Modern Performance Analysis of Optical Systems in Communication and Data Processing Systems

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Summary—Advances in technology are causing many technical disciplines to merge. This is also true of optics and electronics which today often meet in the same system.

The present paper is meant to serve as an introduction to analyzing classical optical components in communications and data processing systems and also as a practical aid in applying these techniques by collecting the most important data required and filling some of the gaps.

I. Introduction

WITH THE advances of technology, more and more forms of information are displayed and even processed in optical form. These advances have forced an evaluation of the performance of optical components in objective terms interpretable in fields outside optics. This need was recognized quite early and analyzed extensively by Schade [1], but his work has since been greatly extended. As a result, the image-forming ability of optical lenses and films is now commonly evaluated in terms of "optical transfer function",[1] "line spread function", and "granularity" [2], [3].

Application of these useful techniques is severely handicapped by the extent to which the required quantitative information is scattered throughout the literature. The present paper is meant to collect this information and to fill one or two gaps.

The discussion of the basic concepts is followed by a more detailed treatment of transfer function and granularity (Sections II and III, respectively) and a discussion of resolution and information capacity (Section IV). Additional useful data are presented in Section V.

In modern optical system analysis, the concept of resolving power has been replaced by the far more powerful concept of the optical transfer function. Optical transfer function describes the manner in which detail is degraded in the optical system in terms of sinusoidal input signals and is analogous to the "frequency response" of communication engineering. The line-spread function describes the spreading of a line image and is analogous to the "impulse response" of an electrical network. Just like their communications analogs, the transfer and line-spread functions are not independent; specifically, they form a Fourier transform pair. Granularity is a measure of the random fluctuations exhibited by the detector, such as a photographic emulsion, and is clearly analogous to the "noise" concept in communication engineering.

In the above analogies, length has replaced time. In the case of an image formed in air or on a passive screen, radiant flux density has replaced power, while in a photographic recording, power is replaced by density or transmittance. The relationship of flux density to photographic density is given by the exposure time and the "Density-Log Exposure" characteristic, frequently referred to as the "H and D curve" (named after Hurter and Driffield).

To avoid certain ambiguities common in the field, some terms used in this report are defined here as follows:

1) Radiant flux density is proportional to the square of the electromagnetic amplitude—the "intensity" of electromagnetic theory. (In photometry, the term "intensity" refers to flux-per-unit solid angle.)
2) Exposure refers to the time integral of flux density.
3) The ratio of transmitted-to-incident flux density is termed transmittance, and its reciprocal, opacity.

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[1] This term is occasionally referred to in the literature as "sine-wave response," "contrast transfer function," etc. The International Commission on Optics recently recommended the term "optical transfer function." Its amplitude is referred to as "modulation transfer function."

4) Density is the logarithm (to the base ten) of opacity.[2]
5) The reciprocal of the exposure required to produce a certain reference density is called sensitivity.

The only basic aspect of this image formation theory which has no analog in ordinary communication theory is the two-dimensional aspect of the optical image. Thus the "point-spread function," representing the flux distribution in the image of a point object, has no analog in ordinary communication theory. But, since the line spread may be derived from the point-spread function by a simple integration, and the inverse transform, too, involves only one integration [4], it is usually sufficient to evaluate the system in terms of line-spread functions.

The more subjective aspects of the photographic image, commonly referred to as sharpness and graininess, although closely related to transfer function and granularity, respectively, are complicated functions of the latter and of the type of object. Fortunately, these are of interest primarily only to the photographer, who is concerned with the aesthetic aspects of his results, and therefore will not be treated here.

II. Optical Transfer and Spread Functions

A. Definitions

In a linear optical system, a sinusoidal input signal, *i.e.*, an object with sinusoidal brightness variations, will result in an image with sinusoidal variations of flux density or density. (Considered here is a segment of the object so small that the variations of the aberrations and diffraction effects within this segment are negligible.) Aberrations and glare in the lens system and spreading of light in the photographic emulsion will only tend to lower the relative amplitude of this sinusoid.

The ratio of the amplitude of the sinusoidal component to the average value of the brightness, flux density, or density may be termed fractional modulation.

At a given spatial frequency, the ratio of image-to-object modulation will be a constant, independent of the signal level. The variation of this "constant" as a function of spatial frequency is referred to as the optical transfer function of the optical system, or component. Its Fourier transform yields the line-spread function, *i.e.*, the distribution in a transverse section through the image of a line.

