On Contrast in Rear-Projection Screens

By LEO LEVI

Rear-projection screens are known to give poorer preformance in rendering large luminance differences in images than do reflecting screens. This seems to be due primarily to internal reflections in the rear-projection screens. This paper analyzes the effect of internal reflection on the rendition of luminance differences and also the improvement obtainable when an absorbing screen base is used. Some experimental results are presented.

Introduction

In small-scale rear-projection systems, the screen usually consists of a diffusing layer coated onto a clear, rigid substrate, such as glass. In such systems, it is readily observed that the attainable luminance range (contrast*) is less than that in front projection. In the following, one important factor responsible for this difference is analyzed and a remedial procedure proposed.

It should be noted that in such small-scale systems it may be important to maintain a maximum of contrast to avoid loss of information. This may happen, for instance, in such technological applications as aerial photography, x-ray recordings and radar recordings. Since in such systems it is often possible to eliminate some of the contrast-reducing causes, such as stray light, it should be of interest to investigate one of the major remaining causes.

Some of the reduction of contrast in projection is due to light incident on bright areas and then scattered, spreading over the rest of the screen, including areas that should be dark. This portion of the scattering can be largely avoided in front projection where the projected light need not enter the screen. In rear projection, however, the light must pass through the substrate and internal reflections will be unavoidable. Whenever light strikes the clear (i.e., smooth) surface, a portion of it is reflected partially or totally, depending on the angle of incidence and the refractive index of the screen material. The reflected light is, then, partially scattered toward the viewer whenever it strikes the diffusing surface. When it happens to strike a region in which the image is dark, the contrast is reduced. A similar phenomenon has been found to lower the contrast obtainable in cathode-ray tubes.[1,2]

One method of reducing this effect has been used successfully with cathode-ray tubes and is applicable to rear-projection screens also. It requires that the screen material be tinted so as to absorb the light partially; as a result, the internally reflected light — with its longer path inside the screen — will be absorbed more than the direct light.

By way of illustration, consider a screen with normal opacity of 2 (density of 0.3) and an index of refraction of 1.5. The mean path length of the internally reflected light is found to be almost five times the screen thickness, so that it is attenuated by a factor

$$O_r = 2^{4.8} = 27.5$$

as compared to the factor of about

$$O_d \approx 2.2$$

for the directly transmitted light, so that there results a net luminance ratio increase of

$$Q = O_r/O_d = 12.5. \qquad (1)$$

The effect of absorption in the screen under these conditions has been analyzed extensively by Law,[1] considering, however, primarily only the totally reflected light. (He approximated the effect of partially reflected light, but only in the case of the clear screen.) The following analysis includes the effect of the partially reflected light because it does contribute appreciably to the total scattered light, especially in the more opaque screens; e.g., for a screen material of refractive index $n = 1.5$ and a transmittance of 0.2, approximately a third of the scattered light is due to partial reflection. Numerical results obtained on an IBM 7090 computer are included.

Analysis

Consider a bright image point on the diffuse surface of a rear-projection screen — at A in Fig. 1.† Of the light (Φ_A) radiated upward from A, a certain portion (Φ_0) will be transmitted at the upper surface and a certain portion will be reflected back toward C.

Note that we may write the luminance at A, relative to the normal luminance, as

$$L(\Theta) = c(\Theta)\cos\Theta \qquad c(0) = 1, \qquad (2)$$

where $c(\Theta)$ is a directionality factor, which is unity for a Lambert law diffuser.

We may now write the flux radiated into a differential cone shell, $\Theta \pm \frac{1}{2}d\Theta$ (see Fig. 1B), as

$$d\Phi_A = (2\pi \sin\Theta\, d\,\Theta)(c\cos\Theta) \qquad (3)$$
$$= 2\pi c \sin\Theta\cos\Theta\, d\,\Theta.$$

For the portion of this flux leaving the screen we find, then,

$$d\Phi_0 = e^{-\alpha s}[1 - r(\Theta)]\, d\Phi_A, \qquad (4)$$

where α is the optical absorption constant of the screen, s is the path length, AB, to the point B at the clear surface and $r(\Theta)$ is the reflectivity of the clear surface.

