

Tables of the Modulation Transfer Function of a Defocused Perfect Lens

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Tables are presented giving the MTF of an aberrationless defocused lens with circular aperture for objects illuminated with incoherent monochromatic light. MTF values are given to six decimal places for reduced spatial frequencies $\nu_r = 0(0.01)1$ and defocusing $\Delta = 0(0.1)0.4, 0.5(0.5)5, 10(5)50$, except that $5/\Delta$ is an upper limit on ν_r . Here Δ is measured in units of Rayleigh's $\frac{1}{4}\lambda$ tolerance on defocusing. The integral of the MTF and $(MTF)^2$ are also given for $\Delta \leq 5$. The numerical integration was performed by Simpson's rule over most of the range and by series expansion over the rest. An error analysis is appended. Tables of the line spread function for the focused case are also given.

The appended tables (I) give the modulation transfer function of the aberrationless lens with circular aperture in the presence of defocusing. The results apply to objects illuminated incoherently with monochromatic light.

The spatial frequency (ν_r) is given in reduced units, which are related to the spatial frequency (ν) by:

$$\nu_r = (\lambda' \nu / 2 \sin \theta) = \lambda' F \nu = (\lambda_0 \nu / 2A), \quad (1)$$

where λ' , λ_0 are the wavelengths of the light in the image space and a vacuum, respectively, θ is the half-angle of the cone subtended by the exit pupil at the image point, and F, A are the lens f /number and numerical aperture, respectively, as measured in the image space.

Defocusing (Δ) is measured in units of Rayleigh's $(\lambda/4)$ tolerance on defocusing, which is $(\lambda'/2 \sin^2 \theta)$. Thus,

$$\Delta = (2 \sin^2 \theta / \lambda') \delta = (2A^2 / n \lambda_0) \delta = (1 / 2F^2 \lambda') \delta, \quad (2)$$

where δ is the defocusing in the units of length in which λ is measured.

The required MTF has been shown¹ to have the form:

$$T(\nu_r, \Delta) = \frac{2}{\pi} \int_0^{(1 - \nu_r^2)^{\frac{1}{2}}} \int_{-(1 - y^2)^{\frac{1}{2}} + \nu_r}^{(1 - y^2)^{\frac{1}{2}} - \nu_r} e^{iay} dx dy, \quad (3)$$

where $a = 2\pi\nu_r\Delta$.

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By integrating first over x and expanding the resulting y integrand in a series of Bessel functions, Hopkins¹ showed that this MTF can be written in series form:

$$T(\nu_r, \Delta) = \frac{4}{\pi a} \cos a \nu_r \left\{ \beta J_1(a) - \sum_{n=1}^{\infty} \frac{(-1)^n}{2n} \sin(2n\beta) [J_{2n-1}(a) - J_{2n+1}(a)] \right\} \\ - \frac{4}{\pi a} \sin a \nu_r \sum_{n=0}^{\infty} \frac{(-1)^n}{2n+1} \sin[(2n+1)\beta] [J_{2n}(a) - J_{2n+2}(a)], \quad (4)$$

where $\beta = \cos^{-1} \nu_r$.

To avoid the inconvenience of evaluating the Bessel functions, the order of integration in Eq. (3) may be reversed, resulting in

$$T(\nu_r, \Delta) = \frac{4}{\pi a} \int_{\nu_r}^1 (1 - x^2)^{\frac{1}{2}} \cos[a(x - \nu_r)] dx, \quad (5)$$

after a simple transformation.

The appended tables were evaluated by integrating Eq. (5) by means of Simpson's rule with intervals of $1/1600$, except in the neighborhood of the upper limit. There the cosine was expanded in a Taylor series and integrated term by term.

The numerical data were obtained on an IBM 7094 computer operated at double precision. This method of evaluating the integral should yield an accuracy of better than 2×10^{-7} for $a \leq 10\pi$ (see Appendix). These tables give $T(\nu_r, \Delta)$ for the following ranges of ν and Δ :

$$\begin{aligned} \nu_r &= 0(0.01)1, \Delta \leq 5 \\ &= 0(0.01)5/\Delta, \Delta > 5, \\ \Delta &= 0(0.1)0.4, 0.5(0.5)5, 10(5)50. \end{aligned}$$

**Table Ia. MTF of Perfect Lens, Defocused
△ Rayleigh Units -2ΔF²λ**

Δ =	0	0.1	0.2	0.3	0.4
ν_r					
0.	1.000000	1.000000	1.000000	1.000000	1.000000
0.01	0.987268	0.987263	0.987249	0.987225	0.987191
0.02	0.974537	0.974519	0.974463	0.974371	0.974242
0.03	0.961809	0.961768	0.961648	0.961448	0.961168
0.04	0.949084	0.949015	0.948809	0.948466	0.947985
0.05	0.936365	0.936261	0.935950	0.935432	0.934707
0.06	0.923651	0.923508	0.923076	0.922356	0.921350
0.07	0.910946	0.910757	0.910191	0.909247	0.907926
0.08	0.898250	0.898012	0.897299	0.896111	0.894449
0.09	0.885563	0.885273	0.884404	0.882295	0.880931
0.10	0.872889	0.872544	0.871510	0.869789	0.867383
0.11	0.860227	0.859825	0.858621	0.856617	0.853816
0.12	0.847579	0.847119	0.845740	0.843446	0.840241
0.13	0.834946	0.834427	0.832871	0.830283	0.826668
0.14	0.822330	0.821752	0.82018	0.817132	0.813105
0.15	0.809733	0.809094	0.807182	0.804001	0.799561
0.16	0.797154	0.796457	0.794367	0.790892	0.786045
0.17	0.784596	0.783841	0.781576	0.777813	0.772564
0.18	0.772061	0.771248	0.769813	0.764766	0.759124
0.19	0.759548	0.758680	0.756078	0.751756	0.745734
0.20	0.747060	0.746138	0.741376	0.738788	0.732397
0.21	0.734598	0.733624	0.730708	0.728565	0.719121
0.22	0.722164	0.72114C	0.718077	0.712990	0.705910
0.23	0.709758	0.708688	0.705484	0.700167	0.692769
0.24	0.697382	0.696268	0.692933	0.687399	0.679703
0.25	0.685038	0.683883	0.680425	0.674689	0.666714
0.26	0.672726	0.671533	0.667962	0.662040	0.653808
0.27	0.660449	0.659221	0.655546	0.649453	0.640986
0.28	0.648208	0.646548	0.643179	0.636931	0.628253
0.29	0.636003	0.634715	0.630863	0.624476	0.615610
0.30	0.623838	0.622525	0.618598	0.612091	0.603059
0.31	0.611712	0.610378	0.606388	0.599776	0.590603
0.32	0.599628	0.598274	0.594232	0.587534	0.578244
0.33	0.587587	0.586220	0.582134	0.575366	0.565982
0.34	0.575599	0.574213	0.570924	0.563274	0.553820
0.35	0.563639	0.562254	0.558113	0.551258	0.541757
0.36	0.551736	0.550347	0.546194	0.539320	0.529795
0.37	0.539883	0.538493	0.534337	0.527460	0.517934
0.38	0.528079	0.526692	0.522544	0.515680	0.506175
0.39	0.516329	0.514946	0.510815	0.503981	0.494518
0.40	0.504638	0.503258	0.499153	0.492363	0.482963
0.41	0.492998	0.491628	0.487559	0.480827	0.471516
0.42	0.48146C	0.480059	0.476032	0.469374	0.460158
0.43	0.469881	0.468511	0.464576	0.458003	0.448909
0.44	0.458416	0.457106	0.453191	0.446717	0.437760
0.45	0.447014	0.445726	0.441878	0.435515	0.426713
0.46	0.435676	0.434413	0.430638	0.424398	0.415766
0.47	0.424404	0.423168	0.419474	0.413366	0.404918
0.48	0.413200	0.411992	0.408385	0.402420	0.394170
0.49	0.402065	0.400889	0.397373	0.391560	0.383521
0.50	0.391002	0.389858	0.386439	0.380787	0.372970
0.51	0.380C013	0.378902	0.375585	0.370101	0.362516
0.52	0.369099	0.368024	0.364812	0.359503	0.352159
0.53	0.352622	0.352724	0.354122	0.348993	0.341899
0.54	0.347505	0.346505	0.343515	0.338572	0.331734
0.55	0.336830	0.335868	0.332999	0.328241	0.321665
0.56	0.326239	0.325316	0.322559	0.317999	0.311691
0.57	0.315733	0.314851	0.3127212	0.307849	0.301812
0.58	0.305317	0.304474	0.301954	0.297791	0.292027
0.59	0.294949	0.294188	0.291791	0.287825	0.282337
0.60	0.284757	0.283995	0.281719	0.277954	0.272740
0.61	0.274619	0.273898	0.271743	0.268177	0.263239
0.62	0.264579	0.263899	0.261864	0.258496	0.253832
0.63	0.254640	0.253999	0.252084	0.248913	0.244520
0.64	0.244805	0.244203	0.242405	0.239428	0.235304
0.65	0.235075	0.234512	0.232830	0.230048	0.226184
0.66	0.225454	0.224929	0.223361	0.220763	0.217161
0.67	0.215945	0.215458	0.214000	0.211585	0.208237
0.68	0.206551	0.206100	0.204750	0.202511	0.199413
0.69	0.197275	0.196859	0.195614	0.193552	0.190690
0.70	0.188120	0.187738	0.186595	0.184700	0.182070
0.71	0.179091	0.178741	0.177695	0.175962	0.173555
0.72	0.170189	0.169871	0.168919	0.167342	0.165148
0.73	0.161420	0.161132	0.160269	0.158839	0.156851
0.74	0.152787	0.152527	0.151749	0.150460	0.148667
0.75	0.144294	0.144061	0.143364	0.142207	0.140599
0.76	0.135945	0.135737	0.135116	0.134086	0.132652
0.77	0.127745	0.127561	0.127012	0.126099	0.124829
0.78	0.119699	0.119537	0.119054	0.118251	0.117134
0.79	0.111811	0.111671	0.111249	0.110548	0.109573
0.80	0.104088	0.103966	0.103760	0.102995	0.102150
0.81	0.096534	0.096430	0.096117	0.095597	0.094872
0.82	0.089157	0.089068	0.088802	0.088361	0.087745
0.83	0.081962	0.081887	0.081664	0.081293	0.080776
0.84	0.074956	0.074894	0.074709	0.074401	0.073972
0.85	0.068147	0.068097	0.067946	0.067694	0.067343
0.86	0.061544	0.061504	0.061382	0.061179	0.060897
0.87	0.055156	0.055124	0.055028	0.054868	0.054644
0.88	0.048993	0.048968	0.048893	0.048769	0.048596
0.89	0.043065	0.043046	0.042990	0.042896	0.042765
0.90	0.037386	0.037372	0.037331	0.037262	0.037166
0.91	0.031970	0.031961	0.031931	0.031883	0.031815
0.92	0.026834	0.026828	0.026808	0.026775	0.026729
0.93	0.021957	0.021933	0.021980	0.021959	0.021930
0.94	0.017483	0.017480	0.017473	0.017460	0.017443
0.95	0.013320	0.013319	0.013315	0.013308	0.013298
0.96	0.009546	0.009545	0.009543	0.009540	0.009535
0.97	0.006209	0.006209	0.006208	0.006207	0.006206
0.98	0.003385	0.003385	0.003385	0.003385	0.003384
0.99	0.001199	0.001199	0.001199	0.001199	0.001199
INTEGRAL CF MTF					
0.424413 0.423776 C.421868 0.418708 0.414324					
INTEGRAL CF MTF-SO.					
0.272421 0.271771 C.269835 0.266651 0.262285					