When the spread function is asymmetrical, the description of phase shift with frequency must be considered part of the transfer function. In that case, the complex Fourier transform must be used. In the symmetrical case, the cosine Fourier transform suffices.

[2] Due to the scattering of light by the silver grains, the density observed depends on the angle subtended at the emulsion by the light used in the measurement. The ratio of density measured with a distant point source (imaged on a small remote photo detector) to that measured with a diffusing screen in contact with the emulsion is called Callier factor and is almost constant for a given emulsion.

B. The Density–Log Exposure Characteristic and Linearity

It should be noted that the definition of the transfer function was based on the assumed linearity of the system and that, similarly, the line-spread function will be constant only in a linear system.

The assumption of linearity seems well justified in a lens system but requires testing in the case of a photographic emulsion. The linearity of these emulsions is conventionally given by the Density–Log Exposure characteristic, a representative curve being shown in Fig. 1.[3] The slope of the linear portion of this curve is referred to as the "gamma" (γ). Thus a linear relationship between exposure and opacity is indicated by unity gamma, but this does not constitute a linear relationship between exposure and transmittance, which is the measured parameter. A system linear in terms of transmittance is most readily obtained by cascading two photographic processes such that the product of the two gammas equals unity. Although the exposure range over which this can be fulfilled even approximately is usually limited to about two decades, this covers a wide range of applications.

The linearity of both a single photographic process and a repeated process has been investigated in detail [5], [6].

In the linear case, the system transfer function is readily obtained from the individual components by simple multiplication. Thus an incoherently illuminated object,[4] a lens system, and a linear photo-detector may have their transfer functions multiplied to yield the system transfer function. (This is not true for components of a lens system which combine nonlinearly, the resulting flux density depending on the relative phase of the various optical wave amplitudes entering the sum.)

Spread functions in such a system combine by convolution. Since convolution is generally a process arithmetically far more involved than multiplication, the transfer function is usually the more convenient tool for analyzing optical systems [6].

A convenient tabulation of the transforms connecting the transfer functions, the line-spread function, and the point-spread function can be found in [4]. Table I gives the most frequently used approximations to transfer functions and the corresponding line-spread functions. Other sets, giving more complicated transform pairs have been published [7], [8].

[3] The A. S. A. (arithmetic) speed S for continuous tone photographic emulsions is given by $0.8\ E_m$, where E_m is the exposure (in meter—candle—seconds) required to raise the density 0.1 units above fog. The emulsion is assumed to be developed so that an exposure $E_n = 20\ E_m$ raises the density 0.9 units above fog.

The D. I. N. rating S^* popular in Europe, also has been recently revised, making it comparable to the A. S. A. rating. The following equality holds approximately

$$S^* = 10 \log_{10} S + 1.$$

[4] By incoherent illumination, we refer to a situation where the phase of the electromagnetic radiation at one point of the object is purely randomly related to that at any other point.