The reflected light reaching the annular ring at C is

$$d\Phi_C = e^{-2\alpha s}\, r(\Theta)\, d\Phi_A. \qquad (5)$$

We note that, for a screen thickness t,

$$s = t/\cos\Theta \qquad (6)$$

and we substitute this and Eq. (3) into Eqs. (4) and (5) to find

$$d\Phi_0 = 2\pi c \sin\Theta\cos\theta\, e^{-\alpha t/\cos\Theta} \times [1 - r(\Theta)]\, d\Theta, \qquad (7)$$

$$d\Phi_C = 2\pi c \sin\Theta\cos\theta\, e^{-2\alpha t/\cos\Theta}\, r(\Theta)\, d\Theta. \qquad (8)$$

The portion of $d\Phi_C$ re-radiated in the upward direction will be

$$d\Phi_{A1} = r'(\Theta)\, d\Phi_C, \qquad (9)$$

where $r'(\Theta)$ is the reflectivity of the diffuse surface and Θ is the angle of incidence.

† The present analysis is based on a screen whose diffuse surface faces the projector; however, an analogous analysis will show that the theoretical results will be identical when the diffuse surface faces the viewer.

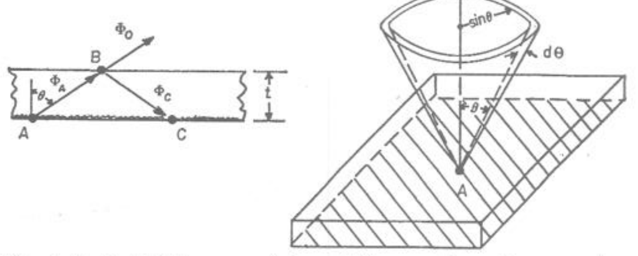

Fig. 1. Left: (A) Image point on diffuse surface of rear-projection screen. Right: (B) Flux radiated into a differential cone shell.

A contribution submitted first on April 17, 1967, and in revised form on September 28, 1967, by Leo Levi, Consulting Physicist, 435 Fort Washington Ave., New York, N.Y. 10033. This work was supported by IBM Corp., ASDD.

* In this paper, the term "contrast" will be used for the luminance range in an image.

Copyrighted, 1967, by the Society of Motion Picture and Television Engineers, Inc.

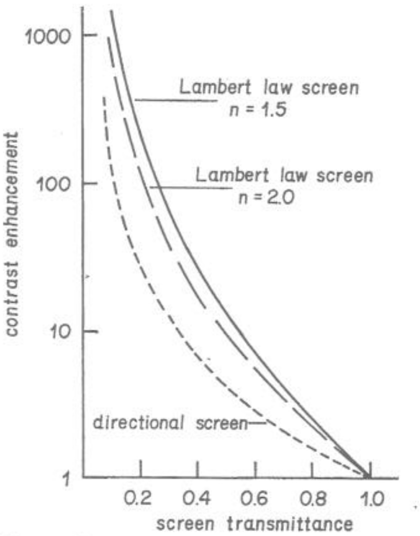

Fig. 2. Contrast enhancement in dense rear-projection screens.

(Note that direction is no longer preserved beyond this point.) Of this light, in turn, a certain portion ($d\Phi_{01}$) will escape and another part ($d\Phi_{C_1}$) will be returned to the screen again. When integrated over all angles Θ, we find, then, that these will be related to Φ_{A1} as:

$$\Phi_{01}/\Phi_{A1} = \Phi_0/\Phi_A, \qquad (10)$$

$$\Phi_{C1}/\Phi_{A1} = \Phi_C/\Phi_A. \qquad (11)$$

Equations (10) and (11) are based on the assumption that the total diffused light re-radiated at C will be directionally distributed similarly to the original light radiated at A. This assumption is quite reasonable, especially with highly diffuse screens.

The light transmitted the next time around (Φ_{02}) will be related to Φ_{01} as

$$\Phi_{02}/\Phi_{01} = \Phi_{01}/\Phi_0 = \Phi_{A1}/\Phi_A = R, \qquad (12)$$

where the second equality follows from Eq. (10).

