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Photometric Resolving Power of Photographic Emulsions

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When photometric methods for detecting photographic images are employed, the methods used to predict visual resolving power of photographic materials[1] do not apply. Photometric resolving power is defined, and an expression is derived enabling one to predict the resolution performance of a given emulsion in terms of its modulation-transfer function, granularity, and gamma. Detailed curves are given for one representative photographic emulsion.

Although the concepts of spread function and granularity have generally replaced resolving power as a criterion for evaluating optical components and systems, the ultimate performance criterion for many optical systems is still resolving power. It should therefore be of interest to determine resolving power as a function of the spread function (or transfer function) and granularity.

A thorough investigation of visual resolving power of photographic emulsions, both theoretical and experimental, is reported in a now classic paper by Selwyn.[1] This paper was necessarily based on certain psychological parameters such as threshold brightness difference.

Since modern techniques rely increasingly on readout by photoelectric devices, it was decided to extend Selwyn's work to such systems. It then becomes possible to predict resolution in fully objective terms.

Such an expression was recently given by Frieser.[2] The present work refines his result to the extent that the optimum slit width (rather than Frieser's somewhat arbitrary half-period slit width) is used. In addition, in view of the one-dimensional nature of the resolving power to be determined, a slit of fixed length, rather than a circular scanning spot, is assumed. This changes the basic form of the result and accounts for the difference with respect to Selwyn's result.[1] Also, some calculated data based on an actual film are presented, rather than using analytic approximations of great convenience but of somewhat questionable validity.

The results presented here permit the prediction of resolving power of a photographic recording system in terms of the transfer functions of the input optical system and the photographic emulsion and the granularity of the latter.

The following resolution test is assumed. The record is scanned by a slit of fixed length and a width adjusted to maximize the ratio of density excursion to grain fluctuation for the image of the sinusoidal object to be detected. The image is said to be detected or "resolved" when in almost all cases the density reading at a peak density position exceeds the reading at the next trough position. Resolving power becomes thus an objectively determinable function of the allowable probability P that a trough density exceed the preceeding peak density of the densitometer slit length and the signal and emulsion parameters.

In the following section we shall determine the form of this function.

Analysis

We consider a photographic emulsion which has been exposed with an aerial image consisting of sinusoidal brightness variations and is scanned by means of a slit oriented parallel to the lines of equal brightness and moving at right angles to its length.

Let k be the spatial frequency (c/mm) of the sinusoidal image;

a the amplitude of the illumination variation in the aerial image divided by the average illumination;

$T(k)$ the transfer function of the photographic emulsion;

G its Selwyn granularity constant, i.e. the standard deviation of the density when the uniformly exposed and processed emulsion is scanned with an aperture of half-unit area;

v the slit length; and

w its width.

The area of the slit is then $A = vw$.

The exposure will be proportional to

$$E \sim (1 + a \cos 2\pi kx)$$

and, in the linear portion of the H & D curve, the relative transmission is given (neglecting granularity) by

$$t/t_0 \approx 1 - a\gamma T(k) \cos 2\pi kx,\ a\gamma T(k) \ll 1$$

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1. E. W. H. Selwyn, *Phot. J.*, 88B: 46 (1948).
2. H. Frieser, *J. Phot. Sci.*, 9: 379 (1961).

where t is the transmittance, t_0 is its average, and the approximation will be valid near the limit of resolution. Thus, the relative transmittance measured at an exposure peak with a slit of width w will be given by:

$$<t_1/t_0> \approx \frac{2}{w}\int_0^{w/2} (1 - \gamma a T \cos 2\pi kx)dx$$
$$= 1 - \gamma a T \sin \pi kw/\pi kw$$

Similarly, the trough relative transmittance reading will be

$$<t_2/t_0> \approx 1 + \gamma a T \sin \pi kw/\pi kw$$

In view of the fact that we are dealing with small amplitudes we may write

$$D_1 - D_0 \approx (\log_{10}e)\ \gamma a T\ \sin \pi kw/\pi kw \approx D_0 - D_2$$

where $D_{0.1.2}$ are the densities corresponding to $t_{0.1.2}$, respectively. Thus the mean density difference d between peak and trough will be approximately

$$m(d) = a'T \sin y/y$$

where we have written*

$$a' \triangleq 2\gamma a \log_{10}e \text{ and } y \triangleq \pi kw$$

Variations around these values due to granularity are Gaussian with a standard deviation of $G/(2A)^{1/2}$. Thus the standard deviation of the density difference is

$$\sigma(d) = G/(vw)^{1/2}$$

Knowing mean and standard deviation plus its normal form we can write the distribution function of this density difference:

$$F(d) = \int_{-\infty}^{d} \left\{\exp[-(u - m)^2/2\sigma^2]/(2\pi)^{1/2}\sigma\right\}du$$
$$= \tfrac{1}{2} + \Phi\left(\frac{d - m}{\sigma}\right),\ \Phi(x) \triangleq \frac{1}{(2\pi)^{1/2}}\int_0^x e^{-u^2/2}du$$

Hence

$$F(0) = \tfrac{1}{2} - \Phi(m/\sigma) = \tfrac{1}{2} - \Phi((vw)^{1/2}a'T \sin y/Gy) \triangleq P$$

the probability of a trough density exceeding a peak density.

