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Photographic Emulsions as Computer Storage Media—Especially with CRT Readout

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When long term storage is desired, photographic films are attractive storage media for electronic computers. Their storage capacity is limited primarily by granularity and turbidity of the emulsion and the spot size of a practical readout device. The relations between the element size and the number of resolvable density steps are derived as a function of the emulsion granularity and point-spread function and the light distribution in the scanning spot. Simplifications for special limiting cases of interest are derived and one such case is worked out numerically.

I. Introduction

As computer technology advances, more information storage devices compatible with high-speed computers are required. When permanent storage is required, photographic films with cathode-ray tube scanners are among the most promising. It should therefore be of some interest to evaluate the storage capabilities of actual photographic emulsions from the point of view of such a computer application. A general analysis in terms of information capacity was made in an excellent paper by Jones,[1] but his results are not readily applied to the present problem.

In the system envisioned, the information to be stored is expressed in terms of optical density and position, so that a given quantity of information corresponds to a unique density pattern. This density pattern is impressed on the photographic emulsion by appropriately exposing each area element. The information is then read out by striking the appropriate element with a constant spot of light from a cathode-ray tube* and receiving the transmitted light on a phototube.

The limitations inherent in this system are primarily due to the following factors:

1. It is impossible to impress a given density on a given film element by exposure and processing control alone. The resulting density is determined by the chance concentration of developable grains in addition to the exposure and processing. The random fluctuations occurring around the average density are called "granularity".

2. It is impossible to limit the exposing light to a given area element. Refractive nonuniformities in the emulsion, generally termed "turbidity", will scatter it into neighboring areas, whose density is thus affected by irrelevant information.

3. The readout scanning spot will cover an area much larger than the area element of interest, due primarily to reflections inside the CRT face plate, and to the finite electron beam size. Although the brightness of the spot may fall off almost completely over a very small area, there may be still significant contributions from points far removed from the bright spot center.

4. The scanning spot intensity itself may be modulated in a random manner by the chance distribution of grains in the phosphor screen.

The implication of all these factors is simply that the area element on the photographic film corresponding to one item of information must be made large enough to make these effects negligible. In other words, if one specifies a certain number of discernible density levels for the system, one has implicitly specified the maximum number of information items that can be accommodated on a given piece of photographic film (using a given CRT).

It is the purpose of this paper to analyze the basic relationships governing area element, size, and number of discernible information levels as a function of film and CRT parameters. A similar analysis, considering only granularity, has been made earlier.[2] The first step is, of course, to translate the system requirements

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* Clearly other readout spot images may be used and the following work can readily be applied to them when their point-spread function is known.

into a tolerance on the density. This translation will generally be a function of the specific system, but the average tolerance can never be more than

$$\overline{2\delta D} = (D_{\max} - D_0)/N, \tag{1}$$

where $D_{\max,\,0}$ are the maximum and minimum usable density values, respectively, and N is the number of information levels which must be discernible.

We shall now determine what the minimum area element must be so that granularity effects permit us to maintain the $\pm\delta D$ tolerance.

II. Granularity

Granularity is a statistical phenomenon so that requirements on it can properly be expressed only in terms of probability. In other words, we cannot require that the density fluctuations *never* exceed D, only that the probability of their exceeding D be less than some small number $(1 - P)$.

On the basis of theoretical considerations, Selwyn[3] found that the density fluctuations were normally distributed and that their standard deviation varied with the square root of the area element a. He gives for the probability of finding a density deviation in the interval $u, u + du$:

$$P(u, du) = \sqrt{\frac{a}{G^2\pi}}\, e^{-u^2a/G^2}\, du, \tag{2}$$

where G is a granularity constant of the emulsion. (G is effectively the standard deviation of density observed when an area element of area one-half is viewed at a time.) G is found to be constant over wide ranges of area,[4] and is therefore a useful parameter in our investigation. It does vary with density.[1,3]

We can now write immediately the expression for the probability of finding at any spot a density within the tolerance range $D_1 \pm \delta D$ when the emulsion is exposed to the average density D_1. It is

$$P(D_1, \delta D) = \operatorname{erf}\left[\frac{\sqrt{a}\,\delta D}{G(D_1)}\right], \tag{3}$$

where

$$\operatorname{erf}(x) = \frac{1}{\sqrt{\pi}} \int_{-x}^{x} e^{-u^2} du. \tag{4}$$

Solving for a we find explicitly:

$$a = \left[\frac{G}{\delta D} \operatorname{erf}^{-1}(P)\right]^2, \tag{5}$$

where G is the known granularity of the material, δD the tolerance on density, and P the specified probability of remaining within this tolerance.