Thus, the fraction of light leaving the screen from point A without reflection will be related to the total light which finally leaves the screen in the forward direction.

$$\Phi_0/\Phi_T = \Phi_0/(\Phi_0 + \sum_{i=1}^{\infty} \Phi_{0i}).$$

On dividing both numerators and denominators on the righthand side by Φ_0, we find

$$\Phi_0/\Phi_T = (\sum_i R^i)^{-1} = 1 - R \qquad (13a)$$

and, therefore, the ratio of scattered light to the total light will be

$$\Phi_s/\Phi_T = (\Phi_T - \Phi_0)/\Phi_T = R \qquad (13b)$$

We are, of course, interested primarily in how this scattered light affects the image contrast. This, in turn, depends on the distribution of illumination in the image on the screen. To rate screen performance, we analyze here the worst possible case: a uniformly illuminated screen with a small unilluminated patch in the middle. What will be the luminance ratio observed for the dark patch relative to the rest of the screen? This, we find, is R (see appendix).

To find R, we evaluate Φ_A, Φ_{A1} by integrating the corresponding differentials. Thus

$$\Phi_A = 2\pi \int_0^{\pi/2} c(\Theta) \sin\Theta \cos\Theta \, d\Theta,$$

$$\Phi_{A1} = 2\pi \int_0^{\pi/2} r'(\Theta)\, d\Phi_C$$

$$= 2\pi \int_0^{\pi/2} c(\Theta) r(\Theta) r'(\Theta) \sin\Theta \times \cos\Theta \, e^{-2\alpha t/\cos\Theta} \, d\Theta \qquad (14)$$

With $c(\Theta) \equiv 1$, i.e., Lambert-law diffusion,

$$\Phi_A = \pi, \qquad (15)$$

and the expression for R (Eq. 12) becomes

$$R = 2 \int_0^{\pi/2} r(\Theta) r'(\Theta) \sin\Theta \cos\Theta \times e^{-2\alpha t/\cos\Theta} \, d\Theta. \qquad (16)$$

If we assume, further, that the diffuse reflectivity, $r'(\Theta)$, is constant with the angle of incidence, we may remove it from the integral. Then, writing

$$I = 2 \int_0^{\pi/2} r(\Theta) \sin\Theta \cos\Theta \, e^{-2\alpha t/\cos\Theta} \, d\Theta, \qquad (17)$$

we find the ratio of total to scattered light to be

$$C = 1/R = 1/r'I. \qquad (18)$$

This is also the upper limit on the attainable luminance ratio for the small dark patch in the uniform field.

The integral of Eq. (17) must be broken up into two ranges: For angles less than the critical angle (Θ_0), $r(\Theta)$ is given by Fresnel's formulas[3] and for angles equal to, and greater than, Θ_0, there is total internal reflection, making $r(\Theta)$ equal to unity. Thus

$$r(\Theta) = r_F(\Theta) = (1/2) \times \left[1 + \frac{\cos^2(\Theta + \Theta')}{\cos^2(\Theta - \Theta')}\right] \times \frac{\sin^2(\Theta - \Theta')}{\sin^2(\Theta + \Theta')} \qquad \Theta < \Theta_0 \qquad (19a)$$

$$r(\Theta) = 1 \qquad \Theta > \Theta_0 \qquad (19b)$$

where $\Theta_0 = \sin^{-1}(1/n)$ and (20a)

$\Theta' = \sin^{-1}(n \sin\Theta)$ is the angle of refraction. (20b)

In addition, it is more convenient to express the exponential in Eq. (17) in terms of the normal screen transmittance

$$T = e^{-\alpha t}. \qquad (21)$$

On substituting Eqs. (19) and (21) into (17), we find

$$I(T) = 2\left[\int_0^{\Theta_0} r_F(\Theta) \sin\Theta \cos\Theta \, T^{2/\cos\Theta} \, d\Theta + \int_{\Theta_0}^{\pi/2} \sin\Theta \cos\Theta \, T^{2/\cos\Theta} \, d\Theta\right]. \qquad (22)$$

This is the value that must be substituted into Eq. (18) to find C, the ratio of total to scattered light, i.e., the maximum contrast in the case of the small dark patch in the uniform field. I is a function of the refractive index, but is even more significantly affected by the screen transmittance.

Unfortunately, the integrals of Eq. (22) could not be evaluated in closed form. When the screen is perfectly clear ($T = 1$), the second integral is readily evaluated‡:

$$\int_{\Theta_0}^{\pi/2} \sin\Theta \cos\Theta \, d\Theta = (\tfrac{1}{2})(1 - \sin^2\Theta_0) = (n^2 - 1)/2n^2. \qquad (23)$$

However, even in this simple case, the first integral could not be evaluated.