Each column gives the MTF for one value of Δ to six decimal places. Such tables have been published previously, but these have been far less extensive, detailed, and precise.* For all values of Δ not exceeding 5, the integral of the MTF and the $(MTF)^2$ are given at the bottom of each column.

The line spread function $L(x)$ can be found from the optical transfer function $T(\nu)$ by means of the Fourier transform. When the optical transfer function is real, the line spread function is symmetrical and takes the form:

$$L(|x|) = \int_0^\infty T(\nu) \cos 2\pi\nu x \, d\nu / \int_0^\infty T(\nu) \, d\nu. \quad (6)$$

In the case of a perfect lens with a circular aperture, we have in terms of the reduced frequency

$$L(x_r) = \frac{2}{\pi} \int_0^1 [\cos^{-1}\nu_r - \nu_r(1 - \nu_r^2)^{\frac{1}{2}}] \cos 2\pi\nu_r x_r \, d\nu_r / \int_0^1 \frac{4}{3\pi}, \quad (7)$$

where the reduced displacement

$$x_r = (2 \sin \theta / \lambda') x = x/F\lambda' = (2A/\lambda_0)x. \quad (8)$$

For convenience we also use, in the following, $\xi \equiv 2\pi x_r$ as a shorthand notation.

Equation (7) can be shown⁴ to yield

$$L(x_r) = 3 \int_0^\infty \left[\frac{1}{1^2 \cdot 3} - \frac{\xi^2}{2^2 \cdot 4^2} + \frac{\xi^4}{3^2 \cdot 5^2} - \dots \right] \sin^2 u \sin(\xi \cos u) du = (3\pi/2\xi^2) H_1(\xi), \quad (9)$$

From this, the series expansion for the line spread function Eq. (9) is readily found to be

$$\begin{aligned} L(x_r) \sim & \frac{3}{\xi^2} \left\{ 1 + \sum_{n=1}^{\infty} \frac{(-1)^n}{2n-1} \left[\frac{(2n)!}{2^n n!} \right]^2 \xi^{-2n} \right. \\ & - \left(\frac{\pi}{\xi} \right)^{\frac{1}{2}} (\sin \xi + \cos \xi) \left[1 + \sum_{n=1}^{\infty} \frac{(-1)^n}{(8\xi)^{2n+1}} \right] \\ & \times \frac{(4-1^2)(4-3^2)\dots[4-(4n-1)^2]}{(2n+1)!} \left. \right\}. \quad (12) \end{aligned}$$

* Barakat and Houston² cover $\nu_r = 0(0.05)$, $\Delta = 0.8(0.8)4.8$, to five decimal places; Steel³ covers $\nu_r = 0(0.1)$, $\Delta = 0(2/\pi)28/\pi$ to four decimal places.