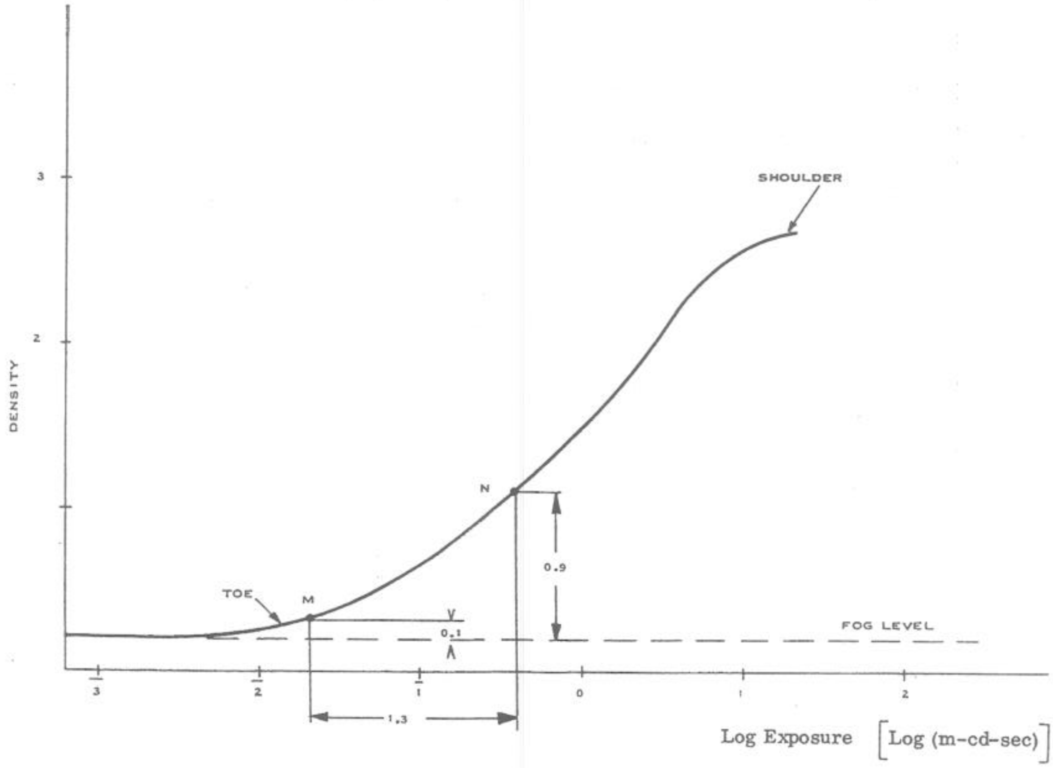

Fig. 1—Representative density—log exposure curve.

TABLE I
USEFUL PAIRS OF TRANSFER AND LINE-SPREAD FUNCTIONS

	Transfer Function	Line-Spread Function
1	$\frac{b^2}{b^2+k^2} = \frac{a^2}{a^2+(2\pi k)^2}$	$\pi b e^{-2\pi b\|x\|} = \frac{a}{2} e^{-a\|x\|}$
2	$e^{-ak} = e^{-2\pi bk}$	$\frac{2a}{a^2+(2\pi x)^2} = \frac{b/\pi}{b^2+x^2}$
3	$e^{-sk^2} = e^{-(\sqrt{2}\pi k\sigma)^2}$	$\sqrt{\frac{\pi}{s}}\, e^{-(\pi x)^2/s} = \frac{e^{-(x/\sigma)^2/2}}{\sqrt{2\pi}\,\sigma}$

Note: 1) k is spatial frequency (cycles/cm). a, b, σ, s are constants.
2) x is distance from spread function origin (cm).

C. Ideal and Actual Transfer and Spread Functions

1) Lenses: When an optical system consists of a perfectly corrected lens system, the transfer function is limited only by diffraction effects at the aperture. It can be shown [3] that for that case the transfer function on axis has the form of the convolution of the aperture with itself, the spatial coordinate being replaced by the product of optical wave length (λ), focal length, and the spatial frequency. Thus, when the aperture is rectangular, the transfer function drops linearly from unity (at zero spatial frequency) to zero at the spatial frequency $k = (\lambda n)^{-1}$, where n is the effective f/number of the lens. When the aperture is circular, the response has the form

$$T(k) = \frac{2}{\pi}\left(\cos^{-1} y - y\sqrt{1-y^2}\right), \qquad y \leq 1 \qquad (1)$$

$$T(k) = 0, \qquad y \geq 1$$

where $y = \lambda kn$. This response curve is shown in Fig. 2(a).

Since this response is a function of optical wave length, it must be integrated over the spectral range (with proper weighting) when heterochromatic light is used. The result of such integrations for a P16 phosphor-S11 phototube combination and an incandescent tungsten lamp at 2848°K combined with the photopic eye are shown in Fig. 2(b). The abscissae are $k' = kn$.

An approximate idea of the point-spread function for actual lens designs may be obtained by means of a spot diagram, which is obtained by tracing rays from one object point through an equispaced large array of points in the lens entrance pupil and plotting the resulting point density in the image plane [9], [10].

Such spot diagrams give useful results only when geometrical effects are much greater than diffraction effects. When this is not true, no simple way of predicting the net result seems to be available and the actual wavefront aberration must be used [11a].

It should be noted that with a given relative aperture the contributions to the transfer function due to diffraction effects are independent of focal length while those due to geometrical optical effects (aberrations) are directly proportional to focal length.

