a/G^2]dD, \qquad (1)$$

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[1] P. Fellgett, J. Opt. Soc. Am. **43**, 271 (1953).
[2] H. R. Blackwell, J. Opt. Soc. Am. **36**, 624 (1946).
[3] E. W. H. Selwyn, Photo. J. **75**, 571 (1935).

where a is the area of the scanning spot and D_0 is the average density.

We shall simplify this expression somewhat by substituting $w=\sqrt{a}$ (if a square area would be considered, w would be the side and is, therefore, proportional to the reciprocal of the resolving power), $w/G=k$, a constant determined by the allowable density fluctuation ΔD, and $\delta=D-D_0$, the density deviation due to graininess. On performing these substitutions we obtain

$$dp=\frac{k}{\pi^{\frac{1}{2}}}\exp(-\delta^2k^2)d\delta.$$

To obtain the probability for a density between $D_0+\Delta D$ and $D_0-\Delta D$ we must integrate this expression

$$p_{\Delta D}=\int_{-\Delta D}^{\Delta D}\frac{k}{\pi^{\frac{1}{2}}}\exp(-\delta^2k^2)d\delta.$$

To put this expression into the standard error function form, we substitute

$$\sqrt{2}\delta k=t,\quad d\delta=dt/\sqrt{2}k,$$

and also take twice the integral with zero as the lower limit. This yields

$$p_{\Delta D}=\left(\frac{2}{\pi}\right)^{\frac{1}{2}}\int_0^{\sqrt{2}k\Delta D}\exp(-t^2/2)dt. \tag{2}$$

This probability is 0.955 when the upper limit is 2 and we find, therefore, that, if

$$\sqrt{2}/\Delta D=k=w/G, \tag{3}$$

only 4.5% of random density fluctuations will exceed the required limit ΔD.

The resolution (r) in lines per unit length, will be given by

$$r=1/2w=\Delta D/2\sqrt{2}G. \tag{4}$$

The factor two enters to permit the scanning aperture to leave the element completely before covering the gap between elements. (In the photographic field resolution is usually given in terms of line pairs/mm.)

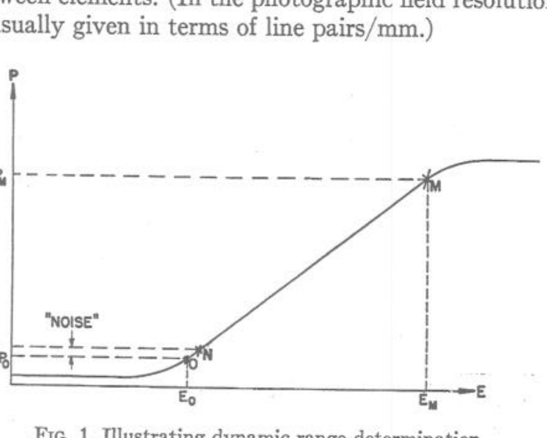

FIG. 1. Illustrating dynamic range determination with unidirectional recording.

It should be noted again that resolution, as used in this analysis, is defined much more severely than is customary.

The probability of detecting a signal corresponding to a density increase D will be given by Eq. (2) provided that the scanning aperture is identical to the element size and is w^2 in area.

DYNAMIC RANGE

Let us now turn to the problem of determining the allowable density fluctuations. We shall treat first the case where the dynamic range of the system is specified.

The dynamic range of a system is defined as the ratio of the maximum signal (before saturation sets in) to the minimum signal (detectable above noise).

The maximum signal may be readily determined from the system transfer characteristic (output *vs* input) and the linearity requirements. The determination of the minimum detectable signal is somewhat more difficult since it involves the uncertain noise characteristics and must therefore be handled on a probability basis. In addition to this, psychological factors play an important role in deciding whether a signal is detected. For the present analysis it will be assumed that a signal equal in amplitude to the root-mean-square noise level is detectable, with the noise amplitude assumed to have a Gaussian distribution. (There is a 4.5% probability of noise exceeding twice the rms value).

The "noise" in the final recording can thus be seen to be an important factor in determining the dynamic range of the system. The following analysis shows how the noise contribution of the recording medium, say photographic film, may be specified in terms of the dynamic range requirement. We shall first assume that all other factors make a negligible contribution to the noise and show later how other noise sources may be accounted for.