Table Ib. MTF of Perfect Lens, Defocused Δ Rayleigh Units - $2\Delta F^2\lambda$

$\Delta =$	0	0.5	1.0	1.5	2.0	2.5	3.0	3.5	4.0	4.5	5.0
ν_r											
0.	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000
0.01	0.987268	0.987149	0.986791	0.986195	0.985361	0.984290	0.982982	0.981437	0.979657	0.977642	0.975393
0.02	0.974537	0.974076	0.972695	0.970396	0.967183	0.963063	0.958044	0.952134	0.945345	0.937690	0.929183
0.03	0.961809	0.960808	0.957809	0.952826	0.945879	0.936996	0.926215	0.913580	0.899144	0.882966	0.865113
0.04	0.949084	0.947367	0.942228	0.933704	0.921857	0.906773	0.888560	0.867351	0.843295	0.816566	0.787353
0.05	0.936365	0.933776	0.926039	0.913241	0.895521	0.873076	0.846153	0.815045	0.780093	0.741672	0.700193
0.06	0.923651	0.920057	0.909331	0.891640	0.867260	0.836568	0.800039	0.758231	0.711780	0.661382	0.607786
0.07	0.910946	0.906231	0.897184	0.889096	0.837446	0.797884	0.751220	0.698397	0.640472	0.578585	0.513934
0.08	0.898250	0.892316	0.874675	0.845792	0.806430	0.757621	0.700631	0.638920	0.568994	0.495855	0.421944
0.09	0.885563	0.878332	0.856877	0.821901	0.774544	0.716334	0.649120	0.575038	0.496334	0.415391	0.334535
0.10	0.872889	0.864296	0.838860	0.797585	0.742089	0.674527	0.597480	0.513836	0.426646	0.338984	0.253804
0.11	0.860227	0.850225	0.827687	0.772992	0.709346	0.632655	0.546360	0.454235	0.360182	0.268008	0.181230
0.12	0.847579	0.836133	0.802418	0.748260	0.676565	0.591121	0.496345	0.396994	0.297853	0.203437	0.117715
0.13	0.834946	0.822036	0.784108	0.723516	0.643973	0.550274	0.447920	0.342711	0.240317	0.145876	0.063648
0.14	0.822330	0.807946	0.765809	0.698872	0.611770	0.510413	0.401476	0.291835	0.188900	0.095604	0.018986
0.15	0.809733	0.793878	0.747566	0.674431	0.580129	0.471787	0.357318	0.244675	0.141124	0.052621	-0.016659
0.16	0.797154	0.779842	0.729424	0.650285	0.549200	0.434599	0.315674	0.201420	0.099739	0.016705	-0.043935
0.17	0.784596	0.765850	0.711421	0.626514	0.519109	0.399009	0.276697	0.162147	0.063747	-0.012542	-0.063668
0.18	0.772061	0.751913	0.693592	0.603189	0.489962	0.365136	0.240477	0.126845	0.032935	-0.035642	-0.076790
0.19	0.759548	0.738038	0.675969	0.580371	0.461843	0.333064	0.207048	0.095427	0.007032	-0.053200	-0.084275
0.20	0.747060	0.724235	0.65579	0.558111	0.434819	0.302845	0.176398	0.067742	-0.014414	-0.065864	-0.087092
0.21	0.734598	0.710513	0.614449	0.536453	0.408938	0.274503	0.148474	0.043596	-0.031717	-0.074295	-0.086167
0.22	0.722124	0.699816	0.624598	0.515432	0.384234	0.248037	0.123190	0.022761	-0.045333	-0.079142	-0.082351
0.23	0.709758	0.683335	0.608046	0.495076	0.360728	0.223425	0.109437	0.05987	-0.055692	-0.081023	-0.076409
0.24	0.697382	0.669892	0.591808	0.475407	0.338428	0.200630	0.080085	0.009989	-0.063215	-0.080508	-0.069003
0.25	0.685038	0.656555	0.575899	0.456440	0.317332	0.179599	0.061991	0.022436	-0.068308	-0.078117	-0.060696
0.26	0.672726	0.643326	0.560328	0.438184	0.297428	0.160266	0.046003	0.032618	-0.071350	-0.074307	-0.051951
0.27	0.660449	0.630211	0.545104	0.420645	0.278698	0.142561	0.031965	0.040793	-0.072692	-0.069479	-0.043137
0.28	0.648208	0.617213	0.533234	0.403823	0.261115	0.126403	0.019720	0.047206	-0.072650	-0.063970	-0.034539
0.29	0.636003	0.604336	0.515721	0.387715	0.246469	0.111711	0.009112	0.052089	-0.071508	-0.058065	-0.026368
0.30	0.623838	0.591580	0.501570	0.372313	0.229264	0.098397	0.000099	0.055656	-0.069515	-0.051993	-0.018769
0.31	0.611712	0.578950	0.487799	0.357607	0.214923	0.086377	0.007790	0.058103	-0.066888	-0.045940	-0.011834
0.32	0.599628	0.566446	0.474350	0.343585	0.201584	0.075564	0.014367	0.059610	-0.063812	-0.040046	-0.005613
0.33	0.587587	0.554071	0.461279	0.330232	0.189204	0.065874	0.019872	0.060336	-0.060442	-0.034418	-0.000120
0.34	0.575590	0.541824	0.448563	0.317531	0.177739	0.057223	0.024425	0.060423	-0.056909	-0.029132	0.004656
0.35	0.563639	0.529706	0.436199	0.305463	0.167144	0.049533	0.028136	0.059997	-0.053319	-0.024238	0.008747
0.36	0.551736	0.517718	0.424181	0.294009	0.157376	0.042729	0.031110	0.051666	-0.049755	-0.019764	0.012196
0.37	0.539883	0.505859	0.412501	0.283147	0.148390	0.036737	0.0033437	0.050825	-0.046286	-0.015724	0.015055
0.38	0.528079	0.494130	0.401154	0.272857	0.140142	0.031491	0.0035204	0.056653	-0.042962	-0.012118	0.017381
0.39	0.516329	0.482530	0.390130	0.263116	0.132589	0.026927	0.0036483	0.055120	-0.039820	-0.008936	0.019231
0.40	0.504632	0.471058	0.379422	0.253900	0.125691	0.022985	0.0037344	0.053482	-0.036887	-0.006162	0.020662
0.41	0.492990	0.459712	0.369020	0.245188	0.119406	0.019611	0.0037843	0.051785	-0.034180	-0.003775	0.021726
0.42	0.481466	0.448493	0.358914	0.236956	0.113697	0.016754	0.0038035	0.050067	-0.031709	-0.001749	0.022476
0.43	0.469881	0.437398	0.349509	0.229181	0.108526	0.014369	0.0037963	0.048358	-0.029477	-0.000061	0.022954
0.44	0.458816	0.426626	0.339551	0.221839	0.103854	0.012411	0.0037666	0.046681	-0.027483	0.001317	0.023203
0.45	0.447014	0.415575	0.330273	0.214908	0.099658	0.010843	0.0037179	0.045054	-0.025722	0.002410	0.023256
0.46	0.435676	0.404845	0.321249	0.208365	0.095895	0.009629	0.0036528	0.043488	-0.024186	0.003241	0.023145
0.47	0.424404	0.394232	0.312467	0.202187	0.092537	0.008739	0.0035738	0.041933	-0.022885	0.003835	0.022893
0.48	0.413200	0.383735	0.303918	0.196352	0.089555	0.008142	0.0034820	0.040571	-0.021748	0.004211	0.022523
0.49	0.402065	0.373352	0.295598	0.190839	0.086921	0.007814	0.0033813	0.039223	-0.020821	0.004390	0.022050
0.50	0.391002	0.363082	0.287468	0.185626	0.084609	0.007730	0.0032705	0.037947	-0.020072	0.004388	0.021488
0.51	0.380013	0.352921	0.279545	0.180691	0.082593	0.007870	0.0031515	0.036740	-0.019487	0.004222	0.020844
0.52	0.369099	0.342869	0.271809	0.176015	0.080849	0.008216	0.0030248	0.035594	-0.019050	0.003905	0.020127
0.53	0.358622	0.332924	0.264247	0.171576	0.079355	0.008750	0.0028910	0.034502	-0.018747	0.003451	0.019340
0.54	0.347505	0.323083	0.256848	0.167355	0.078088	0.009457	0.0027503	0.033454	-0.018562	0.002871	0.018483
0.55	0.336830	0.313345	0.249602	0.163332	0.070726	0.0010322	0.0026029	0.032439	-0.018481	0.002177	0.017558
0.56	0.326239	0.303708	0.242948	0.159489	0.067150	0.011333	0.0024847	0.031446	-0.018486	0.001378	0.016563
0.57	0.315733	0.294170	0.235525	0.155806	0.075440	0.012477	0.0022877	0.032046	-0.018661	0.000486	0.015496
0.58	0.305317	0.284730	0.228672	0.152265	0.074876	0.013744	0.0021196	0.029471	-0.018691	0.000490	0.014354
0.59	0.294990	0.275386	0.221929	0.148848	0.074439	0.015121	0.0028463	0.018856	-0.001537	0.013136	
0.60	0.284757	0.266137	0.215287	0.145538	0.074111	0.016598	0.0027422	0.019338	-0.002645	0.011839	
0.61	0.274619	0.256983	0.208736	0.142317	0.073873	0.018164	0.002510	0.015710	0.0026334	0.019218	0.003799
0.62	0.264579	0.247921	0.202766	0.139169	0.070707	0.019808	0.0021372	0.015718	0.0019376	0.004985	0.009011
0.63	0.254646	0.238951	0.195869	0.136077	0.073595	0.012157	0.0011664	0.023958	0.0019491	0.006186	0.007483
0.64	0.244805	0.230073	0.189537	0.133026	0.073519	0.0123281	0.0019521	0.022643	0.0019543	0.007383	0.005889
0.65	0.235075	0.221286	0.183261	0.130000	0.073461	0.0125087	0.0017299	0.021225	0.0019509	0.008555	0.004236
0.66	0.225654	0.212591	0.177034	0.126983	0.073404	0.0126919	0.0015001	0.019695	0.0019367	0.009678	0.002539
0.67	0.215945	0.203986	0.170850	0.123962	0.073327	0.0128764	0.001				