We now seek the value of w which will minimize P. This minimum coincides with the maximum of Φ and, therefore, its argument. Hence we seek

$$\frac{d}{dw}(\sin \pi kw/w^{1/2}) = (\pi kw^{1/2} \cos \pi kw - \sin \pi kw/2w^{1/2})/w = 0$$

or $2y \cos y - \sin y = 0$.

This yields a value $y_0 = 1.16556\ldots$, $\sin y_0 = .919015\ldots$, and the corresponding value of w is

* The symbol $\triangleq$ indicates a definition.

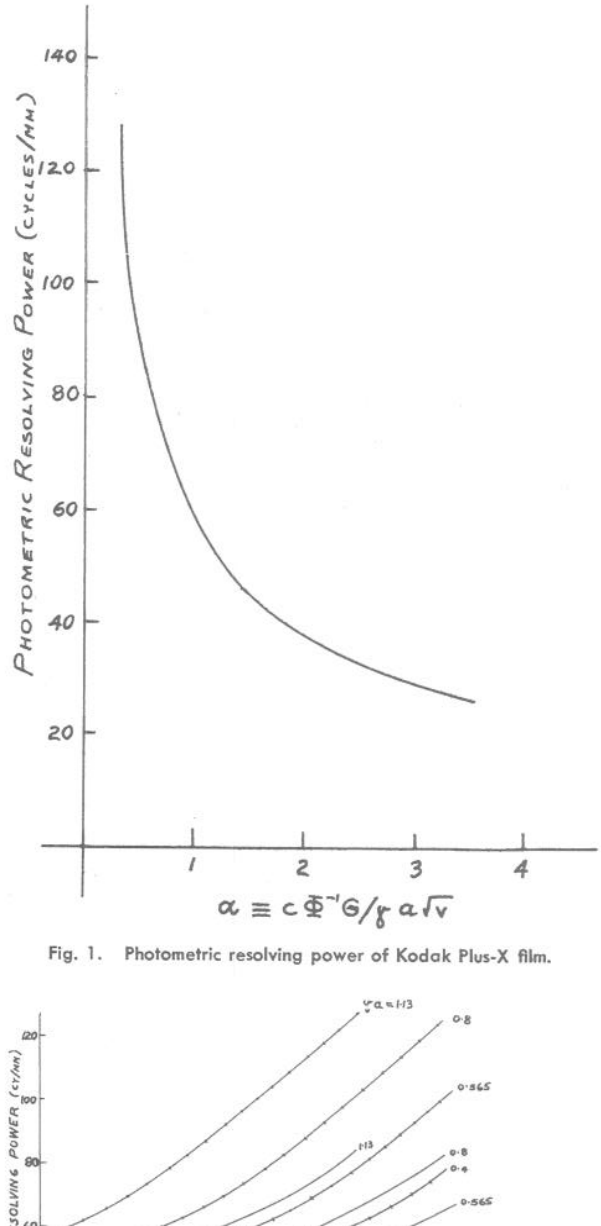

Fig. 1. Photometric resolving power of Kodak Plus-X film.

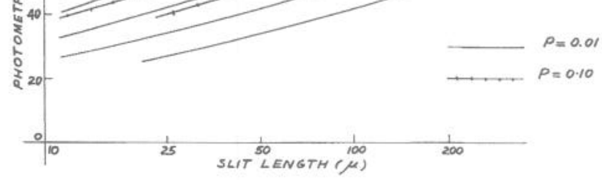

Fig. 2. Photometric resolving power of Kodak Plus-X film – special cases.

$w_0 = y_0/\pi k$. Substituting these values into the expression for P and rearranging terms we find that

$$\gamma a T(k) = c''Gk^{1/2},$$
$$c'' \triangleq \Phi^{-1}(\tfrac{1}{2} - P)c'/v^{1/2},$$
$$c' \triangleq (\pi y_0)^{1/2}/2 \sin y_0 \log_{10}e = 2.3972.$$

This should be compared with Selwyn's expression for visual resolving power, which is

$$\gamma a T(k) = cGk$$

The essential difference between these two results is that the photometric resolving power is more sensitive to changes in image amplitude, gamma, and granularity since here these enter the expression for k as squares.

Numerical Example

To illustrate the values of resolution obtainable with photometric techniques as outlined above, some data for Eastman Kodak Plus-X film were worked out graphically. The transfer function, as given by Lamberts[3] was used, and the Selwyn granularity constant was assumed $G = 1.2$ density-microns. This figure is based on Higgins and Stultz data[4] for density near unity and a small scanning area. Figure 1 permits readily the determination of resolving power for any given combination of probability P, image contrast a, γ, and slit length v. The specific cases of $P = 0.1$ and $P = 0.01$ have been worked out for $\gamma a = 0.4$, 0.565, 0.8, and 1.13. These are shown in Fig. 2.

Conclusions

Photometric resolving power is more sensitive to image and emulsion parameter changes than is visual resolving power.

In the case of Plus-X film with $P = 0.1$ a slit length of 80 μ yields a resolving power about equal to the accepted visual resolving power (based on a modulation, gamma product of 0.9).

3. R. L. Lamberts, *J. Opt. Soc. Am.*, **49:** 425 (1959).
4. G. C. Higgins and K. F. Stultz, *J. Opt. Soc. Am.*, **49:** 925 (1959).