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NOTES AND DISCUSSION

Further Modification of e/m Experiment

George W. Ficken, Jr.

Department of Physics, Cleveland State University, Cleveland, Ohio

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For several years now, we have been using a method, first reported in the December 1960 issue of this Journal,[1] to heat the filament of the Welch e/m tube.[2] The method has additional advantages over the one described in the February 1967 issue.[3] The 1967 note describes the use of ac in place of dc to overcome the troublesome effects of (1) the filament voltage on the accelerating potential and (2) the magnetic field of the filament. Unaware of the 1960 note, we tried first dc and then ac, but the latter was discarded because of the resulting low intensity of the beam when spread out by the magnetic field of the filament. To overcome this intensity problem, our version has the output of a 10-V filament transformer connected to a simple half-wave rectifier, which employs a silicon diode. The heating current is controlled by using a 2.3-Ω rheostat as a series dropping resistor.

The big advantage is that for half of each cycle the filament emits electrons without the above-mentioned troublesome effects. By far the most intense part of the beam is that half produced during the "off" half of the cycle, and this part is focused on the target pins and used for the actual experiment. The other half of the beam is deflected out of the main plane of the experiment and disregarded. Thus, the best features of both the dc and ac methods are retained.

An interesting sidelight is to ask the students to observe the unused half of the beam and to explain why the portion (produced at the peak of the sine wave) which is deflected farthest out of the main plane is also the most intense portion of this half of the beam.

A comment on errors caused by a manufacturing defect might also be in order. In some of our tubes, the cylindrical anode is rotated by as much as approximately 5° from its proper orientation, so that a straight beam emerges at, say, a 95° angle from the support arm for the target pins. Not realizing this, the students, when "balancing" the earth's field, tend to cause appreciable error by sending too much current through the Helmholtz coils and producing a slight arc which looks roughly perpendicular to the support arm. The alternative is to obtain a straight beam, with the correct balancing current (one way is to center the electron beam within the light beam which the glowing filament sends through the slit to the tube wall), and then bend the beam through 185° to the pins. A check was made on the error caused by (1) the distance to the pin being a chord rather than a diameter and (2) the corresponding change in final current through the coils. We found that by using the smaller current, correct for the now larger circle, and by merely dividing the manufacturer's value for the pin distance by cos 5°, the error could be eliminated within the accuracy of the angle measurement. Neglecting this correction, which amounts to less than 1%, is justified in view of the ammeter tolerances inherent in measuring the Helmholtz currents.

[1] R. W. Christy and W. P. Davis, Jr., Am. J. Phys. 28, 815 (1960).

[2] e/m tube, Cat. 623, The Welch Scientific Company, 1515 Sedgewick St., Chicago, Ill. 60610.

[3] M. Iona, H. Westdal, and P. Williamson, Am. J. Phys. 35, 157 (1967).

The "Twin Paradox" Revisited

Leo Levi

The City College of New York, New York, New York

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The "twin paradox" demonstrates an asymmetry between inertial and noninertial reference systems. It has been pointed out frequently[1-3] that no real paradox exists, even in terms of special relativity. However, it does not seem to have been shown quantitatively—in purely special relativity terms—how both twins agree.

It is the purpose of the present paper to analyze quantitatively the asymmetry introduced in special relativity when an accelerated reference system is used. The results of this analysis will, then, enable us to resolve the "twin paradox", and, incidentally, to establish a method enabling an observer to measure his own acceleration using purely kinematical observations.

Accelerated systems in special relativity. In this section it is shown how accelerated systems can be treated in terms of special relativity to yield quantitatively consistent results and how special relativity does predict kinematical asymmetry when an accelerated reference system is used. As pointed out previously[1], this asymmetry involves the synchronization (simultaneity) of clocks.

Specifically, consider two reference systems, both at rest, and each carrying a pair of widely separated, synchronized clocks. When System A is accelerated (parallel to the direction defined by the pairs of clocks), it will be found that each pair of clocks will remain internally synchronized when viewed from the inertial reference frame I but that the synchronization will have been destroyed as viewed by an observer in A.

To demonstrate this result, we consider the clocks arranged with C_{A1} next to C_{I1} and C_{A2} next to C_{I2}, where $C_{A1,A2}$ is the clock pair in A and $C_{I1,I2}$ that in I. Now consider an instantaneous acceleration—i.e., one occurring in an interval Δt:

$$\Delta t << T, \quad L/\Delta v,$$

where T is the order of magnitude of the time intervals measured in the experiment; L is the distance between the clocks C_{I1}, C_{I2}, and Δv is the total change in velocity of A. We assume that, in I, the readings of all the clocks will then not change significantly during the acceleration. To an observer in I, the clocks $C_{I1,I2}$ clearly remain synchronized—nothing concerning them has changed. Thus C_{A12} (who kept step with $C_{I1,2}$, resp.