Table Ic. MTF of Perfect Lens, Defocused Δ Rayleigh Units - $2\Delta F^2\lambda$

$\Delta =$	10	15	20	25	30	35	40	45	50
ν_r									
0.	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000	1.000000
0.01	0.94338	0.883782	C.808408	0.717748	0.615971	0.507639	0.397449	0.289963	0.189363
0.02	0.801513	0.614762	0.401707	0.197349	0.031428	-0.077392	-0.124343	-0.117889	-0.076001
0.03	0.613337	0.302560	0.040933	-0.102387	-0.118066	-0.053566	G.023082	0.360947	0.047757
0.04	0.409469	0.049385	C.115944	-0.083190	0.017658	0.061238	G.025562	-0.026037	-0.033811
0.05	0.222129	-0.089777	C.086291	0.031972	0.052541	-0.01160	-0.034844	0.03464	0.025900
0.06	0.067081	-0.116306	0.006558	0.054253	-0.017842	-0.026093	0.020785	0.011709	-0.018270
0.07	-0.038520	-0.070751	C.057192	-0.01455	-0.028445	0.022461	0.002676	-0.016734	0.011647
0.08	-0.095660	-0.004660	C.041015	-0.034583	0.014614	0.006444	-0.015678	0.013983	-0.004308
0.09	-0.111138	0.042977	C.003930	-0.012786	0.020028	-0.017830	G.012425	-0.004607	-0.001708
0.10	-0.096378	0.056284	-0.031630	0.019078	-0.008844	0.002945	G.001997	-0.004535	0.006462
0.11	-0.064218	0.040155	C.025794	0.020215	-0.015198	0.012892	-0.010396	0.009157	
0.12	-0.026395	0.010533	-0.000882	-0.001792	0.004496	-0.005011	0.005881		
0.13	0.008109	-0.016105	C.019316	-0.016229	0.013328	-0.008765			
0.14	0.033705	-0.029614	0.021651	-0.009666	0.001051	0.005242			
0.15	0.048071	-0.028071	0.009049	0.0C5892	-0.009899				
0.16	0.051519	-0.015827	C.006625	0.012954	-0.004990				
0.17	0.046111	0.000998	C.014892	0.0C6824					
0.18	0.034776	0.013349	C.017967	-0.004055					
0.19	0.025752	0.020194	-0.003346	0.009388					
0.20	0.006169	0.019930	C.006329	-0.006070					
0.21	-0.006443	0.014179	C.011476						
0.22	-0.016041	0.005698	C.010605						
0.23	-0.022114	-0.002742	C.005360						
0.24	-0.024725	-0.009106	C.001161						
0.25	-0.024329	-0.012390	C.006153						
0.26	-0.021605	-0.012545							
0.27	-0.017303	-0.010201							
0.28	-0.012141	-0.006315							
0.29	-0.006733	-0.001876							
0.30	-0.001552	0.002298							
0.31	0.003071	0.005659							
0.32	0.006944	0.007934							
0.33	0.009986	0.009085							
0.34	0.0122C0								
0.35	0.013653								
0.36	0.014444								
0.37	0.014693								
0.38	0.014520								
0.39	0.014041								
0.40	0.013356								
0.41	0.012552								
0.42	0.011698								
0.43	0.010847								
0.44	0.010040								
0.45	0.009303								
0.46	0.008656								
0.47	0.008108								
0.48	0.007662								
0.49	0.007319								
0.50	0.007073								
0.51									
0.52									
0.53									
0.54									
0.55									

Tables of this function have been given previously.⁶ In Tables IIa and b we give considerably more extensive and precise tables, computed from Eqs. (11) and (12), respectively, and correct to six decimal places.*

The last column in Table IIa gives the second difference (central, for the last entry in the row). This is useful in accurate interpolation.

Appendix. Error Estimates

A. Part of Integral Evaluated by Simpson's Rule

The error in Simpson's rule integration is less than $E \leq nh^5 f_{\max}^{IV}(x)/180$, where n is the number of points used in applying Simpson's rule, h is the interval between these points, and f_{\max}^{IV} is the maximum value attained by the fourth derivative of the integrand in the range of integration.

* Incidentally, for ξ appreciably greater than 8, the partial sums in Eq. (11) take on large negative values before they converge to a small positive value. For example, with $\xi = 20$ ($x_r = 3.18$) the partial sum becomes -4×10^4 before it begins to increase. It was therefore necessary to use double precision to maintain six-place accuracy in the latter part of Table IIa.

Referring to Eq. (5), we find that the fourth derivative of the integrand will have a term $15 \cos a(x - \nu_r)$ $(1 - x^2)^{-\frac{1}{2}} < 15(2)^{-\frac{1}{2}} (1 - x)^{-\frac{1}{2}}$, all other terms being of higher order in $(1 - x^2)$. For very small values of $(1 - x)$, all other terms will be negligible, provided $a \ll (1 - x^2)^{-1}$. Writing ϵ for $1 - x$, we find $f_{\max}^{IV}(x) < 1.4 \epsilon^{-\frac{1}{2}}$. Taking $n = 1600$, $h = 1/1600$, and $\epsilon = 0.005$, we find, from Eq. (6), that the error is less than 1.4×10^{-7} .

B. Part of Integral Evaluated by Series Expansion

The part of the integral evaluated by series expansion is

$$\Delta T = \frac{4}{\pi} \int_{1-\epsilon}^1 (1-x^2)^{\frac{1}{2}} \cos[a(x - \nu_r)] dx.$$

On substituting $u = 1 - x$ and expanding, this becomes

$$\begin{aligned} \Delta T = \frac{4}{\pi} \int_0^\epsilon (u)^{\frac{1}{2}} (2-u)^{\frac{1}{2}} [\cos a(1-\nu) \cos au \\ + \sin a(1-\nu) \sin au] du. \end{aligned}$$