To obtain numerical answers, the integrals of Eq. (22) were evaluated by means of Simpson's rule.

Maximum luminance ratios (C) for refractive indexes ranging from 1.45 to 2.0 are shown in Table I. These are given for $r' = 0.5$; ratios for other values of screen reflectance are readily derived from these by reference to Eq. (18).

The contrast enhancements due to screen absorption:

$$Q = C(T)/C(1) = I(1)/I(T) \qquad (24)$$

are also tabulated there. These are, of course, independent of the diffuse screen reflectance, r'. They are shown in Fig. 2, for $n = 1.5$ and $n = 2.0$.

The effect of diffuse screen reflectance is illustrated in Fig. 3, which shows the maximum attainable contrast (C), as a function of diffuse screen reflectance (r') for a perfectly clear screen material ($T = 1$).

Most practical rear-projection screens are far from perfectly diffuse.[4] To cover this situation, we calculate here the contrast enhancement obtainable when the screen is extremely directional. Any practical screen should then fall between these new results and those obtained for the Lambert's law diffuser.

We simulate the directionality by assuming that $c(\theta)$ is negligible for $\theta < \Theta$, where Θ is chosen small enough to permit setting $\cos\Theta < 1$. We also assume that, within the cone with half-angle Θ, the *reflectance* of the diffusing surface is sufficiently uniform to permit taking r' outside the integral. With these as-

‡ For $T < 1$, the second integral can be transformed into an "exponential integral" which cannot be evaluated in closed form but has been tabulated.

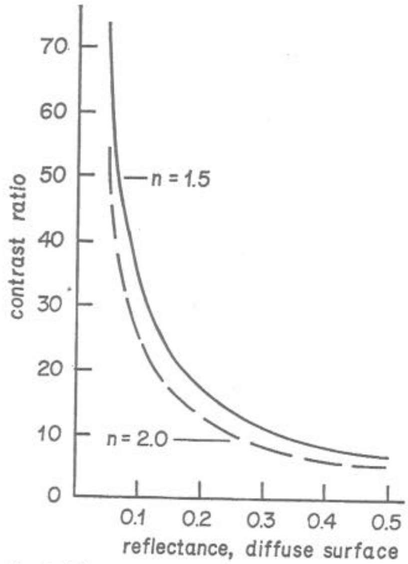

Fig. 3. Clear rear-projection screen; maximum contrast with small, dark image; Lambert-law reflectance.

sumptions, and noting Eq. 21, Eqs. (14–18) become

$$\Phi_{A1} = 2\pi r(0) r' T^2 \int_0^{\Theta} c(\Theta) \sin\Theta \, d\Theta$$

$$= 2\pi r' T^2 \left(\frac{n-1}{n+1}\right)^2 \times \int_0^{\Theta} c(\Theta) \sin\Theta \, d\Theta, \quad (14')$$

$$\Phi_A = 2\pi \int_0^{\Theta} c(\Theta) \sin\Theta \, d\Theta. \quad (15')$$

$$R = \Phi_{A1}/\Phi_A = r' T^2 \left(\frac{n-1}{n+1}\right)^2, \quad (16')$$

$$C = 1/R = \left(\frac{n+1}{n-1}\right)^2 / (r' T^2), \quad (18')$$

and Eq. (24) becomes

$$Q = C(T)/C(1) = 1/T^2. \quad (24')$$

A plot of Eq. (24) is shown as a dotted line in Fig. 2. Any actual contrast enhancement may be expected to fall somewhere between this line and the Lambert-law line correspondng to the refractive index of the screen base material.

Experimental Results

To check the practical implications of the above analysis, the maximum luminance ratio of a number of screen materials used in rear projection was measured under the extreme conditions described above (small dark patch in uniformly illuminated field). Ground glass was found to be capable of a luminance ratio of 39, while two good "high contrast" screens (using refractive nonuniformities[4] and exhibiting a very low value of r') were capable of ratios 85 and 53.

The latter material was available on a 60% transmitting neutral plexiglas base. In that form it exhibited a luminance ratio of 480 — an improvement by a factor of nine. Incidentally, the luminance ratio in the aerial image was 630.