The best photographic objectives seem to approach (within 5 per cent) the perfect lens response on axis at

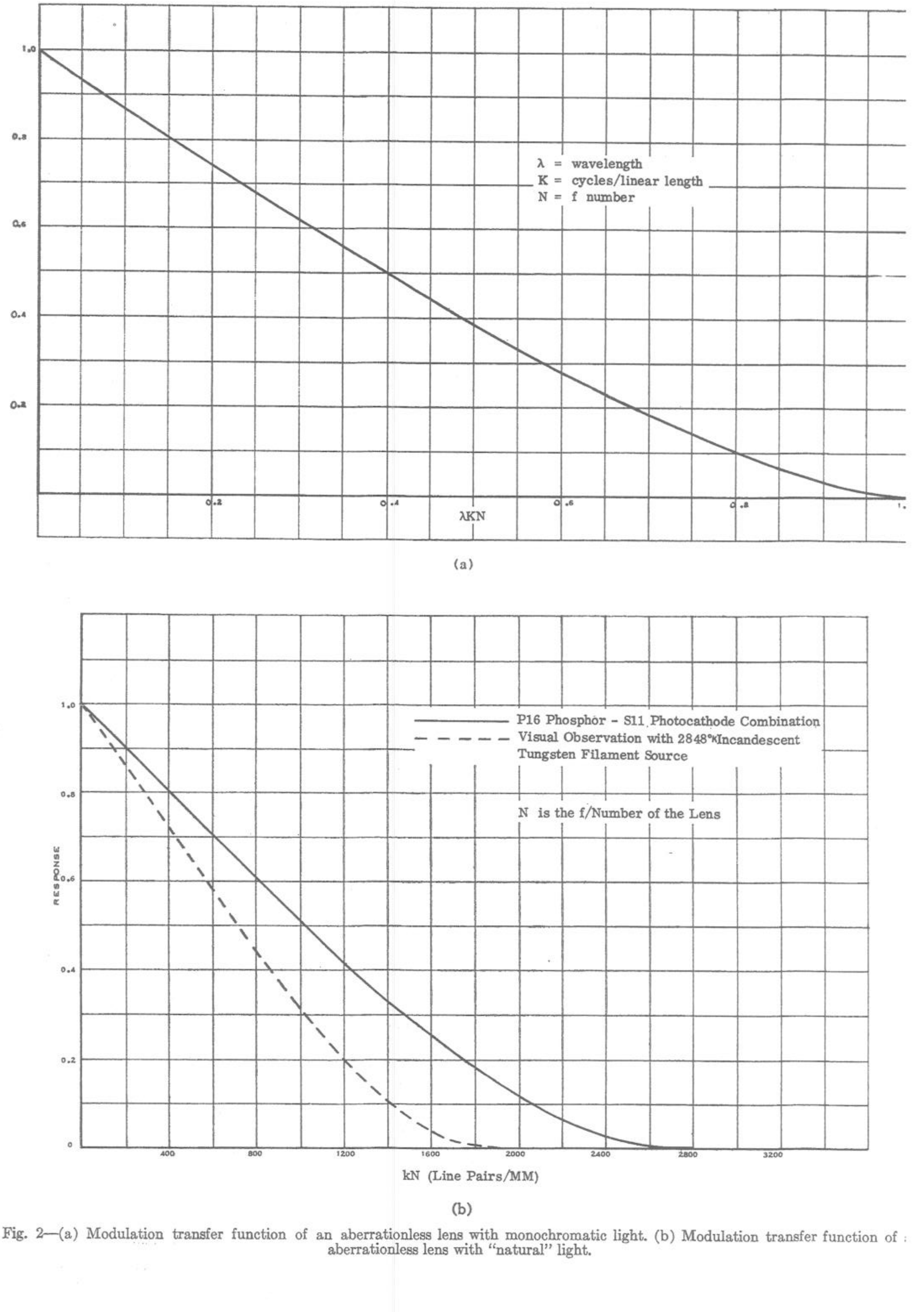

Fig. 2—(a) Modulation transfer function of an aberrationless lens with monochromatic light. (b) Modulation transfer function of aberrationless lens with "natural" light.