Consider a plot of opacity *vs* exposure of a representative photographic material, Fig. 1. Opacity (P) is the ratio of incident illumination to transmitted illumination ($P\geq 1$); and exposure is measured in energy per unit area (e.g., meter-candle-seconds). Points O and M define the beginning and end, respectively, of the segment linear within specifications. N represents the point due to rms noise. Thus E_0 is the exposure required to bring the material to the beginning of the linear portion, where it will exhibit an opacity P_0. An exposure E_m will carry it to saturation with an opacity P_m.

If operation is limited to the region OM, with point O as the quiescent or "no-signal" point, the maximum input signal (S_m) will be proportional to E_m-E_0; this, in turn, will be proportional to P_m-P_0, so that we may write $S_m=k(P_m-P_0)$. If the probable (rms) value of the random noise fluctuations are proportional to P_n-P_0, then—in accordance with our earlier assumption concerning detectability—the minimum detectable signal (S_n) will be proportional to P_n-P_0 also, i.e.,

$S_n=k(P_n-P_0)$. Since the factor of proportionality (k) is the same in both cases, we find that the dynamic range (R) will be given

$$R=\frac{S_m}{S_n}=\left(\frac{P_m-P_0}{P_n-P_0}\right). \tag{5}$$

From this relationship we find that the allowable density fluctuation ΔD is given by

$$\Delta D=\log_{10}r_n=\log_{10}\left(\frac{r_m-1}{R}+1\right) \tag{6}$$

where

$$r_n=P_n/P_0 \quad \text{and} \quad r_m=P_m/P_0.$$

If $r_m\gg 1$ and $r_m/R\ll 1$, we may write

$$\Delta D\doteq 0.43\left(\frac{r_m}{R}\right). \tag{7}$$

Note.—If other noise sources are present, producing exposure $E_n'-E_0$ and opacity fluctuation $P_n'-P_0$, these will combine with the fluctuations of the emulsion opacity so that the resulting opacity fluctuation will have an rms value

$$P_n''-P_0=[(P_n'-P_0)^2+(P_n-P_0)^2]^{\frac{1}{2}}.$$

We now have

$$R=\frac{P_m-P_0}{P_n''-P_0}=\frac{r_m-1}{r_n''-1}.$$

This leads to

$$r_n=\left[\left(\frac{r_m-1}{R}\right)^2-(r_n'-1)^2+1\right]^{\frac{1}{2}} \tag{8}$$

as the permissible opacity fluctuation.

The above analysis was based on the assumption that signals of only positive polarity would have to be accommodated so that the quiescent (no signal) level was chosen at the lower limit of linear portion of the opacity *vs* exposure curve. If both positive and negative signals must be recorded with opposite polarity, the quiescent point must be chosen at point C, halfway between the upper and lower limit of the linear portion, see Fig. 2. The dynamic range now becomes

$$R=\frac{P_m-P_c}{P_n-P_c}. \tag{9}$$

This is identical to the relationship obtained in the first case with P_c taking the place of P_0. The tolerable density fluctuation is, therefore,

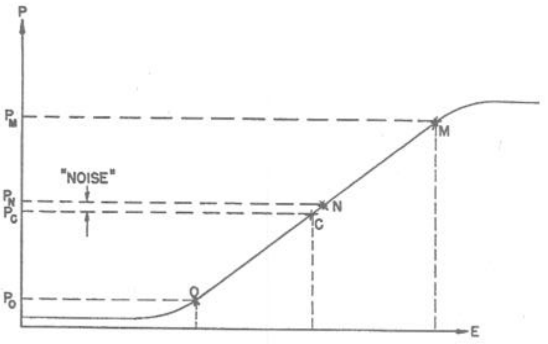

FIG. 2. Illustrating dynamic range determination with bipolar recording.

$$\Delta D=\log\frac{P_n}{P_c}=\log\left(\frac{P_m/P_c-1}{R}+1\right)$$
$$\doteq\log\left(\frac{1}{R}+1\right)\doteq\frac{0.43}{R} \tag{10}$$

since $P_m/P_c\doteq 2$ and $R\gg 1$.

INFORMATION CONTENT

If the number of discernible shades or "gray-steps" are specified, the allowable density fluctuation is again determined.

It is given by

$$\Delta D=(D_m-D_0)/2N, \tag{11}$$

where N is the number of shades discernible (with a 95.5% certainty). Or, if ΔD has been determined on some other basis, the number of shades is

$$N=(D_m-D_0)/2\Delta D. \tag{11a}$$

If granularity varies with the density and a fixed scanning aperture is used, ΔD will vary with density. In the previous analysis ΔD at the quiescent point had to be used. In determining N, though, an average ΔD must be used. If we assume that G does not change appreciably over a region ΔD, we can use the mean value

$$\langle\Delta D\rangle_{Av}=\left\{\int_{D_0}^{D_m}\Delta D(G)dD\right\}\Big/ D_m-D_0 \tag{12}$$

and consequently

$$N=\frac{(D_m-D_0)^2}{2\int_{D_0}^{D_m}\Delta D(G)dD}. \tag{13}$$

Now that the number of shades of intensity discernible and the number of resolution elements have been determined we can write an expression for the information capacity of a grainy photograph as a function of the scanning aperture used.