Table IIa. Line Spread Function of Perfect Lens

x_r	0.000	0.001	0.002	0.003	0.004	0.005	0.006	0.007	0.008	0.009
G.00	.000000	.999997	.999981	.99976	.999958	.999934	.999905	.999871	.999832	.999787
C.01	.999737	.999682	.999621	.999555	.999484	.999408	.999326	.999240	.999148	.999050
C.02	.998948	.998840	.998727	.998609	.998485	.998356	.998222	.998083	.997938	.997789
C.03	.997634	.997473	.997308	.997137	.996961	.996780	.996594	.996402	.996206	.996004
C.04	.995797	.995584	.995367	.995144	.994916	.994683	.994444	.994201	.993952	.993698
C.05	.993439	.993174	.992905	.992630	.992351	.992066	.991775	.991480	.991180	.990874
C.06	.990564	.990248	.989927	.989601	.989269	.988933	.988592	.988245	.987893	.987537
C.07	.987175	.986808	.986436	.986059	.985676	.985289	.984897	.984499	.984097	.983689
C.08	.983277	.982859	.982437	.98209	.981576	.981139	.980696	.980248	.979796	.979338
C.09	.978875	.978408	.977935	.977458	.976488	.975995	.975498	.974488	.974488	-5
C.10	.973976	.973459	.972937	.972410	.971878	.971342	.970800	.970254	.969703	.969146
C.11	.968585	.968020	.967449	.966673	.966293	.965708	.965118	.964524	.963924	.963320
C.12	.962711	.962097	.961478	.960855	.960227	.959594	.958957	.958315	.957668	.957016
C.13	.956360	.955699	.955033	.954363	.953688	.953009	.952324	.951636	.950942	.950244
C.14	.949541	.948834	.948122	.947406	.946685	.945960	.945230	.944495	.943756	.943012
C.15	.942264	.941512	.940755	.939993	.939227	.938457	.937682	.936902	.936119	.935331
C.16	.934538	.933741	.932940	.932134	.931324	.930510	.929691	.928886	.928041	.927209
C.17	.926373	.925533	.924689	.923840	.922987	.922130	.921268	.920403	.919533	.918659
C.18	.917781	.916898	.916012	.915121	.914226	.913327	.912424	.911517	.910606	.909690
C.19	.908771	.907848	.906920	.905989	.905053	.904114	.903170	.902223	.901272	.900316
C.20	.899357	.898394	.897427	.896455	.895481	.894502	.893519	.892533	.891542	.890548
C.21	.889550	.888548	.887543	.886533	.885520	.884504	.883483	.882459	.881431	.880399
C.22	.879364	.878325	.877282	.876236	.875186	.874132	.873075	.872014	.870950	.869882
C.23	.868811	.867736	.866657	.865575	.864490	.863401	.862309	.861213	.860113	.859011
C.24	.857905	.856795	.855682	.854566	.853437	.852324	.851198	.850368	.848935	.847799
C.25	.846660	.845517	.844371	.843222	.842070	.840915	.839756	.838595	.837430	.836262
C.26	.835090	.833916	.832739	.831559	.830375	.829189	.827999	.826807	.825611	.824413
C.27	.823211	.822007	.820800	.819589	.818376	.817160	.815941	.814720	.813495	.812268
C.28	.811037	.809804	.808569	.807330	.806089	.804645	.803598	.802349	.801096	.799842
C.29	.798584	.797324	.796061	.794796	.793528	.792258	.790984	.789709	.788431	.787150
C.30	.785867	.784581	.783293	.782003	.781070	.779414	.778117	.776816	.775514	.774209
C.31	.772902	.771592	.770280	.768966	.767650	.766331	.765010	.763687	.762362	.761034
C.32	.759704	.758373	.757039	.755702	.754364	.753024	.751681	.750337	.748990	.747642
C.33	.746291	.744939	.743584	.742228	.740869	.739509	.738147	.736783	.735417	.734049
C.34	.736267	.731307	.729934	.728559	.727182	.725803	.724422	.723040	.721656	.720270
C.35	.718883	.717494	.716104	.714711	.713317	.711922	.710525	.709126	.707726	.706324
C.36	.704921	.703516	.702110	.700702	.699293	.697883	.696471	.695257	.693643	.692227
C.37	.690809	.689390	.687970	.686549	.685126	.683702	.682277	.680850	.679423	.677994
C.38	.676564	.675133	.673700	.672267	.670832	.669396	.667960	.666522	.665083	.663643
C.39	.662202	.660760	.659317	.657873	.656428	.654983	.653536	.652088	.650640	.649190
C.40	.647740	.646289	.644837	.643385	.641931	.640477	.639022	.637566	.636110	.634653
C.41	.633195	.631736	.630277	.628818	.627357	.625896	.624435	.622972	.621510	.620047
C.42	.618583	.617119	.615654	.614189	.612723	.611257	.609790	.608323	.606856	.605388
C.43	.603920	.602452	.600983	.599514	.598045	.596575	.595105	.593635	.592164	.590694
C.44	.589223	.587752	.586281	.584810	.583338	.581867	.580395	.578923	.577452	.575980
C.45	.574508	.573036	.571564	.570092	.568620	.567149	.565677	.564205	.562733	.561262
C.46	.559791	.558319	.556848	.555377	.553906	.552436	.550965	.549495	.548025	.546556
C.47	.545086	.543617	.542149	.540680	.539212	.537744	.536277	.534810	.533343	.531877
C.48	.530411	.528946	.527481	.526017	.524553	.523089	.521626	.520164	.518702	.517241
C.49	.515780	.514320	.512861	.511402	.509943	.508486	.507029	.505573	.504117	.502662
C.50	.501208	.499755	.498302	.496850	.495399	.493949	.492499	.491051	.489603	.488156
C.51	.486710	.485264	.483820	.482377	.480934	.479492	.478052	.476612	.475173	.473736
C.52	.472299	.470863	.469429	.467995	.466562	.465131	.463701	.462271	.460843	.459416
C.53	.457970	.456565	.455142	.453719	.452298	.450987	.449459	.448042	.446626	.445211
C.54	.443797	.442384	.440973	.439563	.438155	.436748	.435342	.433937	.432534	.431132
C.55	.429732	.428333	.426936	.425540	.424145	.422752	.421360	.419977	.418581	.417194
C.56	.415809	.414425	.413042	.411661	.410282	.409044	.407528	.406153	.404780	.403409
C.57	.402039	.400671	.399305	.397940	.396577	.395216	.393856	.392498	.391142	.389788
C.58	.388435	.387084	.385735	.384388	.383042	.381699	.380357	.379017	.377679	.376343
C.59	.375008	.373676	.372345	.371016	.369690	.368365	.367042	.365721	.364402	.363085
C.60	.361770	.360457	.359145	.357836	.356529	.355224	.353921	.352620	.351321	.350024
C.61	.346730	.347437	.346146	.344858	.343572	.342287	.341005	.339725	.338448	.337172
C.62	.335898	.334627	.333359	.332091	.330827	.329564	.328304	.327046	.325790	.324537
C.63	.323286	.322037	.320790	.319546	.318303	.317064	.315826	.314591	.313358	.312128
C.64	.310900	.309674	.308451	.307230	.306011	.304795	.303581	.302370	.301161	.299954
C.65	.298750	.297548	.296349	.295152	.293958	.292766	.291577	.290390	.289205	.288023
C.66	.286844	.285667	.284493	.283321	.282152	.280985	.279821	.278659	.277500	.276343
C.67	.275158	.274038	.272889	.271743	.270599	.269458	.268320	.267184	.266051	.264920
C.68	.263793	.262667	.261545	.260265	.259308	.258193	.257081	.255972	.254865	.253762
C.69	.252660	.251562	.250466	.249373	.248283	.247195	.246111	.245028	.243949	.242873
C.70	.241799	.240728	.239659	.238594	.237531	.236471	.235414	.234359	.233307	.232259
C.71	.231213	.230169	.229129	.228091	.227056	.226024	.224995	.223969	.222945	.221925
C.72	.220907	.219892	.218880	.217870	.216864	.215860	.214860	.213862	.212867	.211875
C.73	.210884	.209899	.208616	.207935	.206958	.205983	.205011	.204042	.203076	.202113
C.74	.201153	.200195	.199241	.198289	.197341	.196395	.195452	.194513	.193576	.192642
C.75	.191711	.190783	.189858	.188935	.188016	.187100	.186187	.185276	.184349	.183465
C.76	.182563	.181664	.180769	.179876	.178987	.178100	.177216	.176335	.175458	.174583
C.77	.173711	.172842	.171976	.171113	.170254	.169397	.168543	.167692	.166344	.165999
C.78	.165157	.164318	.163481	.162648	.161818	.160991	.160167	.159346	.158528	.157713
C.79	.156901	.155285	.154482	.153682	.152885	.152090	.151299	.150511	.149726	-2
C.80	.148943	.148164	.147388	.146614	.145844	.145077	.144312	.143551	.142793	.142037
C.81	.141285	.140536	.139789	.139046	.138305	.137568	.136833	.136102	.135373	.134648
C.82	.133925	.133205	.132489	.131775	.131064	.130357	.129652	.128950	.128251	.127555
C.83	.126862	.126172	.125485	.124801	.124120	.123442	.122767	.122095	.121425	.120759
C.84	.120094	.119435	.118773	.118123	.117471	.116823	.116177	.115534	.114894	.114257
C.85	.113623	.112992	.112363	.111738	.111116	.110496	.109879	.109266	.108655	.108047
C.86	.107442	.106840	.106246	.105644	.105056	.104460	.103872	.103287	.102705	.102126
C.87	.101549	.100976	.100405	.99836	.992739	.98711	.			

