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Doubling of the cylinder contribution in the topological expansion and Pomeron-f identity

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This note examines how the various intermediate-state configurations and different regions of phase space contribute to building up the planar and cylinder contributions of the topological expansion. Various factors of 2 are generated on the planar and cylinder level. For the cylinder, the doubling mechanism is different depending on whether the number of intermediate-state clusters is even or odd. Despite the cylinder doubling, the Pomeron-f identity is retained.

I. INTRODUCTION

In the topological expansion[1,3] or dual unitarization,[2,3] the Pomeron contribution to high-energy scattering first occurs at the cylinder level. Several calculations[4,5] show that, when the planar and cylinder terms are combined, the singlet-exchange Regge pole in the planar amplitude is replaced by a single leading pole with vacuum-exchange quantum numbers and trajectory intercept near one. This promotion of the f trajectory by the cylinder insertion—without generating a new unique pole in addition to the planar f—is called the Pomeron-f identity.[4,5] Intense questioning of the validity of the Pomeron-f identity has led the present authors to a careful study of the manner in which the cylinder and planar amplitudes combine. In particular, we have examined the contributions of "flipped" ordered amplitudes (i.e., different ordered amplitudes having the same channel structure) to see whether including both amplitudes and their flips has any influence on the Pomeron-f identity. It turns out that various factors of 2 are introduced on both the planar level and cylinder level. On the cylinder level the mechanism which produces the factor of 2 is different, depending on whether the number of intermediate-state clusters is odd or even. Despite these factors of 2 required by the presence of flipped amplitudes, the phenomenon of the Pomeron-f identity is not altered.

In the process of examining the role of flipped amplitudes, we also present some practical understanding of the relation of hadron and quark diagrams.

II. THE FLIPPED AMPLITUDE

The basic ingredient of the topological expansion is the ordered amplitude,[3] examples of which are shown in Fig. 1 for a four-line process. The labels A, B, C, and D include not only the particle type but also its momentum, spin, etc. The clockwise arrow in Fig. 1 indicates a "sense" or orientation and the six different noncyclic permutations of the labels gives six different ordered amplitudes. Ordered amplitudes possess energy cuts only in channels where particles are adjacent in the order. In Fig. 1(a), for example, cuts exist corresponding to the AB channel (the s channel) and the CD channel (the t channel) but not the AC channel (the u channel).

The amplitude in Fig. 1(b) is the flip of the one in Fig. 1(a). Both amplitudes in Fig. 1 are cut in the same channels yet they represent different ordered amplitudes. Both diagrams in Fig. 1 have an s-channel discontinuity, thus making them relevant for an s-channel unitary calculation. The other four ordered amplitudes having different permutations of the labels A, B, C, D are either strongly damped for the large-s, fixed-t asymptotic limit we consider or else have no s-channel discontinuity.

In a standard quark model the hadron amplitudes in Fig. 1 have corresponding quark diagrams shown in Fig. 2. For simplicity, we assume only two quarks. With the standard assumptions about SU(2) symmetry, the two amplitudes in Figs. 2(a) and 2(b) are equal.

In both Figs. 1(b) and 2(b), the orientation of the flipped amplitudes is still given by clockwise arrows. Thus these graphs are not literally flips of Figs. 1(a) and 2(a) but simply have the inverted cyclic order for the labels A, B, C, D.

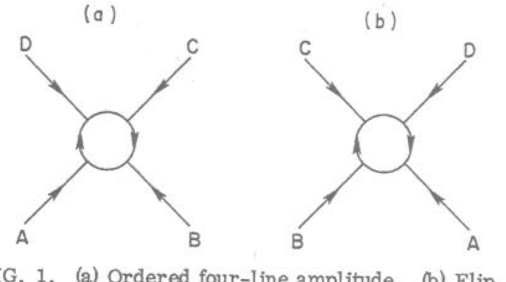

FIG. 1. (a) Ordered four-line amplitude. (b) Flip amplitude of (a).

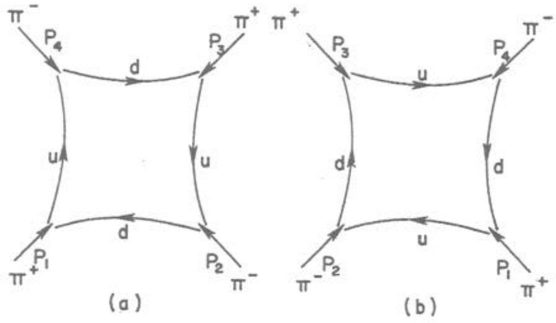

FIG. 2. (a) Quark-line model example of Fig. 1(a). (b) Quark-line model example of Fig. 1(b).