The above expression specifies the area element that must be viewed at once and presupposes that all parts of this element are illuminated equally. On the other hand, in the system under consideration the illumination distribution is that of the CRT spot image and is, therefore, far from uniform. It can be shown (see Appendix) that the effective area of the CRT spot is given by:

$$a = \frac{\left[\int_A\int B(x, y)dx\, dy\right]^2}{\int_A\int B^2(x, y)dx\, dy}, \tag{6}$$

where $B(x, y)$ represents the brightness of the CRT spot image at point (x, y), and the integration is performed over the whole effective CRT face (or all portions of significant brightness of the image). For instance, if the brightness of the scanning spot has the form:

$$B(r) = \frac{B_0}{2\pi s^2}\, e^{-r^2/2s^2} \tag{7}$$

then the effective area is

$$a_e = 4\pi s^2. \tag{8}$$

Equation (5) then specifies the minimum *scanning spot* size, if density fluctuations are to be kept below D.

If phosphor grain is to be taken into account, the distribution of fluctuations in phosphor efficiency must be known for the given spot size. Unfortunately, it was impossible to obtain these data.

Assuming (1) that this distribution, too, is Gaussian, and (2) that its variance varies inversely with the spot area, we may combine phosphor grain effects with those of the photographic emulsion by using a combined granularity constant

$$G = \sqrt{G_E^2 + G_P^2}, \tag{9}$$

where G_E is the granularity constant of the emulsion and G_P is the standard deviation of the logarithm of brightness when the phosphor is scanned with a spot of effective area one-half.

III. Turbidity

The amount of error introduced into the record due to internal scattering of light is far more difficult to evaluate due to the nonlinear character of the H and D curve: $D = f(E)$, where E is the logarithm of exposure. We assume that the point spread function of the light in the emulsion is known to be of the form $S(r)$, where $S(r)$ represents the amount of exposure at a point a distance r from a point on the emulsion having received unit exposure.

We simplify the algebraic work by writing directly the transmissivity as a function of the exposure:

$$T = g(e), \tag{10}$$

where

$$T = 10^{-D} \quad (11)$$

Then the exposure e at any point will be, in terms of the impressed exposure e^*,

$$e(x, y) = \int_A\int e^*(x', y')S[(x - x')^2 + (y - y')^2]^{1/2}dx'\,dy' \quad (12)$$

and the amount of light transmitted with the scanning spot at (x_0, y_0)

$$L(x_0, y_0) = \int_A\int T(x, y)B[(x - x_0), (y - y_0)]dx\,dy$$
$$= \int_A\int g\{\int\int e^*(x', y')S[(x - x')^2 + (y - y')^2]^{1/2}dx'\,dy'\}B[(x - x_0), (y - y_0)]dx\,dy. \quad (13)$$

On the other hand, the amount of light that would be transmitted in the absence of turbidity effects, and with the total spot luminosity $B_0 \equiv \int BdA$ concentrated in a point, would be

$$L^*(x_0, y_0) = B_0\, g[e^*(x_0, y_0)]. \quad (14)$$

When the exposure e^* in the region surrounding x_0, y_0 is given a constant value, say e, L will approach L^* as this region is increased in area, a. The value of a for which $10^{-\delta D} < L/L^* < 10^{+\delta D}$ represents the minimum element area sought.