Table IIa. Line Spread Function of Perfect Lens (continued)

x_r	0.000	0.001	0.002	0.003	0.004	0.005	0.006	0.007	0.008	0.009
1.00	.049265	.049021	.048779	.048539	.048301	.048066	.047832	.047600	.047371	.047143
1.01	.046918	.046695	.046474	.046255	.046037	.045822	.045609	.045398	.045189	.044982
1.02	.044777	.044574	.044373	.044174	.043977	.043782	.043588	.043397	.043208	.043020
1.03	.042835	.042651	.042469	.042290	.042112	.041936	.041761	.041589	.041419	.041250
1.04	.041083	.040918	.040755	.040594	.040435	.040277	.040121	.039967	.039819	.039664
1.05	.039516	.039369	.039223	.039080	.038938	.038798	.038660	.038523	.038388	.038255
1.06	.038124	.037994	.037866	.037739	.037614	.037491	.037370	.037250	.037131	.037015
1.07	.036900	.036786	.036674	.036564	.036455	.036348	.036242	.036138	.036036	.035935
1.08	.035836	.035738	.035641	.035546	.035453	.035361	.035271	.035182	.035094	.035008
1.09	.034924	.034841	.034759	.034679	.034600	.034522	.034446	.034372	.034299	.034227
1.10	.034156	.034087	.034019	.033953	.033888	.033824	.033762	.033701	.033641	.033582
1.11	.033525	.033469	.033414	.033361	.033309	.033258	.033209	.033160	.033113	.033067
1.12	.033022	.032979	.032937	.032895	.032855	.032817	.032779	.032743	.032707	.032673
1.13	.032640	.032608	.032578	.032548	.032519	.032492	.032466	.032440	.032416	.032393
1.14	.032371	.032350	.032330	.032311	.032293	.032276	.032260	.032245	.032231	.032218
1.15	.032207	.032196	.032186	.032177	.032168	.032161	.032155	.032150	.032146	.032142
1.16	.032140	.032138	.032137	.032138	.032140	.032143	.032147	.032151	.032156	0
1.17	.032162	.032169	.032186	.032195	.032205	.032216	.032228	.032246	.032254	0
1.18	.032268	.032282	.032298	.032314	.032331	.032349	.032367	.032386	.032406	.032427
1.19	.032448	.032470	.032492	.032516	.032539	.032564	.032589	.032615	.032641	.032668
1.20	.032696	.032724	.032753	.032783	.032813	.032843	.032874	.032906	.032939	.032971
1.21	.033005	.033039	.033073	.033108	.033144	.033180	.033217	.033254	.033291	.033329
1.22	.033368	.033407	.033446	.033486	.033526	.033567	.033608	.033650	.033692	.033735
1.23	.033778	.033821	.033865	.033909	.033948	.033998	.034044	.034089	.034135	.034182
1.24	.034229	.034276	.034323	.034371	.034419	.034467	.034516	.034565	.034614	.034664
1.25	.034714	.034764	.034814	.034865	.034916	.034967	.035019	.035071	.035123	.035175
1.26	.035227	.035280	.035333	.035386	.035439	.035493	.035564	.035600	.035654	.035709
1.27	.035763	.035818	.035872	.035927	.035982	.036038	.036093	.036148	.036204	.036260
1.28	.036316	.036372	.036428	.036484	.036540	.036597	.036653	.036709	.036766	.036823
1.29	.036880	.036936	.036993	.037050	.037107	.037164	.037221	.037278	.037335	.037392
1.30	.037449	.037507	.037564	.037621	.037678	.037735	.037792	.037849	.037906	.037963
1.31	.038020	.038077	.038134	.038191	.038248	.038305	.038362	.038418	.038475	.038531
1.32	.038588	.038644	.038700	.038757	.038813	.038869	.038924	.038980	.039036	.039091
1.33	.039147	.039202	.039257	.039312	.039367	.039422	.039477	.039531	.039586	.039640
1.34	.039694	.039748	.039801	.039854	.039908	.039961	.040014	.040067	.040120	.040172
1.35	.040224	.040276	.040328	.040379	.040431	.040482	.040533	.040584	.040634	.040684
1.36	.040734	.040784	.040834	.040883	.040932	.040981	.041030	.041078	.041126	.041174
1.37	.041221	.041269	.041316	.041362	.041409	.041455	.041501	.041546	.041592	.041637
1.38	.041681	.041726	.041770	.041814	.041857	.041900	.041943	.041986	.042028	.042070
1.39	.042112	.042153	.042194	.042235	.042275	.042315	.042354	.042394	.042433	.042471
1.40	.042510	.042547	.042585	.042622	.042659	.042696	.042732	.042767	.042803	.042838
1.41	.042873	.042907	.042941	.042974	.043008	.043040	.043073	.043105	.043136	.043168
1.42	.043199	.043229	.043259	.043289	.043318	.043347	.043376	.043404	.043431	.043459
1.43	.043486	.043512	.043538	.043564	.043589	.043614	.043638	.043663	.043686	.043709
1.44	.043732	.043754	.043776	.043798	.043819	.043840	.043860	.043880	.043899	.043918
1.45	.043954	.043972	.043989	.044006	.044042	.044084	.044095	.044069	.044099	.044083
1.46	.044097	.044111	.044124	.044137	.044149	.044161	.044173	.044184	.044194	.044204
1.47	.044214	.044223	.044240	.044248	.044255	.044262	.044269	.044275	.044280	0
1.48	.044286	.044290	.044294	.044298	.044302	.044304	.044307	.044309	.044310	.044311
1.49	.044312	.044312	.044311	.044311	.044309	.044307	.044305	.044303	.044299	.044296
1.50	.044292	.044287	.044282	.044277	.044271	.044265	.044258	.044251	.044243	.044235
1.51	.044226	.044217	.044207	.044197	.044187	.044176	.044164	.044153	.044140	.044128
1.52	.044114	.044101	.044086	.044072	.044057	.044041	.044025	.044009	.043992	.043975
1.53	.043957	.043939	.043920	.043901	.043881	.043861	.043841	.043820	.043799	.043777
1.54	.043755	.043732	.043709	.043685	.043661	.043637	.043612	.043586	.043561	.043534
1.55	.043508	.043481	.043453	.043425	.043397	.043368	.043339	.043309	.043279	.043248
1.56	.043217	.043186	.043154	.043124	.043089	.043058	.043022	.042988	.042954	.042919
1.57	.042884	.042848	.042812	.042776	.042739	.042702	.042664	.042626	.042588	.042549
1.58	.042509	.042470	.042430	.042389	.042348	.042307	.042265	.042223	.042181	.042138
1.59	.042094	.042051	.042007	.041962	.041917	.041872	.041827	.041781	.041734	.041688
1.60	.041841	.041593	.041545	.041497	.041449	.041400	.041350	.041301	.041251	.041200
1.61	.041150	.041098	.041047	.040995	.040943	.040891	.040838	.040785	.040731	.040677
1.62	.040623	.040568	.040513	.040458	.040403	.040347	.040290	.040234	.040177	.040120
1.63	.040062	.040004	.039946	.039888	.039829	.039777	.039710	.039651	.039591	.039530
1.64	.039470	.039409	.039347	.039286	.039224	.039162	.039099	.039037	.038974	.038910
1.65	.038847	.038783	.038719	.038654	.038590	.038525	.038459	.038394	.038328	.038262
1.66	.038196	.038129	.038062	.037995	.037928	.037860	.037793	.037724	.037656	.037588
1.67	.037519	.037450	.037380	.037311	.037241	.037171	.037101	.037030	.036960	.036889
1.68	.036818	.036746	.036675	.036603	.036531	.036459	.036387	.036314	.036241	.036168
1.69	.036095	.036022	.035948	.035874	.035800	.035726	.035652	.035577	.035503	.035428
1.70	.035353	.035277	.035202	.035127	.035051	.034975	.034899	.034823	.034746	.034670
1.71	.034593	.034516	.034439	.034362	.034285	.034207	.034130	.034052	.033974	.033896
1.72	.033818	.033740	.033662	.033583	.033505	.033426	.033347	.033268	.033189	.033110
1.73	.033031	.032951	.032872	.032792	.032713	.032633	.032553	.032473	.032393	.032313
1.74	.032323	.032152	.032072	.031992	.031911	.031838	.031750	.031689	.031622	.031507
1.75	.031426	.031345	.031264	.031188	.031102	.031021	.030939	.030858	.030777	.030695
1.76	.030614	.030532	.030451	.030369	.030288	.030206	.030124	.030043	.029961	.029879
1.77	.029798	.029716	.029634	.029552	.029471	.029389	.029307	.029225	.029143	.029062
1.78	.028980	.028898	.028816	.028738	.028653	.028571	.028489	.028407	.028326	.028244
1.79	.028162	.028081	.027999	.027918	.027836	.027755	.027673	.027592	.027510	.027429
1.80	.027348	.027266	.027185	.027104	.027023	.026942	.026861	.026780	.026699	.026618
1.81	.026537	.026457	.026376	.026296	.026215	.026135	.026054	.025974	.025894	.025814
1.82	.025574	.025554	.025574	.025495	.025415	.025335	.025256	.025177	.025097	.025018
1.83	.024939	.024860	.024781	.024703	.024624	.024546	.024467	.024389	.024311	.024233
1.84	.024155	.024077	.023999	.023924	.023844	.023767	.023690	.023613	.023536	.023459
1.85	.023383	.023306	.023230	.023154	.023078	.023002	.022926	.022850	.022775	.022700
1.86	.022625	.022550	.022475	.022400	.022326	.022251	.022177	.022103	.022029	.02

