Reprinted from:

NYCLEAR PHYSICS, Volume B90 (1975) No. 2

POMERON DECOUPLING IN INCLUSIVE MULTI-REGGE MODELS*

J.R. FREEMAN**
Department of Physics, University of Washington, Seattle, Washington 98195 USA

NORTH-HOLLAND PUBLISHING COMPANY – AMSTERDAM

Nuclear Physics B90 (1975) 307–312

POMERON DECOUPLING IN INCLUSIVE MULTI-REGGE MODELS*

J.R. FREEMAN**
Department of Physics, University of Washington, Seattle, Washington 98195 USA

Received 30 September 1974

Inclusive charge conservation sum rules and naive duality are used to constrain the parameters of a multi-Regge-Mueller model with factorizable J-plane singularities including a leading pomeron trajectory (of intercept not necessarily one) and a countable number of secondary trajectories. Consistency requires the vanishing of the central coupling of particle $\overline{\pi^+}$ to two pomeron trajectories, that is, for single particle π^+ spectra, the rapidity plateau must disappear! This unpalatable result may be a warning not to push oversimplified models too far; nevertheless, we show in two examples, a possible way of avoiding the decoupling through modification of the pomeron propagator to vanish for small rapidity separations.

A natural first stage in the attempt to unravel the mysteries of multiparticle hadronic interactions has been to test various simple models of inclusive reactions against the growing collection of experimental data. One such model, the Regge-Mueller [1] model, with factorizable J-plane poles has served as a useful guide for inclusive phenomenology [2]. But it does have limitations since inclusive spectra do show long-range rapidity correlations [3] which are absent in the factorizable Regge-Mueller model. To disentangle the long- and short-range correlations, it is popular to adopt the view that there are two distinct mechanisms [4] of particle production – a long-range diffractive component and a short-range multiperipheral mechanism. If these components are indeed distinct, then inclusive conservation sum rules [5] may be applied separately to each.

The purpose of this note is to point out that a number of assumptions that are common in the inclusive multi-Regge literature and that do have some phenomenological support are in fact incompatible with inclusive charge conservation. The apparent lesson impled by this result is not to overextend simplified models beyond their phenomenological limitations.

The assumptions we make are listed below together with appropriate comments:

(i) The short-range component of inclusive rapidity spectra may be described by a Regge-Mueller model with factorizable poles consisting of a leading singularity, the pomeron of intercept α_P (not necessarily equal to one) and a countable number of

* Supported by the US Atomic Energy Commission.
** Present address: Department of Nuclear Physics, Weizmann Institute of Science, Rehovot, Israel.

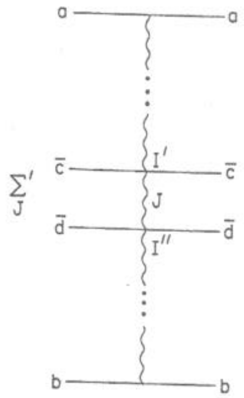

Fig. 1. Many-leg Regge-Mueller graph. Trajectories J are non-pomeron trajectories; I', I'' are arbitrary.

secondary trajectories J of intercept $\alpha_J < \alpha_P$. Phenomenological analyses are common in which the pomeron is given as unit intercept and the secondary trajectories are restricted to the leading meson trajectories of intercept one-half.

(ii) In integrating inclusive rapidity spectra over a domain away from the kinematic boundaries, the multi-Regge version of the Mueller model may be used, even for small rapidity separations. This is in keeping with the observed two-particle correlations in the central region at ISR energies [3] which do seem to exhibit exponential fall-off with correlation length $\simeq 2$.

(iii) Harara-Freund duality [6] may be extended to many leg Mueller graphs in the sense that the sum of graphs shown in fig. 1 vanishes when the time-like $(\overline{cd})$ channel has exotic quantum numbers. Nothing need be assumed about the "space-like" channels at both ends of the chain.

Assumption (iii) corresponds to the notion that if resonance production is impossible in the $(\overline{cd})$ channel, then there must be a corresponding cancellation among cross channel secondary trajectories. In particular, it suggests there should be no short-range rapidity correlations between negative pions in non-diffractive production, a frequent assumption in inclusive models [7]. Of course, as we emphasized earlier, the world is more complicated than pure SRC models. Nevertheless two particle $(\pi^-\pi^-)$ correlations are indeed much smaller than $(\pi^-\pi^+)$ correlations [8] so that in the context of two component models, assumption (iii) may not be unreasonable.

Now let us see to where (i), (ii) and (iii) lead us.