In general it will be neither practical nor necessary to evaluate the convolution integral (12) for the exposure at point (x, y). It will suffice to take the most severe situation of an element with minimum exposure e_1 surrounded by a region with maximum exposure e_2. By breaking up the integral (12) into two regions, e^* may be treated as a constant and taken out of the integral, yielding, for A exceeding the region of significant S,

$$e(x, y) = e_2 \int_A\int S[(x - x')^2 + (y - y')^2]^{1/2}dx'\,dy' + (e_1 - e_2)\int_a\int S[(x - x')^2 + (y - y')^2]^{1/2}dx'\,dy'$$
$$= e_2 + (e_1 - e_2)U(a, \sqrt{x^2 + y^2});$$

or

$$e(r) = e_2 + (e_1 - e_2)\,U\,(a, r), \quad (15)$$

where $U\,(a, r)$ represents the integral over a region a, situated a distance r from the center of the point-spread function. The resulting expression for the transmission ratio is, when the origin is taken at the center of the scanning spot ($x_0 = y_0 = 0$) and using Eqs. (10) and (15) in the second member of Eq. (13):

$$\lambda = \frac{L}{L^*} = \frac{1}{B_0\, g(e_1)}\int_A\int B(x, y)\, g[e_2 + (e_1 - e_2)U(a, \sqrt{x^2 + y^2})]dx\,dy. \quad (16)$$

If B, too, has circular symmetry and becomes negligible before the edge of the CRT is reached, this may be written:

$$\lambda = \frac{2\pi\left\{\int_0^\infty r\,B(r)\,g[e_2 + (e_1 - e_2)\,U\,(a, r)]dr\right\}}{B_0\, g(e_1)} \quad (17)$$

At this point it becomes necessary to introduce some specialization. We shall assume that the area element is a square $2b \times 2b$. (The alternative, a hexagonal area element, would result in a slightly smaller resolution element, but the resultive increased difficulty in the numerical analysis appears unwarranted, especially in view of the approximate nature of the other assumptions.) Realistic approximations for the other parameters are somewhat more difficult to make. The form of the effective point-spread function is complicated by adjacency effects in processing and that of the CRT spot by reflections from the face plate surface. The following assumptions should therefore be taken as illustrations rather than recommendations:

If $S\,(r)$ is taken to be Gaussian

$$S(r) = \frac{1}{2\pi\sigma^2}\, e^{-r^2/2\sigma^2}, \quad (18)$$

we find that

$$U(r, b) = {}^1/_2 \operatorname{erf}\left(\frac{b}{\sqrt{2}\sigma}\right)\left[\operatorname{erf}\left(\frac{r + b}{\sqrt{2}\sigma}\right) - \operatorname{erf}\left(\frac{r - b}{\sqrt{2}\sigma}\right)\right]. \quad (19)$$

$B(r)$ is usually taken as

$$B(r) = \frac{B_0}{2\pi s^2}\, e^{-r^2/2s^2} \quad (20)$$

and, in the linear portion of the H and D curve, we may take

$$T = g(e) = T_0\left(\frac{e_0}{e}\right)^\gamma,\ e > e_0. \quad (21)$$

With these assumptions, λ becomes:

$$\lambda = \frac{e_1{}^\gamma}{s^2}\int_0^\infty re^{-r^2/2s^2}\left\{e_2 + \frac{(e_1 - e_2)}{2}\operatorname{erf}\left(\frac{b}{\sqrt{2}\sigma}\right)\left[\operatorname{erf}\left(\frac{r + b}{\sqrt{2}\sigma}\right) - \operatorname{erf}\left(\frac{r - b}{\sqrt{2}\sigma}\right)\right]\right\}^{-\gamma} dr. \quad (22)$$

λ may now be evaluated as a function of b, and that value of b for which $\lambda = 10^{-\delta D}$ gives the minimum element size. If conditions are such that $s << \sigma$, we may set $r = 0$ in the factor in braces and take this factor out of the integral. This yields

$$\lambda = \left(\frac{e_1}{e_2 + (e_1 - e_2)\operatorname{erf}^2(b/\sqrt{2}\sigma)}\right)^\gamma = 10^{-\delta D} \quad (23)$$

or, letting $e_2/e_1 = R$,

$$b = \sqrt{2}\,\sigma\,\mathrm{erf}^{-1}\left(\frac{R - 10^{\delta D/\gamma}}{R - 1}\right)^{1/2}, \qquad (24)$$

again giving the size of the area element in closed form.