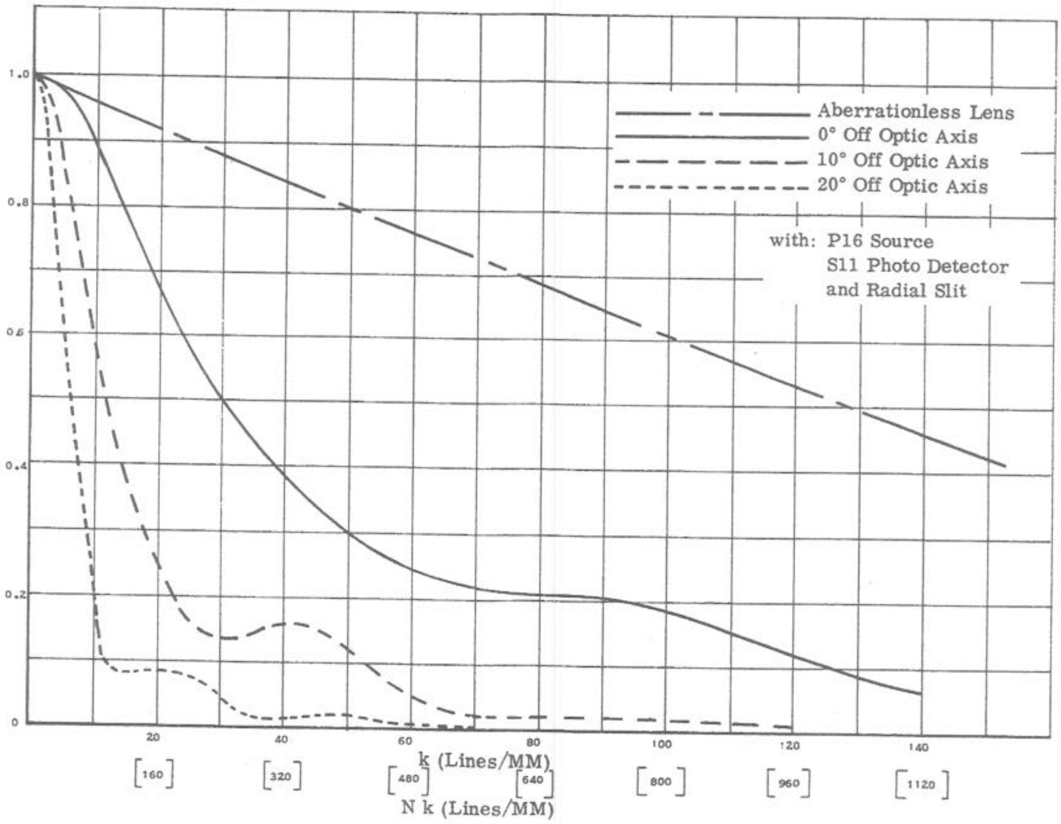

Fig. 3—Modulation transfer function of a 100 mm, F/8 enlarging lens.

f/8 in the 50-mm focal length range. At f/2.8, the transfer function may be down to 20 per cent at 80 cycles/mm where the aberrationless lens would have 85 per cent transfer [12]. Off-axis performance seems to be considerably poorer [13].

To illustrate the effect of using a lens which was corrected for the visible spectrum with an ultraviolet phosphor, the transfer function of a 100 mm focal length f/8 enlarging lens is shown in Fig. 3. This transfer function was obtained with light corresponding to the output of a P16 phosphor. The response of an aberrationless lens is shown for comparison.

2) Photographic Emulsions: On the basis of the simplest physical models, it would seem that the spread functions of photographic emulsions should be Gaussian, so that the transfer function, too, would be Gaussian. In general, though, this assumption is not in good agreement with results obtained in practice. The form taken by these results varies from emulsion to emulsion, with development procedure, and, possibly, with storage. A discussion of these approximations, together with extensive results for Agfa films, is available [7].

If a single, one-parameter approximation is to be used, it would seem that an exponentially decaying spread function is optimum [14]. This corresponds to a transfer function of the form

$$T(k) = b^2(b^2 + k^2)^{-1}.$$

See Table II for values of b for actual emulsions [17].

TABLE II

DATA FOR COMPUTING INFORMATION CAPACITY OF VARIOUS EASTMAN KODAK FILMS AND THE RESULTING CAPACITIES

Film	W_G(cm²)	b(cycle/cm)	C(bits/cm²)
Royal-X Pan	314×10^{-10}	248	0.499×10^6
Tri-X	123×10^{-10}	252	0.845×10^6
Plus-X	38.5×10^{-10}	270	1.86×10^6
Panatomic-X	38.5×10^{-10}	470	2.85×10^6

For transfer functions of Eastman Kodak films, see [5], [8], [15], and [16]. For line-spread functions see [4], [17], and [18].

3) The Cathode Ray Tube: In engineering applications of optics, the most common inputs to an optical system are 1) a sharply outline field, such as an ink line drawing or an illuminated slit, 2) a processed photograph, 3) a cathode ray tube display. Of these, the first may be treated by means of step functions and the second by the methods just given. When the input is a cathode ray tube display, knowledge of the transfer function of the cathode ray tube may be important.