First we write down the Reggeized spectra. The total a + b cross section at sufficiently high energies is

$$^{ab}\sigma(Y) = \beta_P^b \beta_P^a e^{Y(\alpha_P - 1)} + \sum_J \beta_J^b \tau_J \beta_J^a e^{Y(\alpha_J - 1)}, \tag{1}$$

where $Y \equiv y^{a} - y^{b}$ is the rapidity separation of a and b; β_I^i is the particle i-trajectory I residue at zero momentum transfer, τ_I is the trajectory signature. For convenience we will normalize reggeized spectra by the pomeron piece σ_P of (1). The single particle rapidity spectrum of a + b → c + ... in the double Regge domain $y^{ac} \equiv y^{a} - y^{c} \geqslant L$ and $y^{cb} \geqslant L$ is simply

$$\frac{1}{\sigma_P}\frac{d\sigma}{dy^c} = \sum_{\lambda \geqslant 0}\sum_{\lambda' \geqslant 0} e^{-y^{ac}\lambda} e^{-y^{cb}\lambda'} \sum_I{}' \sum_{I'}{}' \gamma^{b}_{I'}\tau_{I'} h^{c}_{I'I}\tau_I \gamma^{a}_{I}, \quad \text{with} \quad \gamma^i_I \equiv \frac{\beta^i_I}{\beta^i_P}, \tag{2}$$

where Σ_I' sums over trajectories of common intercept $(\alpha_P - \lambda)$ (thus $\lambda = 0$ corresponds to the pomeron and $\lambda > 0$ to secondary trajectories); $h^i_{I'I}$ is the central coupling of particle i to the I' and I trajectories, for example, h^c_{PP} gives the height of the rapidity plateau of (2). Similarly the two particle spectrum of a + b → i + j + ... for $\{y^{ai}, y^{ij}, y^{jb}\} \geqslant L$ has the form

$$\frac{1}{\sigma_P}\frac{d\sigma}{dy^i dy^j} = \sum_{\lambda}\sum_{\lambda'}\sum_{\lambda''} e^{-y^{ai}\lambda} e^{-y^{ij}\lambda'} e^{-y^{jb}\lambda''} \sum_I{}' \sum_{I'}{}' \sum_{I''}{}' [\dots\dots] . \tag{3}$$

The reggeized spectra may be contrained by equating terms with the same y^c and Y dependence in the inclusive charge conservation sum rule [5]

$$-Q^c \frac{1}{\sigma}\frac{d\sigma}{dy^c} = \sum_{d} Q^d \int dy^d \left[\frac{1}{\sigma}\frac{d\sigma}{dy^c dy^d} - \frac{1}{\sigma^2}\frac{d\sigma}{dy^c}\frac{d\sigma}{dy^d}\right], \tag{4}$$

where Q^i is the charge of particle i. Taking $\{y^{ac}, y^{cb}\} \geqslant L$ the energy-independent pieces of (4) yield

$$-Q^c h^c_{PP} = \sum_d Q^d \sum_{\lambda > 0} \frac{2}{\lambda} \sum_J{}' h^c_{PJ}\tau_J h^d_{JP} , \tag{5}$$

a result found also by Webber [9]. By combining (5) with similar constraints from strangeness and baryon number conservation and restricting the number of trajectories, one might try to solve the conservation equations [9]. Instead, we focus on the additional constraint of duality.

Although what duality entails for inclusive cross sections is controversial [10,11], we will assume the previously stated analogue of the usual Harari-Freund hypothesis. That is (with reference to fig. 1)

$$\sum_J{}' h^c_{I'J}\tau_J h^d_{JI''} = 0 \quad \text{for cd exotic} . \tag{6}$$

To exploit this relation, consider the inclusive charge conservation sum rule connecting the three-particle spectrum of a + b → i + j + d + ... and the two-particle spectrum of a + b → i + j + ...,

$$-(Q^i + Q^j)\frac{1}{\sigma}\frac{d\sigma}{dy^i dy^j} = \sum_d \int dy^d \left[\frac{1}{\sigma}\frac{d\sigma}{dy^i dy^j dy^d} - \frac{1}{\sigma^2}\frac{d\sigma}{dy^i dy^j}\frac{d\sigma}{dy^d}\right] Q^d , \tag{7}$$

with the restrictions $\{y^{\mathrm{a}i}, y^{ij}, y^{j\mathrm{b}}\} \geqslant L$. In the domain $0 \leqslant y^{i\mathrm{d}} \leqslant y^{ij}$ we take

$$\frac{1}{\sigma_{\mathrm{P}}} \frac{\mathrm{d}\sigma}{\mathrm{d}y^i \mathrm{d}y^j \mathrm{d}y^{\mathrm{d}}} = \sum_{\lambda_1} \sum_{\lambda_2} \sum_{\lambda_3} \sum_{\lambda_4} \mathrm{e}^{-y^{\mathrm{a}i}\lambda_1} \mathrm{e}^{-y^{i\mathrm{d}}\lambda_2} \mathrm{e}^{-y^{\mathrm{d}j}\lambda_3} \mathrm{e}^{-y^{j\mathrm{b}}\lambda_4} [......] , \tag{8}$$

with similar forms in $0 \leqslant y^{ij} \leqslant y^{i\mathrm{d}}$ and $0 \leqslant y^{\mathrm{d}i} \leqslant y^{ij}$.