Table I. Maximum Contrast and Contrast Enhancement in Dense Rear-Projection Screens.

Trans-mittance	$n = 1.45$ C_{mx} (0.5)	C_{mx}/C_{mx0}	$n = 1.50$ C_{mx} (0.5)	C_{mx}/C_{mx0}	$n = 1.75$ C_{mx} (0.5)	C_{mx}/C_{mx0}	$n = 2.0$ C_{mx} (0.5)	C_{mx}/C_{mx0}	Directional screen C_{mx}/C_{mx0}
0.1	11,348	1,610	9,257	1,386	4,717	848	3,224	645	100.00
0.2	1,573	223	1,326	198.9	757.6	136.1	554	110.8	25.00
0.3	468.2	66.4	404.0	60.6	249.5	44.9	190.7	38.14	11.11
0.4	191.6	27.19	168.4	25.2	110.5	19.88	87.28	17.46	6.67
0.5	93.43	13.25	83.39	12.49	57.48	10.34	46.66	9.33	4.00
0.6	50.76	7.20	45.92	6.88	33.04	5.94	27.46	5.49	2.78
0.7	29.62	4.20	27.12	4.06	20.28	3.65	17.21	3.44	2.04
0.8	18.11	2.57	16.77	2.51	12.99	2.34	11.24	2.25	1.56
0.9	11.36	1.61	10.63	1.59	8.53	1.53	7.52	1.50	1.23
1.0	7.05	1.00	6.67	1.00	5.56	1.00	5.00	1.00	1.00

Note: C_{mx} (0.5) is the maximum contrast obtainable with screen reflectance = 0.5.
C_{mx0} is the maximum contrast obtainable with a clear screen with the same reflectance.

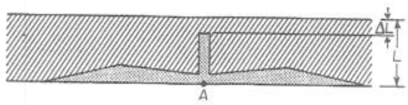

Fig. 4. Illumination due to two hypothetical sources.

Conclusions

In conclusion, it appears that screen materials with low reflectance are essential to achieving high image contrast in rear projection. But even these seem to limit performance to ratios below 100. The highest luminance ratios may be obtained by using absorbing ("tinted") screen base materials.

It must, of course, be noted that these techniques provide a gain only in contrast *potential*. This gain can be utilized only if other factors degrading contrast are eliminated. Chief among these are ambient illumination from other light sources and light scattered at the surfaces of the projection lens.

Acknowledgments: The experimental work was done in a highly efficient manner by A. Dreyfoos of the Photo Electronics Corp., Byram, Conn. The cooperation of L. M. Heath of Polacoat, Inc., Blue Ash, Ohio, in providing test samples is also greatly appreciated.

APPENDIX

Relationship Between the Contrast Maximum and the Scattered Light Ratio

Consider a screen which is uniformly illuminated (luminance L) by means of two light sources: S_A illuminates uniformly a small patch at A, and nothing else; S_0 illuminates uniformly the rest of the screen, but not the patch at A.

Due to internal reflections, the luminances resulting will extend beyond the assigned regions. However, the luminance due to S_0 must complement that due to S_A to the uniform level, as shown in Fig. 4 in schematic form. There the cross-hatched portion shows the luminance due to S_A and the shaded portion that due to S_0.

Since the total flux density at A is the same as that on the rest of the screen, the flux missing at A, which is the scattered part, equals the flux scattered into A from source S_0. Assuming the patch to be chosen sufficiently small for the scattered light in it to be uniformly distributed, the scattered light is related to the total incident light as

$$R = \Delta L / L,$$

where ΔL is the decrease in luminance at A due to scattering in the absence of S_0. Hence, when the patch at A remains unilluminated (S_A eliminated) — corresponding to our "dark patch in a uniform field" — the luminance ratio will again be given by

$$\Delta L / L = R.$$

References

1. R. R. Law, "Contrast in kinescopes," *Proc. IRE*, *27:* 511–524, Aug. 1939.
2. J. H. Haines, "Contrast in CRT's," *Tele. Tech.*, *12:* 100, June 1953.
3. M. Born and E. Wolf, *Principles of Optics*. Pergamon Press, New York, 1959 (see Sec 1.5.2).
4. P. Vlahos, "Selection and specification of rear-projection screens," *Jour. SMPTE*, *70:* 89–95, Feb. 1961.