Table IIa. Line Spread Function of Perfect Lens (continued)

x_r	0.000	0.001	0.002	0.003	0.004	0.005	0.006	0.007	0.008	0.009
2.00	.014222	.014180	.014138	.014097	.014056	.014015	.013975	.013934	.013894	.013855
2.01	.013816	.013776	.013738	.013699	.013661	.013623	.013586	.013548	.013511	.013474
2.02	.013438	.013402	.013366	.013330	.013295	.013260	.013225	.013191	.013156	.013123
2.03	.013089	.013056	.013023	.012990	.012957	.012925	.012893	.012861	.012830	.012799
2.04	.012768	.012738	.012708	.012678	.012648	.012619	.012589	.012561	.012532	.012504
2.05	.012476	.012448	.012421	.012394	.012367	.012340	.012314	.012288	.012262	.012237
2.06	.012211	.012187	.012162	.012138	.012113	.012090	.012066	.012043	.012020	.011997
2.07	.011975	.011953	.011931	.011909	.011888	.011867	.011846	.011825	.011805	.011785
2.08	.011765	.011746	.011727	.011708	.011689	.011671	.011653	.011635	.011617	.011600
2.09	.011583	.011566	.011549	.011533	.011517	.011501	.011486	.011471	.011456	.011441
2.10	.011426	.011412	.011398	.011384	.011371	.011358	.011345	.011332	.011320	.011307
2.11	.011295	.011284	.011272	.011261	.011250	.011239	.011229	.011219	.011209	.011199
2.12	.011189	.011180	.011171	.011162	.011154	.011145	.011137	.011130	.011122	.011115
2.13	.011107	.011101	.011094	.011087	.011081	.011075	.011069	.011064	.011059	.011053
2.14	.011049	.011044	.011040	.011035	.011031	.011028	.011024	.011021	.011018	.011015
2.15	.011012	.011010	.011007	.011005	.011003	.011002	.011000	.010999	.010998	.010997
2.16	.010997	.010996	.010996	.010996	.010997	.010998	.010998	.010999	.011001	.011001
2.17	.011002	.011003	.011005	.011007	.011009	.011012	.011014	.011017	.011020	.011023
2.18	.011026	.011030	.011033	.011037	.011041	.011045	.011050	.011054	.011059	.011064
2.19	.011069	.011074	.011079	.011085	.011090	.011096	.011102	.011108	.011115	.011121
2.20	.011128	.011135	.011142	.011149	.011156	.011164	.011171	.011179	.011187	.011195
2.21	.011203	.011212	.011202	.011229	.011237	.011246	.011255	.011265	.011274	.011283
2.22	.011293	.011303	.011313	.011323	.011333	.011343	.011354	.011364	.011375	.011385
2.23	.011396	.011407	.011419	.011430	.011441	.011453	.011464	.011476	.011488	.011500
2.24	.011512	.011524	.011536	.011549	.011561	.011574	.011586	.011599	.011612	.011625
2.25	.011638	.011651	.011665	.011678	.011692	.011705	.011719	.011733	.011746	.011760
2.26	.011774	.011788	.011803	.011817	.011831	.011846	.011860	.011875	.011889	.011904
2.27	.011919	.011934	.011949	.011964	.011979	.011994	.012009	.012024	.012040	.012055
2.28	.012071	.012086	.012102	.012117	.012133	.012149	.012165	.012181	.012196	.012212
2.29	.012228	.012245	.012261	.012277	.012293	.012309	.012325	.012342	.012358	.012374
2.30	.012391	.012407	.012424	.012440	.012457	.012474	.012490	.012507	.012523	.012540
2.31	.012557	.012574	.012590	.012607	.012624	.012641	.012658	.012675	.012691	.012708
2.32	.012725	.012742	.012759	.012776	.012793	.012810	.012827	.012844	.012861	.012878
2.33	.012895	.012912	.012929	.012946	.012963	.012979	.012996	.013013	.013030	.013047
2.34	.013064	.013081	.013098	.013115	.013132	.013149	.013165	.013182	.013199	.013216
2.35	.013233	.013249	.013266	.013283	.013299	.013316	.013333	.013349	.013366	.013382
2.36	.013399	.013415	.013432	.013448	.013464	.013481	.013497	.013513	.013529	.013545
2.37	.013562	.013578	.013594	.013610	.013626	.013642	.013657	.013673	.013689	.013705
2.38	.013720	.013736	.013751	.013767	.013782	.013798	.013813	.013828	.013843	.013859
2.39	.013874	.013889	.013904	.013919	.013933	.013948	.013963	.013978	.013992	.014007
2.40	.014021	.014035	.014050	.014064	.014078	.014092	.014106	.014120	.014134	.014148
2.41	.014161	.014175	.014189	.014202	.014215	.014229	.014242	.014255	.014268	.014281
2.42	.014294	.014307	.014319	.014332	.014345	.014357	.014369	.014382	.014394	.014406
2.43	.014418	.014430	.014442	.014453	.014465	.014476	.014488	.014499	.014510	.014521
2.44	.014533	.014543	.014554	.014565	.014576	.014586	.014597	.014607	.014617	.014627
2.45	.014637	.014647	.014657	.014667	.014676	.014686	.014695	.014704	.014713	.014722
2.46	.014731	.014740	.014749	.014757	.014766	.014774	.014782	.014791	.014799	.014807
2.47	.014814	.014822	.014830	.014837	.014844	.014852	.014859	.014866	.014872	.014879
2.48	.014888	.014892	.014899	.014905	.014911	.014917	.014923	.014929	.014934	.014940
2.49	.014945	.014950	.014956	.014961	.014965	.014970	.014975	.014979	.014984	.014988
2.50	.014992	.014996	.015000	.015004	.015007	.015011	.015014	.015017	.015021	.015024
2.51	.015026	.015029	.015032	.015034	.015036	.015039	.015041	.015043	.015044	.015046
2.52	.015048	.015049	.015050	.015051	.015053	.015053	.015054	.015055	.015055	.015056
2.53	.015056	.015056	.015056	.015056	.015056	.015055	.015054	.015054	.015053	.015052
2.54	.015051	.015051	.015049	.015048	.015046	.015045	.015043	.015041	.015039	.015037
2.55	.015032	.015029	.015027	.015024	.015021	.015018	.015015	.015011	.015008	.015004
2.56	.015000	.014996	.014992	.014988	.014984	.014979	.014975	.014970	.014965	.014960
2.57	.014955	.014950	.014944	.014939	.014933	.014927	.014921	.014915	.014909	.014903
2.58	.014896	.014890	.014883	.014876	.014869	.014862	.014855	.014847	.014840	.014832
2.59	.014824	.014817	.014809	.014800	.014792	.014784	.014775	.014767	.014758	.014749
2.60	.014740	.014731	.014721	.014712	.014702	.014693	.014683	.014673	.014663	.014653
2.61	.014642	.014632	.014621	.014611	.014600	.014589	.014578	.014567	.014556	.014544
2.62	.014533	.014521	.014509	.014497	.014485	.014473	.014461	.014448	.014436	.014423
2.63	.014411	.014398	.014385	.014372	.014359	.014345	.014332	.014318	.014305	.014291
2.64	.014277	.014263	.014249	.014235	.014220	.014206	.014191	.014177	.014162	.014147
2.65	.014132	.014117	.014102	.014087	.014071	.014056	.014040	.014024	.014008	.013992
2.66	.013976	.013960	.013944	.013928	.013911	.013895	.013878	.013861	.013844	.013827
2.67	.013818	.013793	.013776	.013759	.013741	.013724	.013706	.013688	.013670	.013652
2.68	.013634	.013616	.013598	.013580	.013561	.013543	.013524	.013506	.013487	.013468
2.69	.013449	.013430	.013411	.013392	.013373	.013353	.013334	.013314	.013295	.013275
2.70	.013255	.013236	.013216	.013196	.013176	.013155	.013135	.013115	.013095	.013074
2.71	.013054	.013033	.013012	.012991	.012971	.012950	.012929	.012908	.012887	.012865
2.72	.012844	.012823	.012802	.012780	.012759	.012737	.012715	.012694	.012672	.012650
2.73	.012628	.012606	.012584	.012562	.012540	.012518	.012496	.012473	.012451	.012428
2.74	.012406	.012383	.012361	.012338	.012316	.012293	.012270	.012247	.012224	.012201
2.75	.012178	.012155	.012132	.012109	.012086	.012063	.012040	.012016	.011993	.011970
2.76	.011946	.011923	.011899	.011876	.011852	.011828	.011805	.011781	.011757	.011734
2.77	.011710	.011686	.011662	.011638	.011614	.011590	.011566	.011542	.011518	.011494
2.78	.011470	.011446	.011422	.011398	.011374	.011350	.011325	.011301	.011277	.011252
2.79	.011228	.011204	.011179	.011155	.011131	.011106	.011082	.011058	.011033	.011009
2.80	.010984	.010960	.010935	.010911	.010886	.010862	.010837	.010813	.010788	.010764
2.81	.010739	.010714	.010690	.010665	.010641	.010616	.010592	.010567	.010543	.010518
2.82	.010493	.010469	.010444	.010420	.010395	.010371	.010346	.010322	.010297	.010273
2.83	.010248	.010224	.010199	.010175	.010150	.010126	.010101	.010077	.010052	.010028
2.84	.010004	.009979	.009955	.009931	.009908	.009882	.009858	.009833	.009809	.009785
2.85	.009761	.009737	.009713	.009688	.009664	.009640	.009616	.009592	.009568	.009544
2.86	.009520	.009496	.009473	.009449	.009425	.009401	.009377	.009354</td		