Equating the constant pieces in (7) that are independent of y^i, y^j and Y leads, of course, to nothing new. For the exp $[-\zeta y^{ij}]$ terms with $\zeta > 0$ corresponding to a family of common intercept trajectories Z, a straightforward calculation yields

$$\begin{aligned} -(Q^i + Q^j) \sum_{\mathrm{Z}}{}' h^i_{\mathrm{PZ}} \tau_{\mathrm{Z}} h^j_{\mathrm{ZP}} = \sum_{\mathrm{d}} Q^{\mathrm{d}} \sum_{\mathrm{Z}}{}' \Bigg\{ & \sum_{\lambda>0} \frac{1}{\lambda} \sum_{J}{}' [h^i_{\mathrm{PZ}} \tau_{\mathrm{Z}} h^J_{\mathrm{Z}J} \tau_J h^{\mathrm{d}}_{J\mathrm{P}} \\ & + h^{\mathrm{d}}_{\mathrm{P}J} \tau_J h^i_{J\mathrm{Z}} \tau_{\mathrm{Z}} h^j_{\mathrm{ZP}}] - \frac{1}{\zeta} [h^i_{\mathrm{PP}} h^{\mathrm{d}}_{\mathrm{PZ}} \tau_{\mathrm{Z}} h^j_{\mathrm{ZP}} + h^i_{\mathrm{PZ}} \tau_{\mathrm{Z}} h^{\mathrm{d}}_{\mathrm{ZP}} h^j_{\mathrm{PP}}] \\ & + \sum_{\substack{\lambda \neq \zeta \\ \lambda \neq 0}} \frac{1}{\lambda - \zeta} \sum_{J}{}' [h^i_{\mathrm{P}J} \tau_J h^{\mathrm{d}}_{J\mathrm{Z}} \tau_{\mathrm{Z}} h^j_{\mathrm{ZP}} + h^i_{\mathrm{PZ}} \tau_{\mathrm{Z}} h^{\mathrm{d}}_{\mathrm{Z}J} \tau_J h^j_{J\mathrm{P}}] \Bigg\} . \end{aligned} \tag{9}$$

There are also other constraints, for example, from the $y^{ij}\,\mathrm{e}^{-y^{ij}\zeta}$ piece of (7) one finds

$$0 = \sum_{\mathrm{d}} Q^{\mathrm{d}} \sum_{\mathrm{Z}}{}' \sum_{\mathrm{Z}'}{}' h^i_{\mathrm{PZ}} \tau_{\mathrm{Z}} h^{\mathrm{d}}_{\mathrm{ZZ}'} \tau_{\mathrm{Z}'} h^j_{\mathrm{Z}'\mathrm{P}} , \tag{10}$$

but let us focus on (9). Summing (9) over $\zeta > 0$ and using (5) implies

$$\begin{aligned} & -(Q^i + Q^j) \sum_{\zeta>0} \sum_{\mathrm{Z}}{}' h^i_{\mathrm{PZ}} \tau_{\mathrm{Z}} h^j_{\mathrm{ZP}} \\ & + \sum_{\mathrm{d}} Q^{\mathrm{d}} \sum_{\zeta>0} \sum_{\mathrm{Z}}{}' \sum_{\lambda>0} \sum_{J}{}' \frac{1}{\lambda} [h^i_{\mathrm{PZ}} \tau_{\mathrm{Z}} h^j_{\mathrm{Z}J} \tau_J h^{\mathrm{d}}_{J\mathrm{P}} \\ & + h^{\mathrm{d}}_{\mathrm{P}J} \tau_J h^i_{J\mathrm{Z}} \tau_{\mathrm{Z}} h^j_{\mathrm{ZP}}] = \frac{1}{2} (Q^i + Q^j) h^i_{\mathrm{PP}} h^j_{\mathrm{PP}} . \end{aligned} \tag{11}$$

Now take $i = j = \pi^+$ and use duality assumption (6). The left-hand side of (11) vanishes. Thus,

$$h^{\pi^+}_{\mathrm{PP}} = 0 . \tag{12}$$

This rather unpalatable type of pomeron decoupling means that the (short-range) π^+ spectrum vanishes in the central region except for non-scaling terms that die off with increasing Y!