TABLE I. Twin paradox.[a]

Event	Earth system	Astronaut system
Start	$t_{0E} = t_{0S} = 0$ $t_{0A} = 0$	$t'_{0E} = 0$ $t'_{0A} = 0$ $t'_{0S} = Lv/c^2$
Arrival at S	$t_{1E} = t_{1S} = L/v$ $t_{1A} = RL/v$	$t'_{1E} = R\,t'_{1A} = R^2L/v$ $t'_{1A} = L'/v = RL/v = t''_{1A}$ $t'_{1S} = t'_{0S} + t'_{1E} = Lv/c^2 + R^2L/v = t''_{1S}$
Instantaneous reversal		$t''_{1E} = t'_{1S} + \frac{Lv}{c^2} = \frac{2Lv}{c^2} + \frac{R^2L}{v}$.
Return To earth	$t_{2E} = t_{2S} = 2L/v$ $t_{2A} = 2RL/v$	$t''_{2E} = t''_{1E} + t'_{1E} = 2\left[\frac{Lv}{c^2} + \frac{R^2L}{v}\right] = 2L/v$ $t''_{2A} = 2t'_{1A} = 2RL/v$

[a] $R = [1 - (v/c)^2]^{1/2}$. Subscripts: E—clock on earth, A—clock on space ship, S—clock on star. Prime: Astronaut system.

during the "instantaneous" acceleration), too, remain synchronized to this observer. However, clocks separated by L and synchronized in one inertial system will exhibit a time difference.

$$L = \mathrm{v} \cdot \mathrm{L}/c^2$$

to an observer moving with velocity v with respect to it. Thus, the clocks appearing synchronized to the observer in I will appear to differ by a time

$$\tau = vL/c^2$$

to the observer in A.

We note that this enables the observer in A to measure his acceleration by purely kinematic means. To determine the change in his velocity, he must only measure the time difference shown on his previously synchronized clocks!

We conclude that, in an inertial reference frame, a time interval measured on an accelerated clock, will still be simply:

$$\delta t_A = R\,\delta t_I$$

in I, where $R = [1 - (v/c)^2]^{1/2}$.

However, for an accelerated observer, the time interval shown by a nonaccelerated clock, will be in the limit

$$\delta t_I = R\,\delta t_A + \delta\tau = R\,\delta t_A + (L/c^2)\delta v = (R + La/c^2)\,\delta t_A$$

in A, where

$$a = \lim_{\delta t\to 0} \frac{\delta v}{\delta t}$$

is the acceleration; even clocks in his own system will run at different rates:

$$\delta t_{A2} = (La/c^2)\,\delta t_{A1}\,,$$

in A. This accounts for the Doppler shift observed within an accelerated system.

The "twin paradox." By way of illustration, we use this asymmetry to treat the "twin paradox." This is usually stated as follows: One of a pair of twins leaves earth in a space ship traveling rapidly to a distant star. There he turns around and arrives back on earth many years later. From the point of view of the twin who remained on earth, the astronaut will have aged more slowly (time dilation) during the trip; and therefore the earth twin will not be surprised to find his brother much younger than himself. However, from the astronaut's point of view, the *earth* twin will have aged more slowly during the trip—and yet, surprisingly, he finds him older instead of younger.

Our purpose here is to show that both twins can evaluate the correct final ages using exclusively special relativity concepts. Both during the trip away and back, the astronaut will observe the earth twin as aging more slowly. However, during the acceleration at the turning point, he will observe him to age significantly, even if the acceleration takes place essentially instantaneously. For instance, if the astronaut travels with constant velocity v throughout—except at the turning point, where his velocity reverses "instantaneously—he will, there, observe the earth twin to age "instantaneously" by an amount

$$\tau = 2\,v\,L/c^2.$$

As a result of this, upon the astronaut's arrival back on earth, both twins agreed that the earth twin is older than the astronaut by

$$\Delta t = 2\,L\,(1-R)/v.$$

The detailed calculations, as made by each twin, are presented in Table I.

Note added in proof: A very thorough treatment of the clock paradox has been published by G. Builder [Austral. J. Phys. 10, 246 (1957)]. This treatment, however, does not seem to use the simultaneity relationship on which the (hopefully tutorially valuable) present table—last column—is based. Also, the possibility of the test for acceleration for an observer in an accelerated system is, therefore, not mentioned there.

1 R. H. Romer, Am. J. Phys. 27, 131 (1959).
2 G. D. Scott, Ref. 1, p. 580.
3 G. Builder, Ref. 1, p. 656.