Table IIb. Line Spread Function of Perfect Lens

x_r	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
2.	.014221	.011426	.011128	.012391	.014221	.014992	.014740	.013255	.010984	.008594
3.	.006709	.005703	.005608	.006153	.006898	.007409	.007405	.006836	.005866	.004787
4.	.003905	.003422	.003384	.003678	.004089	.004391	.004424	.004146	.003635	.003048
5.	.002556	.002284	.002267	.002547	.002702	.002897	.002932	.002775	.002469	.002110
6.	.001805	.001635	.001626	.001746	.001917	.002051	.002081	.001984	.001785	.001547
7.	.001343	.001228	.001224	.001309	.001430	.001527	.001552	.001487	.001350	.001183
8.	.001038	.000957	.000995	.001018	.001108	.001181	.001201	.001155	.001056	.000933
9.	.000827	.000768	.000767	.000814	.000883	.000939	.000956	.000923	.000848	.000756
10.	.000675	.000629	.000629	.000666	.000720	.000765	.000779	.000754	.000696	.000624
11.	.000561	.000525	.000525	.000555	.000599	.000635	.000647	.000627	.000582	.000524
12.	.000474	.000445	.000446	.000470	.000506	.000535	.000545	.000530	.000493	.000446
13.	.000405	.000382	.000383	.000403	.000433	.000457	.000466	.000454	.000423	.000385
14.	.000351	.000332	.000332	.000350	.000374	.000395	.000403	.000392	.000367	.000335
15.	.000307	.000291	.000291	.000306	.000327	.000345	.000351	.000343	.000322	.000295
16.	.000271	.000257	.000258	.000270	.000288	.000304	.000309	.000302	.000284	.000261
17.	.000240	.000229	.000229	.000240	.000256	.000269	.000274	.000268	.000253	.000233
18.	.000215	.000205	.000206	.000215	.000229	.000240	.000245	.000240	.000226	.000209
19.	.000193	.000185	.000185	.000194	.000206	.000216	.000220	.000215	.000204	.000188
20.	.000175	.000167	.000168	.000175	.000186	.000195	.000199	.000195	.000184	.000171
21.	.000159	.000152	.000153	.000159	.000169	.000177	.000180	.000177	.000168	.000156
22.	.000145	.000139	.000140	.000146	.000154	.000161	.000164	.000161	.000153	.000143
23.	.000133	.000128	.000128	.000133	.000141	.000148	.000150	.000148	.000140	.000131
24.	.000122	.000118	.000118	.000123	.000130	.000136	.000138	.000136	.000129	.000121
25.	.000113	.000109	.000109	.000113	.000120	.000125	.000127	.000125	.000119	.000112
26.	.000105	.000101	.000101	.000105	.000111	.000116	.000118	.000116	.000110	.000103
27.	.000097	.000094	.000094	.000098	.000103	.000107	.000109	.000107	.000103	.000096
28.	.000090	.000087	.000088	.000091	.000096	.000100	.000101	.000100	.000095	.000090
29.	.000084	.000082	.000082	.000085	.000089	.000093	.000095	.000093	.000089	.000084
30.	.000079	.000076	.000077	.000079	.000083	.000087	.000088	.000087	.000083	.000078
31.	.000074	.000072	.000072	.000074	.000078	.000081	.000083	.000082	.000078	.000074
32.	.000070	.000067	.000068	.000070	.000073	.000076	.000078	.000077	.000073	.000069
33.	.000065	.000063	.000064	.000066	.000069	.000072	.000073	.000072	.000069	.000065
34.	.000062	.000060	.000060	.000062	.000065	.000068	.000069	.000068	.000065	.000062
35.	.000058	.000057	.000057	.000059	.000061	.000064	.000065	.000064	.000062	.000058
36.	.000055	.000054	.000054	.000056	.000058	.000060	.000061	.000061	.000058	.000055
37.	.000052	.000051	.000051	.000053	.000055	.000057	.000058	.000057	.000055	.000052
38.	.000050	.000048	.000048	.000050	.000052	.000054	.000055	.000054	.000052	.000050
39.	.000047	.000046	.000046	.000047	.000050	.000051	.000052	.000052	.000050	.000047
40.	.000045	.000044	.000044	.000045	.000047	.000049	.000050	.000049	.000047	.000045
41.	.000043	.000042	.000042	.000043	.000045	.000047	.000047	.000047	.000045	.000043
42.	.000041	.000040	.000040	.000041	.000043	.000044	.000045	.000045	.000043	.000041
43.	.000039	.000038	.000038	.000039	.000041	.000042	.000043	.000042	.000041	.000039
44.	.000037	.000036	.000036	.000037	.000039	.000040	.000041	.000041	.000039	.000037
45.	.000036	.000035	.000035	.000036	.000037	.000039	.000039	.000039	.000037	.000036
46.	.000034	.000033	.000033	.000034	.000036	.000037	.000038	.000037	.000036	.000034
47.	.000033	.000032	.000032	.000033	.000034	.000035	.000036	.000036	.000034	.000033
48.	.000031	.000031	.000031	.000032	.000033	.000034	.000034	.000034	.000033	.000031
49.	.000030	.000029	.000029	.000030	.000031	.000033	.000033	.000033	.000032	.000030

Expanding $\cos au$ and $\sin au$ in a Taylor series and integrating, we find

$$\Delta T = \frac{4}{\pi} \left[\cos(a(1-\nu)) \sum_{n=0}^{\infty} a^{2n} A_{2n} + \sin(a(1-\nu)) \sum_{n=0}^{\infty} a^{2n+1} A_{2n+1} \right],$$

where

$$A_0 = \left(\frac{\epsilon}{2}\right)^{\frac{1}{2}} \epsilon [(4/3) - (\epsilon/5) - (\epsilon^2/56) - \dots],$$

$$A_1 = \left(\frac{\epsilon}{2}\right)^{\frac{1}{2}} \epsilon^2 [(2/5) - (\epsilon/14) - \dots],$$

$$A_2 = \left(\frac{\epsilon}{2}\right)^{\frac{1}{2}} \epsilon^3 [(2/7) - \dots].$$

With $\epsilon = 0.005$, the last retained term is less than 3×10^{-9} !

C. Numerical Check

As a numerical check, $T(0.5, 1/\pi) = 0.379515$ was evaluated on a desk computer by means of Eq. (4) and was found to agree exactly with the corresponding result obtained on the computer.

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