The brightness distribution of a typical cathode ray scanning spot consists of a near Gaussian central peak surrounded by a faint haze and a series of widely spaced, circular "mounds", called "halation rings". These rings usually exhibit fairly sharp inner edges and fall off slowly on the outside; successive rings decrease rapidly in brightness with distance from the center. These rings are caused by total reflection at the front surface of the face plate and methods of reducing them are available [19], [20]. In modern cathode ray tubes, they contain only 6 per cent of the light and are negligible in most applications.

A representative line-spread function obtained with a high resolution 5 inch P16 tube is compared to a Gaussian in Fig. 4. The very close agreement would seem to warrant this approximation in practice.

III. Granularity

A. Photographic Emulsion

Most photographic emulsions, though processed to a uniform density macroscopically, will exhibit random density fluctuations when viewed under sufficient magnification. Such viewing is equivalent to scanning the emulsion densitometrically with a small aperture.

On the basis of theoretical considerations, Selwyn [21] found that the density fluctuations could be expected to be normally distributed with a variance inversely proportional to the area of the scanning aperture. He defined a constant G which equals the standard deviation of the density when a half-unit area scanning aperture is used. In terms of this constant, the probability of finding a density in the interval D, $D + dD$ when the emulsion is processed to a macroscopically uniform density D_1 and scanned with an aperture of area A is:

$$P(D, dD) = \left[\sqrt{\frac{A}{\pi G^2}} \exp\left[-A(D - D_1)^2/G^2\right]\right] dD. \quad (2)$$

In practice, G has been found to be essentially constant with the area of the scanning spot, but to vary appreciably with density. An empirical analytic expression for the variation of G with density of the form $G = G_1 D^{0.23}$ has been given [17]. It has been found to represent well Kodak Royal-X, Tri-X, and Panatonic-X films, for example, but not Plus-X film. A detailed analysis of the auto correlation function of granularity has been made [22]. For extensive granularity data on Eastman Kodak films, see [23], [24]. Representative figures extracted from these (by interpolation and extrapolation) are given in Table III.

B. Cathode Ray Tubes

In cathode ray tubes, the granular structure of the phosphor screen causes the scanning spot brightness to vary as this spot moves across the screen. The finer the screen, of course, the less pronounced are these fluctuations. They are, on the one hand, sensibly absent in the evaporated transparent phosphor screens and, on the other hand, extremely pronounced in the case of the typical P16 screen.

Unfortunately, no data seem to have been published as yet on the granularity of these screens.

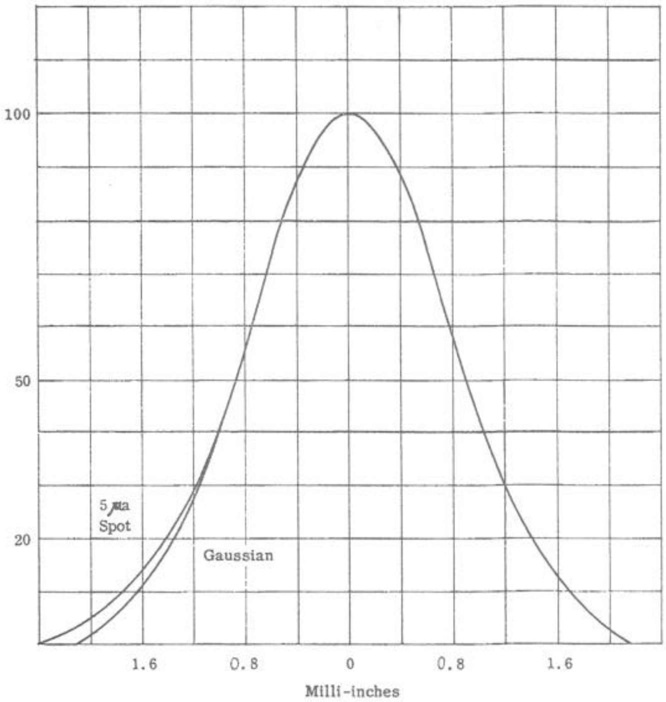

Fig. 4—Luminance distribution in a CRT spot (P 16) compared with a Gaussian distribution.

