AVOIDANCE OF POMERON–f IDENTITY THROUGH SYSTEMATIC TREATMENT OF OVERLAP CONTRIBUTIONS TO THE CYLINDER

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We formulate a unitarity equation for the cylinder term in the topoligical expansion which includes a systematic treatment of the overlap configurations to phase space as well as the periphal configurations. If the overlap contribution has Regge pole behavior, then the singlet cylinder plus planar exchange has two output poles which, for suitable values of the parameters, can be identified with a distinct pomeron and shifted f-trajectories.

An outstanding controversy in the topological expansion/dual unitarization [1–3] framework is whether to identify the leading exchanged cylinder singularity, the pomeron, as the promoted f-trajectory [4] or whether to identify the pomeron as a unique new singularity unrelated to the f-trajectory [5]. Veneziano has suggested that the usual cylinder constructions which generate pomeron–f identity may not adequately account for overlapping configurations (i.e clusters or resonances that are emitted from different quark lines yet having almost the same longitudinal momenta). While there have been, to date, a few attempts [6] to build cylinders with overlapping configurations, these efforts have not included systematically all possible configurations that build up the cylinder. Leaving out configurations may seriously affect the subdominant cylinder singularity structure, thus keeping the question of f-extinction in doubt. The current phenomenological situation does not conclusively verify or contradict the importance of overlapping clusters; however, such configurations have been used to explain high-energy charge exchange data [7].

In this letter we present a cylinder construction that incorporates *all* known important configurations that build up cylinder exchange, including both overlapping and non-overlapping configurations. It turns out that it is possible to generate a unique pomeron together with a distinct renormalized f-trajectory [‡1].

First consider the usual cylinder construction (without overlap). Fig. 1 shows how to build up $\underline{C_B}$ and $\underline{C_F}$ which are those parts of the cylinder in which the leftmost leading particle or cluster is emitted from the back and front of the cylinder, respectively. The quantity N is the number of quarks in an SU(N) symmetric theory. It can be readily shown that the formulation in fig. 1 requires the standard pomeron–f identity phenomenon [9].

One wonders, however, if all possible configurations have been included in fig. 1. For example, is the overlap configuration of fig. 2a some smooth limit of figs. 2b,c as is tacitly assumed in the cylinder of fig. 1?

Suppose overlap configurations like fig. 2a are in-

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[‡1] Early loop calculations in the framework of the dual resonance model [8] also produced a distinct cylinder pole while including effects of overlap configurations. In recent calculations, however, the planar bootstrap condition plays a central role in determining the properties of the cylinder and its singularities and such work is no longer directly tied to the dual resonance model. The recent focus of interest has been on the delicate interplay between planar and cylinder singularities discussed in terms of unitarity and self-consistency within the framework of the topological expansion.

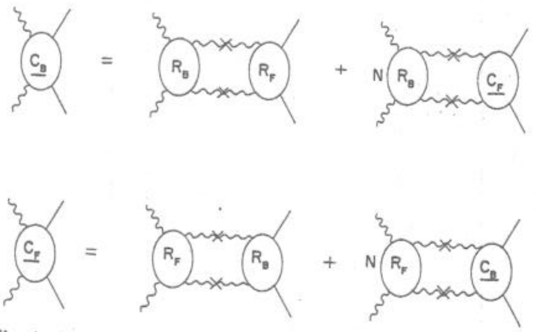

Fig. 1. Clyinder equations without overlap. R_B (R_F) is a planar (ordered) amplitude on the back (front) of the exchanged cylinder.

deed missing in the formulation of fig. 1. In addition to such overlapping particles or resonances having a limited mass, there may also be overlapping clusters of high mass which cannot be broken down into smaller resonances of both the overlapping and non-overlapping variety.

Consider the case where there is no cut-off on the average mass of the overlapping clusters. Call such an irreducible [‡2], overlapping-clusters insertion θ and denote by C_0 the sum of all cylinder graphs built with an overlap configuration at the left-most edge of the exchanged cylinder. Fig. 3 then represents the new cylinder equations that incorporate cluster overlap.

‡2 It cannot be broken down into smaller clusters separated by factorizable Regge exchanges.