IV. Numerical Results

We shall now apply the above results to an actual emulsion. Higgins and Stultz[4] give the Selwyn granularity of Kodak Plus-X film at a density of 0.13 and scanning aperture diameter 24 μ as $G = 0.66$.

Zweig *et al.*[5] give the line-spread function of Plus-X film and Jones[6] both line and point-spread functions of other films. From these two papers it is possible to estimate the value of σ of Plus-X film as 7 μ. (Since the shape of the point-spread function is not truly Gaussian, it is here approximated by a Gaussian curve whose 2.5% points coincide with those of the actual point-spread function. This is the lowest point that could be read off the curve.)

Assuming that it is required that twenty signal levels be distinguishable over a density range $D_1 = 0.2$, $D_2 = 2.2$ and that $\gamma = 1$ ($R = 100$), we find that $\overline{\delta D} = 0.05$. For the present approximation we shall take this* as our operating $\overline{\delta D}$ and require that the probability of exceeding this tolerance be less than 0.034. We have then

$$\mathrm{erf}^{-1}(P) = 1.5,$$

and using Eqs. (5) and (8):

$$a = 400 \text{ sq. } \mu, \qquad s = 5.65\ \mu.$$

Noting that $[(R - 10^{\delta D/\gamma})/(R - 1)]^{1/2} = 0.99939$ and erf^{-1} (0.99939) = 2.423, we find, using Eq. (24): $b = 24\ \mu$. Thus, the resolution element should not be made smaller than 48 μ square.

Working out the case where only two levels (black and white) are to be recorded, we use $G = 1.21$ (corresponding to an operating density 0.96) and $\sigma = 4.6\ \mu$ (corresponding to the 10% point on the point-spread function). We may take δD here as 1. This calls for $b = 9\ \mu$ so that 18-μ elements are possible. This corresponds to 350,000 elements per square cm. Obviously, higher resolution films will yield correspondingly greater storage capacity.

Conclusions

The relationship between the number of spatial resolution elements and the number of discernible levels of gray has been worked out in terms of emulsion granularity and point-spread function and CRT spot parameters. The resulting expressions lend themselves to evaluation by numerical integration methods [Eq. (17) and (22)]. With certain simplifying assumptions, expressions in closed form can be obtained [Eq. (24)]. A numerical example showed that with all the above limitations, the photographic emulsion is a highly efficient information storage medium. Not too much weight should be attached to the numerical examples worked out because of the discrepancy between fact and assumption.

* The actual δD should be a function of the density. Specifically it should be $\delta D(D_1) = C/G(D_1)$ where

$$C = \frac{(D_{\max} - D_0)}{2}\left[\sum_{i=1}^{N}\frac{1}{G(D_i)}\right]^{-1}.$$

Appendix. The Effective Area of a Nonuniformly Illuminated Scanning Spot in Determining Granularity Effects

Let the standard deviation (s.d.) of the transmissivity be given by $t/\sqrt{a_e}$, where a_e is the equivalent area, and let the s.d. of an elementary area α be given by $\tau = t/\sqrt{\alpha}$.

The s.d. of the average transmissivity equals the s.d of the transmitted light over the area covered divided by the incident light.

Clearly, with a scanning spot of a brightness/unit area distribution $B(x, y)$, the s.d. of the transmitted light is the root-sum-square of the s.d.'s of all the elementary area segments α (even though the distribution of the transmissivity fluctuations is *not* normal). That is,

$$\sigma(L) = \sqrt{\sum_N (\alpha B\tau)^2} = \sqrt{\sum_N \alpha^2 B^2 t^2/\alpha} = t\sqrt{\sum_N B^2\alpha} \doteq t\sqrt{\int B^2\, da},$$

where αB is the light incident on each area element. Similarly, the incident light is given by

$$L_0 = \int B da.$$

We have therefore for the s.d. of the transmissivity:

$$\sigma(T) = \frac{t\sqrt{\int B^2\, da}}{\int B\, da} = t/\sqrt{a_e}.$$

We conclude that the equivalent area of the above situation is:

$$a_e = [\int B da]^2/\int B^2\, da.$$

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