TABLE III

Film	D	$G(7\mu)$	$G(50\mu)$	$G(385\mu)$	$G(10\mu)$	$G(20\mu)$
Kodak Royal-X	0.1	(1.46)	(1.9)	(1.77)	1.21	1.28
	0.3	2.08	2.37	2.27	2.17	2.05
	0.8	3.1	3.14	2.97	2.44	2.58
	1.0	3.38	3.33	3.18	2.32	2.74
Kodak Tri-X	0.1				0.75	0.99
	0.3				1.02	1.25
	0.8				1.15	1.40
	1.0				1.35	1.46
Kodak Plus-X	0.1	(0.61)	(0.65)	(0.95)	0.46	0.45
	0.3	0.88	0.86	0.93	0.76	0.73
	0.8	1.36	1.03	0.92	0.93	0.93
	1.0	(1.23)	(1.06)	(0.92)	0.94	0.92
Kodak Panatomic-X	0.1	(0.68)	(1.02)	(1.57)	0.43	0.51
	0.3	1.01	1.14	1.5	0.63	0.70
	0.8	1.57	1.54	1.78	0.75	0.79
	1.0	1.73	1.67	1.94	0.88	0.81

Note: 1) The first three columns of G were obtained from the Table of [23], the last two columns from [24], Table C, by interpolation (and extrapolation).
2) The values following G are the diameters of the scanning apertures used.

IV. Resolving Power and Information Capacity

A. Resolving Power

The classical quality factor for optical systems is resolving power. Basically, resolving power refers to the reciprocal of the separation of two object elements that are barely "resolved", *i.e.*, recognized as two. It is not surprising to find that the value obtained for resolving power depends heavily on the type of test pattern [25] used as well as on the modulation [25], [26]. Although many patterns have been used for resolving power determination, the periodic bar (or columnar) pattern, consisting of a series of bars and gaps of equal width, seems to be by far the most popular. This popularity may be only partly due to the fact that this pattern, consisting of three bars, appears to give values of resolving power higher then those yielded by any other of the conventional test patterns of high-contrast targets [25].

A rational derivation of the visual resolving power (K) for a sinusoidal test pattern in terms of the emulsion granularity (G) and the transfer functions (T_e, T_a) of the emulsion and the aerial image, respectively, resulted in the expression [27]

$$a\gamma T_e(K) = cKG, \tag{3}$$

where

$$a = T_a(K)a_0$$

is the modulation of the aerial image, a_0 is that of the test object, and c is a constant. When G is in microns and K in cycles/mm, $c = 0.003$, approximately.

A similar analysis can be made for resolving power when a sinusoidal density pattern is scanned with a slit photometer [28]. In this case, resolution can be defined objectively in terms of the probability (P) that granularity will make a density trough exceed the adjacent peak when averaged over a photometer slit of length b. (It is assumed that the slit width is chosen to maximize K.) This analysis yields an expression of the form:

$$a\gamma T_e(K) = c'G\sqrt{K}$$

where

$$c = 2.4\,\phi^{-1}(1/2 - P)/\sqrt{b}$$

and

$$\phi(x) = \frac{1}{\sqrt{2\pi}}\int_0^x e^{-u^2/2}\,du. \tag{4}$$

The shortcomings of the "resolution" concept as a quality factor are two-fold.

1) Knowledge of resolution with one pattern and one contrast does not permit prediction of contrast with other patterns and contrast values.
2) The value of resolving power does not correlate well with the system performance well above the threshold. A system with a high resolving power may degrade coarser detail so severely, that it may be considerably inferior in over-all performance to a system with a considerably lower resolving power.

B. Information Capacity

A value which may be more useful to the engineer in rating optical systems is their information capacity. In a linear system this has the form [29]

$$c = \frac{1}{2}\iint \log_2\left(1 + \frac{T^2 W_E}{W_G}\right) dk_x\, dk_y \tag{5}$$

where W_E, W_G are the Wiener spectra of the messages and the granular structure, respectively, and T is the transfer function of the emulsion.

In the case of photographic emulsions, this calculation is complicated by the nonlinear character of the system response and especially by the limited density range of the photographic emulsions. An approximate solution has been found, and, assuming W_G constant and T of the form

$$T(k) = b^2(b^2 + k^2)^{-1},$$

and making allowance for the peak limitation, the information capacities of various photographic emulsions have been calculated [17]. The results are shown in Table II.

Also of interest is the *information efficiency* of photographic emulsions. This is defined as the information capacity per unit power and has been compared with that of other photodetectors [30], [31]. Results are presented in Table IV.

TABLE IV

Detector	Information Capacity (bit/photon)
1P21 Phototube	0.1
Human Vision	7.2×10^{-3}
Royal-X Film	1.3×10^{-3}
6849 Image Orthicon	2.0×10^{-5}
Heat Detector	1.6×10^{-8}

V. Miscellaneous

A. Special Cases

Optical system data in the form of transfer function and granularity permit the calculation of performance in all applications. Two such applications, which may be of special interest and have been treated in the literature, are 1) the minimum element size in a photographic computer storage when the number of distinguishable levels per element is prescribed [32], and 2) the minimum energy necessary to produce a sensible effect [33].