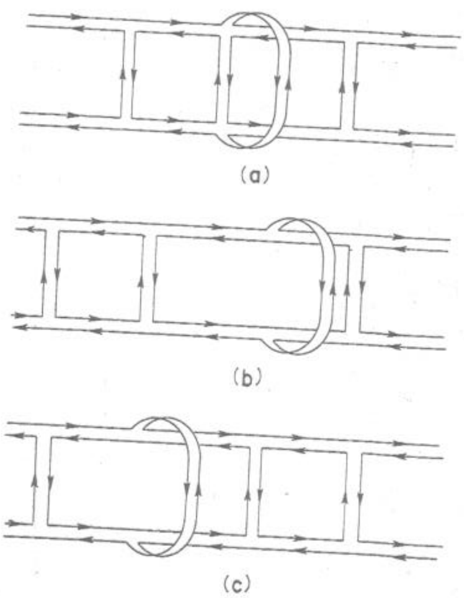

Fig. 2. (a) An overlap configuration. (b), (c) Non-overlapping configurations.

To solve the equations of fig. 3, let us take $R_B = R_F = R$ (this numerical equivalence is to be expected when planar trajectories are degenerate). Thus $C_B = C_F$ and we can then lump together

$$C' \equiv C_B + C_F + N^{-1}R_B + N^{-1}R_F .$$

The amplitude C' now satisfies the equation of fig. 4a and C_0 satisfies the equation of fig. 4b. For simplicity, take $R_{ii'\to jj'}$ to be dominated by a simple factorizable

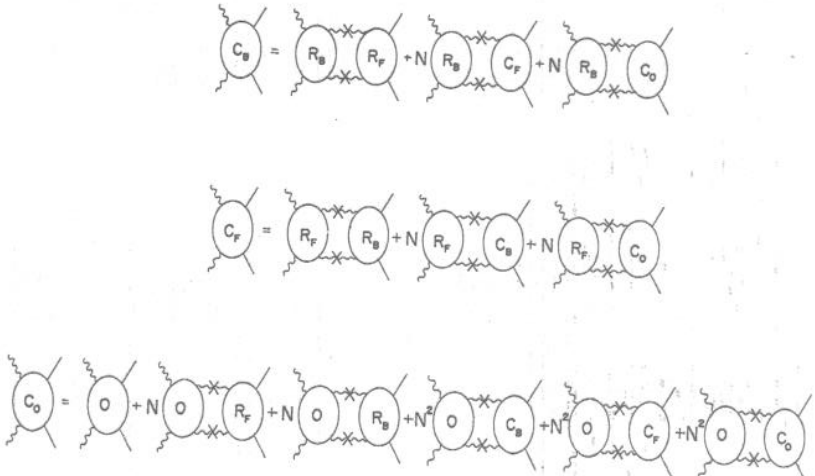

Fig. 3. Cylinder equations including overlap.

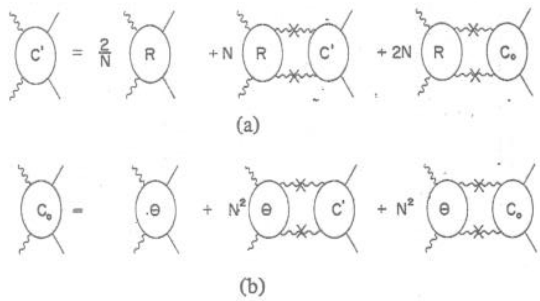

Fig. 4. (a), (b). Equation for cylinder + planar singlet.

pole with trajectory function $\alpha(t)$ and residue

$g^{\alpha}_{ii'}(t, t_i, t_{i'}) g^{\alpha}_{jj'}(t; t_j, t_{j'})$

where ii' and jj' label particles or Reggeons coupling to $\alpha(t)$ on the left and right, respectively. Also take $\theta_{ii'\to jj'}$ to be dominated by a factorizable pole with trajectory function $w(t)$ and residue

$N^{-1} h^{\omega}_{ii'}(t; t_i, t_{i'}) h^{\omega}_{jj'}(t; t_j, t_{j'})$.

(The N^{-1} factor is present because the overlap piece is of cylindrical topology.)

We arrive at the following equations in the j-plane corresponding to figs. 4a,b:

$$\widetilde{C}'_{ll'\to rr'} = \frac{2}{N}\,\frac{g^{\alpha}_{ll'} g^{\alpha}_{rr'}}{j-\alpha(t)} + \frac{N g^{\alpha}_{ll'}}{j-\alpha(t)} \int \frac{\mathrm{d}\phi_{ii'}\, g^{\alpha}_{ii'} \widetilde{C}'_{ii'\to rr'}}{\alpha(t)-\alpha(t_i)-\alpha(t_i')+1} + \frac{2N g^{\alpha}_{ll}}{j-\alpha(t)} \int \frac{\mathrm{d}\phi_{ii'}\, g^{\alpha}_{ii'} \widetilde{C}_0^{ii'\to rr'}}{\alpha(t)-\alpha(t_i)-\alpha(t_i')+1}\,, \tag{1}$$