The information content in bits is, assuming equal probability of all density levels,[4]

$$I=(A/w^2)\log_2 N, \tag{14}$$

where A is the area of the photograph. This differs from the analogous channel capacity as determined by the Hartley-Shannon relationship due to the fact that the noise amplitude is proportional to the signal level.

Since w may be considered a function of the required number of discernible shades as defined by Eqs. (3) and (11),

$$w=\frac{G\sqrt{2}}{\Delta D}=\frac{G\sqrt{2}}{D_m-D_0}2N, \tag{15}$$

and we may write

$$I=\frac{A(D_m-D_0)^2}{8G^2}\frac{\log_2 N}{N^2}. \tag{16}$$

$N\geq 2$ if there is to be any information with our criterion of 95.5% certainty. The information content can thus be seen to be a maximum when $N=2$†,[5] and the scanning aperture

$$w=4\sqrt{2}G/(D_m-D_0). \tag{17}$$

The number of resolution elements is then obviously given by

$$\frac{A}{w^2}=I=\frac{A(D_m-D_0)^2}{32G^2}. \tag{18}$$

In our case—in the limit of two density shades—this uncertainty reduces the information content by a factor of 0.82.

Above all, it should be noted that the information content calculated here is based on limitations of granularity and density range only. Other factors, such as diffusion of light in the emulsion, halation effects and deficiencies of the image-forming optical system may seriously decrease the information content.

QUANTITATIVE EXAMPLE

We shall now briefly investigate the order of magnitude of resolution obtainable with common photographic materials when the contrast (density-difference) capacity is specified. To this end we shall use granularity figures as published by Jones and Higgins.[6]

We shall assume that a dynamic range of $R=1000$ is specified, that Kodak Aeromap Super XX film is to be used and that the opacity *vs* exposure curve is sufficiently linear between the opacity values $P_0=2$ and $P_m=200$, that is $D_0=0.3$ and $D_m=2.3$.

We have, therefore, $r_m=100$. Also, $r_m/R=0.1$ so that the approximate Eq. (7) may be used. We conclude that the allowable density fluctuation is

$$\Delta D=(0.43)(0.1)=0.043.$$

The paper by Jones and Higgins gives the Selwyn granularity for the Super XX film as $G=1.0$ density microns at a density of 0.28 and a scanning aperture of 39.9 microns.

The minimum element size is, therefore, by Eq. (3)

$$w=\sqrt{2}G/\Delta D=\sqrt{2}(1)/0.043=32.9 \text{ microns}$$

This corresponds to a resolving power of about 15 lines per mm.

At this element size, effects of exposure of neighboring elements would be expected to be considerable so that the diffusion of light in the emulsion and halation effects would have to be considered before the information content can be evaluated.

On the other hand, it appears safe to conclude from this result that a 33 micron square element, exposed at one thousandth of maximum exposure, can be determined with 95.5% certainty if it is surrounded by an unexposed area.

CONCLUSIONS

It is concluded that a minimum element size on a photographic recording is determined with a given photographic material when the dynamic range of the recording is specified. This element size may be established from the linear exposure range and the granularity figure of the photographic emulsion with the granularity figure based on frequency of deviation of density from the average density.

Granularity acts as one limitation on the information content of a photographic recording, but in practical cases other deficiencies of photographic materials play an even greater role.

[4] E. G. L. Brillouin, *Science and Information Theory* (Academic Press, Inc., New York, 1956), Eq. (1.7).

† Note that $dI/dN=k/N^3[1-2\ln N]<0$, $(N>e^{\frac{1}{2}})$. Also note that the information content given by Eq. 16 is only approximately correct. It ignores the 4.5% uncertainty in the density level determination. Thus, according to Shannon,[5] the information capacity of a channel in bits per digit is $I=1+p\log_2 p+(1-p)$ where p is the error probability.

[5] C. E. Shannon and W. Weaver, *The Mathematical Theory of Communication* (University of Illinois Press, Urbana, 1949) p. 38.

[6] L. A. Jones and G. C. Higgins, J. Opt. Soc. Am. **36**, 203 (1946).