To dismiss this predicament one could argue that (i) the extreme multi-Regge assumption is unwarranted; (ii) duality is far more complicated [10] than we have assumed; (iii) the idea of a distinct short-range mechanism describable by a Regge-

Mueller model with factorizable poles but without cuts is oversimplified and should not be pushed too far.

However, in the spirit of the inclusive multi-Regge bootstrap program [12], one might try to retain our original assumptions except to modify the pomeron singularity. For example, suppose the pomeron pole propagator $\exp[\alpha_P y^{ij}]$ is replaced instead by $[C + K y^{ij}] \exp[\alpha_P y^{ij}]$ where to retain factorization both K and C are taken to be constants independent of i and j. The inclusive spectra are easily written down. Imposing the conservation sum rule (4), we find that from terms of order† (KY), the constraint (5) again follows. Use of (7) results in separate constraints for the $[K y^a y^b / Y]$ pieces and the $K \exp[-y^{ij}\zeta] y^a y^b / Y$ pieces. Straightforward manipulation yields eq. (11) modified by a multiplicative factor of C on the right-hand side. Naive duality now requires

$$\text{either } C = 0 \quad \text{or } h^{+}_{PP} = 0\,. \tag{13}$$

So by taking the pomeron propagator to be $[y^{ab} e^{y^{ij}\alpha_P}]$ the decoupling may be avoided.††

If the corresponding calculations are repeated with the pomeron propagator $[C + K y^{ij} + D(y^{ij})^2] \exp[\alpha_P y^{ij}]$, the result is again (13). In our examples, the "escape route" from the decoupling result (12) is to modify the pomeron so that it vanishes for small rapidity separation but grows with increasing rapidity separation. There may be a more general depth to this possibility than our crude examples illustrate. Harari and co-workers [13] have pointed out that for K^+p and pp total cross sections and πN elastic scattering, the pomeron does seem to "disappear" below lab momenta of 1 GeV/c. On the other hand, a modest dose of non-factorizability tends to cripple the utility of conservation sum rules, so that a possible moral of our exercises is not to push oversimplified models [14] beyond their phenomenological limitations.

I thank Robert Cahn and Lowell Brown for helpful comments, and Naren Bali for interesting discussions.

References

[1] A.H. Mueller, Phys. Rev. D2 (1970) 2963.
[2] D.P. Roy, Rutherford report RPP/T/41 (1973).
[3] G. Bellettini, Proceedings of the 5th Int. Conf. on high-energy collisions, Stony Brook, N.Y., 1973.
[4] K. Wilson, Cornell report CLNS 131 (1970), unpublished.
[5] C.E. De Tar, D.Z. Freedman and G. Veneziano, Phys. Rev. D4 (1971) 906; B.R. Webber, Nucl. Phys. B43 (1972) 541.

† Note: there are non-cancelling L-dependent pieces that are constant in energy.

†† For such a propogator, the Mueller correlation parameter f_2 is not positive but this poses no problem in two component models.

[6] H. Harari, Phys. Rev. Letters 20 (1968) 1395;
P.G.O. Freund, Phys. Rev. Letters 20 (1968) 235.
[7] W.R. Frazer, R. Peccei, S. Pinsky and C.I. Tan, Phys. Rev. D7 (1973) 2647.
[8] D. Cohen, Report at 4th Int. Symposium on multi-particle hadrodynamics, Pavia, Italy, 1973.
[9] B.R. Webber, Argonne preprint ANL/HEP 7348.
[10] S-H.H. Tye and G. Veneziano, Phys. Letters 38B (1972) 30;
M. Einhorn, M.B. Green and M.A. Virasoro, Phys. Letters 37B (1972) 292.
[11] J.R. Freemen and C. Quigg, Phys. Letters 47B (1973) 39;
J.R. Freemen, Nucl. Phys. B71 (1974) 301;
Chan Hong-Mo, H.I. Miettinen, D.P. Roy and P. Hoyer, Phys. Letters 40B (1972) 406.
[12] M.B. Green and B.R. Webber, Cambridge preprint HEP 73/4;
W.A. Bardeen and R.D. Peccei, Phys. Letters 45B (1973) 353.
[13] H. Harari and Y. Zarmi, Phys. Rev. 187 (1969) 2230;
H. Harari and E. Rabinovici, Weizmann preprint WIS 73/41 Ph.
[14] S. Pinsky and G.H. Thomas, Argonne report ANL/HEP 7345.