ON NOISE ANALYSIS OF ELECTRO-OPTICAL SYSTEMS

Leo LEVI
Jerusalem College of Technology,
P.O.B. 16031, Jerusalem, Israel

Received 30 May 1973,
Revised version received 2 August 1973

A simple relationship is derived to facilitate the calculation of system signal-to-noise ratios when those of the components are known. This leads to particularly simple forms in certain electro-optical systems.

1. Introduction

Modern electro-optical systems are often composed of a series of linear components each a source of a significant amount of noise. We derive here a simple relationship facilitating the calculation of the combined effect of these sources. For brevity we refer to the noise-to-signal power ratio as "specific noise".

2. System noise

Noise power may be defined as the variance of the noise value. The noise power in the output may then be calculated as the sum of the noise powers contributed by each component. This contribution, in turn, is obtained as the noise power as generated at the component multiplied by the power gain of the subsequent stages combined. Thus the total system noise power is

$$W_{\mathrm{nT}} = \sum_{j=1}^{m} W_{\mathrm{n}j}\left[\prod_{j+1}^{m} g_k\right]^2 ,$$

where $W_{\mathrm{n}j}$ is the mean noise power generated at the jth stage, g_j is the gain of the jth stage and m is the total number of stages.

If we denote the output signal power by W_0 we can write☆ for the specific noise, R^{-2},

$$R^{-2} = W_0^{-1}\left[\sum W_{\mathrm{n}j}\left(\prod_{j+1}^{m} g_k\right)^2\right]$$

$$= \sum \frac{W_{\mathrm{n}j}}{W_0/\left(\prod_{j+1}^{m} g_k\right)^2} = \sum_{j=1}^{m} R_j^{-2}, \qquad (1)$$

where R_j^{-2} is the specific noise generated at the jth stage. The last step follows from the fact that

$$W_0/\left(\prod g_k\right)^2$$

represents the signal power at the jth stage. Thus *the value of the system specific noise equals the sum of the values of the component specific noise.*

Note that signal levels and gains do not appear explicitly in this formula, although they are likely to enter in determining the individual specific noise values.

3. Representative noise sources

Since quantum noise sources play a major role in electro-optical systems, we discuss these here in some

☆ We use the symbol R^{-2} for the specific noise to be consistent with the use of the symbol R for the (amplitude) signal-to-noise ratio.

more detail to illustrate the above result. The statistics of many of such noise sources may be represented by the binomial or by the Poisson distribution.

In this context the binomial process may be viewed as having a gain which varies randomly, having the value unity with a probability g and the value zero with a probability $(1-g)$. The mean value of the gain, too, is then g and the value of the variance $g(1-g)$. The variance (σ_{0j}^2) of the gain, i.e. the variance of the output for a single quantum input, is then given by

$$\sigma_{0j}^2 = g_j, \quad g_j(1-g_j) \tag{2}$$

for Poisson and binomial processes, respectively.

To obtain the variance with a number n of identical input quanta, we use the fact that, for independent random variables, the variance of the sum equals the sum of the variances (see, for example, ref. [1]). Since the variances are equal for the n incident quanta, the variance of the sum equals n times the variance for a single quantum. Thus

$$R_j^{-2} = \sigma_j^2/n_j^2 = (n_j/g_j)\sigma_{0j}^2/n_j^2$$
$$= \sigma_{0j}^2/g_j n_j = \sigma_{0j}^2/g_j n_i G_j, \tag{3}$$

where $n_j = n_i G_j$ is the number of signal quanta at the output of stage j, n_i is the number of quanta at the system input and

$G_j = \prod_{k=1}^{j} g_k$ is the cumulative gain through stage j.

Using (2) we find

$$R_j^{-2} = 1/G_j n_i, \quad (1-g_j)/G_j n_i \tag{4}$$

for Poisson and binomial process, respectively.

Specifically, if there is a series of m, consecutive components which may be treated as following binomial statistics, their combined specific noise may be represented in terms of their total gain $(G = G_m)$, as

$$R_\mathrm{T}^{-2} = \frac{1}{n_i}\sum_{j=1}^{m}\frac{1-g_j}{G_j} = \frac{G}{n_i G}\sum_{j=0}^{m}(1-g_j)/G_j$$
$$= \frac{G}{n_0}\sum_{j=0}^{m}\left(\frac{1}{G_j}-\frac{1}{G_{j-1}}\right), \tag{5}$$

where $n_0 = Gn_i$ is the number of quanta at the system output, and $G_0 = 1$. In the sum (5), each term $(1/G_j)$ is canceled out by the following $(1/G_{j-1})$-term, except for the first $(1/G_{j-1} = 1)$ and the last $(1/G_j = 1/G)$. Thus (5) becomes

$$R_\mathrm{T}^{-2}(\text{binom}) = (1-G)/n_0, \tag{6}$$

i.e., *the series of components may be treated as a single component with a gain equal to the product of the component gains.*

If the series is headed by a component following Poisson statistics, this eliminates the G-term in the numerator, so that simply

$$R_\mathrm{T}^{-2}(\text{Poisson} - \text{binom}) = n_0^{-1}. \tag{7}$$

Obviously this approximation is valid also whenever $G \ll 1$. This fact broadens the applicability of (7). To illustrate: light from a distant object entering an objective lens (binomial, $g \ll 1$) which images it at the input surface of a fiber bundle (binomial), conducting the light to a photocathode (binomial (?)), constitute such a series. If this is followed by a phosphor with quantum gain, g_p, the temporal specific noise, overall, will be simply

$$R^{-2} = (g_\mathrm{p} + 1)/n_0, \tag{8}$$

where n_0 now is the number of photons radiated by a single image element during the integration period of the detector.

On the other hand, for a series of m Poisson processes, each of gain m, (4) yields

$$R_\mathrm{T}^{-2} = \frac{1}{n_i}\sum_{0}^{m} g^{-j} = \frac{1}{n_i}\frac{g - 1/G_m}{g-1}, \tag{9}$$

the well-known relationship for the multiplier phototube.

Reference

[1] H. Cramer, Mathematical Methods of Statistics, Princeton, 1946; Eq. 15.6.2.