Representative results show that about 350,000 binary elements may be accommodated on a square centimeter of Kodak Plus-X emulsion with a very small error rate. The noise equivalent energy for a highly concentrated exposure is about 5.5×10^{-9} ergs for Kodak Plus-X film and 1.5×10^{-9} ergs for Kodak Royal-X film.

B. Factors Affecting Transfer Function

1) Spectral Character of Light: It should be noted that the transfer function of both lenses and photographic emulsions must be expected to depend heavily on the wavelength of the light making up the image.

In the lens system, this is due to the change of the refractive index, with wavelength, of the optical media. Although, in a well-corrected system, this change should be negligible over the design spectral region, this effect must be considered when the system is used with radiation outside this region. (Compare, *e.g.*, Fig. 3.)

The transfer function of the emulsion is determined primarily by the scattering of light and its absorption in the emulsion. These processes, too, must be expected to be sensitive to changes in wavelength [7], [34].

2) Adjacency Effects: Another important factor affecting the form of the transfer function in a photographic emulsion is due to the diffusion of the developing solutions through the emulsion and its surface. As a result, areas close to a highly exposed region will tend to be underdeveloped due to the diffusion of exhausted developing solution there, while the reverse will be true of regions close to unexposed areas.

This phenomenon is called adjacency effect and accounts for the dark fringe observed near the edge on the dense side of a knife-edge exposure and the light fringe on the light side of such an edge. These fringes lend the typical knife-edge exposure density trace, the appearance of "over-shoot" on both sides of the transition. In the case of a small bright object on a dark background, adjacency effects cause the density in the image to be significantly higher than would be expected from the Density–Log Exposure characteristic alone [35].

In the transfer function, this phenomenon shows up as an increase of the response with frequency over a certain range of spatial frequencies. It is understandably sensitive to agitation of the solution during development.

C. Spatial Filtering

When a coherent "parallel" beam of light enters an aperture with a given transmittance distribution, the emerging beam will have an angular distribution of amplitude which is the (two-dimensional) Fourier transform of the aperture transmittance distribution. Thus the image of a point source will have an intensity distribution representing the square of the Fourier transform of the imaging lens aperture transmittance distribution.

A second lens may now be placed into the plane of this image, to image the first aperture on a screen. If this second lens is optically perfect and large enough, a faithful rendition of the transmittance distribution of the first aperture will appear on the screen. On the other hand, any absorption of light at this second lens will reduce the amplitude of certain spatial frequency components in the image. The absorbed frequency components will be the ones corresponding to the location of the absorbing region. In this manner, the image may be frequency filtered simply by placing an opacity mask at the aperture of the imaging lens [36], [37].

Such filtering may be used, for instance, to increase the signal-to-noise ratio in an image if the spatial frequency spectrum of the signal differs in a known manner from that of the noise.

The granularity spectra of several photographic emulsions have been measured by this method [38].

D. CRT Light Output

When a *CRT* raster serves as input to an optical system, a knowledge of its brightness is required. It is important to recognize that brightness may depend heavily not only on the raster area but also on spot size and writing speed, at least when the spot-dwell time is short compared to the decay time [39].

It has been found that in such cases the phosphor efficiency is a function of the energy density delivered during a single exposure of a phosphor element. In addition to the beam power and raster parameters, the expression for the raster brightness involves two constants of the phosphor which may be readily determined from a single plot of brightness vs beam current obtained with any given raster [39].

VI. Conclusions

Optical systems, as electronic communications systems, suffer from limitations analogous to bandwidth and noise, the corresponding analogs being optical transfer function and granularity.

A knowledge of the transfer function of an optical system for the spectral region used permits the calculation of the image for any given object, *i.e.*, the system output for any given input. In addition, the characteristics of the eye must be known for a visual system; granularity and the Density–Log Exposure curve (for exposure time, developing process, and spectral region) must be known in the case of photographic recording. For regions of units gamma, linear communications systems analysis techniques can be used, especially for "one dimensional" images. By working with exposure, rather than density, this can be extended to nonunit gamma.

The classical concept of "resolving power" is far less valuable in analyzing such a system. Satisfactory analytic techniques for predicting the transfer function of optical systems seem to have been developed.

VII. Acknowledgment

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