$$\widetilde{C}_0^{ll'\to rr'} = \frac{h^{\omega}_{ll'} h^{\omega}_{rr'}}{N(j-\omega(t))} + \frac{N h^{\omega}_{ll'}}{j-\omega(t)} \int \frac{\mathrm{d}\phi_{ii'}\, h^{\omega}_{ii'} \widetilde{C}'_{ii'\to rr'}}{\omega(t)-\alpha(t_i)-\alpha(t_i')+1} + \frac{N h_{ll'}}{j-\omega(t)} \int \frac{\mathrm{d}\phi_{ii'}\, h^{\omega}_{ii'} \widetilde{C}_0^{ii'\to rr'}}{\omega(t)-\alpha(t_i)-\alpha(t_i')+1}\,, \tag{2}$$

where $\mathrm{d}\phi_{ii'}$ integrates over the internal momentum transfers and the wiggle denotes the j-plane transform.

Solving eqs. (1) and (2) one finds that the total singlet exchange contribution (planar plus cylinder) $\bar{C}' = C' + C_0$ is given by

$$\widetilde{\bar{C}}'_{ll'\to rr'} = F(t)\,[(j-\alpha(t)-NG(t)) \times (j-\omega(t)-NH(t)) - 2N^2 A(t) B(t)]^{-1}\,,$$

where

$$F(t) = \{2N^{-1}[j-\omega(t)-NH(t)]\, g^{\alpha}_{ll'} g^{\alpha}_{rr'} + 2A(t) g^{\alpha}_{ll'} g^{\alpha}_{rr'} + N^{-1}[j-\alpha(t)-NG(t)]\, h^{\omega}_{ll'} h^{\omega}_{rr'} + 2B(t) h^{\omega}_{ll'} g^{\omega}_{rr'}\}\,,$$

$$G(t) = \int \frac{\mathrm{d}\phi_{ii'}\, g^{\alpha}_{ii'} g^{\alpha}_{ii'}}{\alpha(t)-\alpha(t_i)-\alpha(t_i')+1}\,,$$

$$H(t) = \int \frac{\mathrm{d}\phi_{ii'}\, h^{\omega}_{ii'} h^{\omega}_{ii'}}{\omega(t)-\alpha(t_i)-\alpha(t_i')+1}\,,$$

$$A(t) = \int \frac{\mathrm{d}\phi_{ii'}\, g^{\alpha}_{ii'} h^{\omega}_{ii'}}{\alpha(t)-\alpha(t_i)-\alpha(t_i')+1}\,,$$

$$B(t) = \int \frac{\mathrm{d}\phi_{ii'}\, h^{\omega}_{ii'} g^{\omega}_{ii'}}{\omega(t)-\alpha(t_i)-\alpha(t_i')+1}\,.$$

Note that $\widetilde{C}'$ has two singularities – the leading singularity may be identified as a distinct pomeron while the subdominant singularity may be considered to be the "shifted" f-trajectory. Thus it appears that pomeron –f identity is avoided.

Now suppose one wants to generate a pomeron with an intercept arount unity. With no overlap, the resultant pomeron [10] has an intercept given by $\alpha(0) + NG(0)$. This is slightly bigger than unity by virtue of the planar bootstrap condition. Including overlap raises the pomeron intercept; so, to avoid too large an intercept, the overlap couplings h^{ω} must be small. With $A, B, H \ll 1$ the resultant pomeron has intercept just above one and a residue of order $N^{-1}(g^{\alpha})^2$ while the other cylinder singularity (possibly the "new" f-trajectory) is a pole near $\omega(t)$ with a weak residue of order $(g^{\alpha})^2(h^{\omega})^2$. Thus it appears that with overlap it is possible to maintain a distinct f-trajectory although its couplings will be small. This contrasts with the no overlap case, $h^{\omega} \to 0$, where pomeron–f identity previals. Some formulations of the planar bootstrap [2,11,12] introduce parameters that characterize planar cluster production – for example, average cluster size. While a completely consistent planar bootstrap that avoids generating Regge cuts and that depends explicitly on these physical cluster parameters, has not yet been achieved, such formulations as in refs. [2,11,12] do allow a lower, no-overlap pomeron intercept [12] than in those constructions where planar clusters are treated only as mathematical entities that avoid double-counting in unitarity. If one could perfect those formulations that include physical cluster parameters, so that a no-overlap pomeron of intercept less than unity is generated, then adding in the overlap cylindrical configurations could conveivably build up a final pomeron of intercept around one, *without weak residues* for the renormalized f-trajectory.

If, instead of allowing the overlapping clusters to extend to large masses, we had introduced a cut-off for the overlap mass, then, it turns out that pomeron–f identity is preserved – the overlap serves merely to raise the pomeron singularity.

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