In Figs. 2(a) and 2(b) the amplitude and its flip correspond to different quark diagrams so that there is no confusion about the need to include both terms. Confusion may arise when both the amplitude and its flip have the same quark diagram, as is shown in Figs. 3(a) and 3(b). It might appear that including both Figs. 3(a) and 3(b) is double counting but this is not the case. We shall see later that both graphs must be kept to consistently combine planar and cylinder terms.

One final point relevant in our later discussion is that a quark diagram has a nonzero flip amplitude only if every other quark line in the diagram (as one follows the quark lines around the diagram) is of the same quark type.

III. UNITARITY FOR ORDERED AMPLITUDES

Ordered amplitudes are postulated to satisfy unitarity. In this section we illustrate by a simple example how unitarity "works" for the two-particle discontinuity in hadron or quark diagrams.

Unitarity for Fig. 2(a) is shown in Fig. 4(a). Assigning, for simplicity, the coupling g^2 to each intermediate-state planar insertion, we see that Fig. 4 requires $g^2 \sim 2g^4$ or $g^2 \sim \frac{1}{2}$. This is the familiar result that $g^2 \sim 1/N$, where N is the number of quark flavors.

It is interesting to reexamine the unitarity condition of Fig. 4 in terms of hadron diagrams as shown in Fig. 5. Note that (i) both orders for the intermediate-state particles should be summed over unless the particles are identical, (ii) exotic channels (i.e., $\pi^+\pi^+$) are absent, (iii) the π^0 is the neutral member of the triplet, namely

$$\pi^0 = \frac{u\bar{u} - d\bar{d}}{\sqrt{2}}, \tag{3.1}$$

whereas the $\pi^{0\prime}$ is the singlet

$$\pi^{0\prime} = \frac{u\bar{u} + d\bar{d}}{\sqrt{2}}. \tag{3.2}$$

To achieve consistency between Fig. 4 and Fig. 5, there must be weighting factors which relate the various amplitudes. If g^2 is the weight for a planar quark diagram insertion, then, from Eqs. (3.1) and (3.2) it follows that all the ordered amplitudes for $\pi^+\pi^- \to 2$ neutrals have a weight $g^2/2$ (see, for example, Fig. 6). In orders of g^2, Fig. 5 becomes

$$g^2 \sim g^4 + 4\times(\tfrac{1}{4}g^4),$$

or

$$g^2 \sim 2g^4, \tag{3.3}$$

which agrees with the results of Fig. 4.

IV. THE EXCHANGED CYLINDER FOR TWO-BODY INTERMEDIATE STATES

Let us now turn to the cylinder correction to Figs. 1(a) and 1(b). In particular consider the two-particle intermediate state shown in Figs. 7(a) and 7(b) where as before, we go to high s and fixed t.

Figures 7(a) and 7(b) include two terms corres-

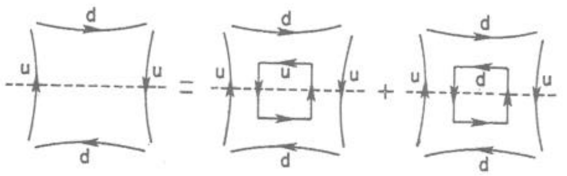

FIG. 4. Unitarity for amplitude in Fig. 2(a).

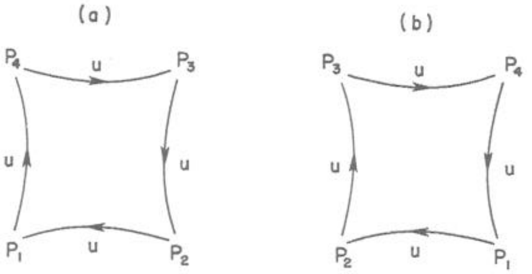

FIG. 3. (a) Quark-line diagram with a single quark. (b) Flip amplitude of (a).

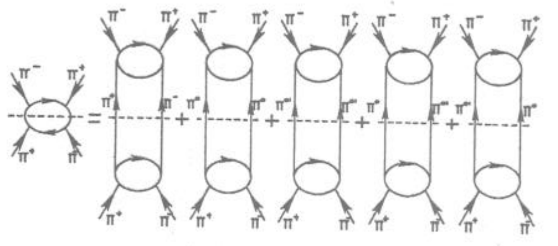

FIG. 5. Unitarity for hadron diagrams.

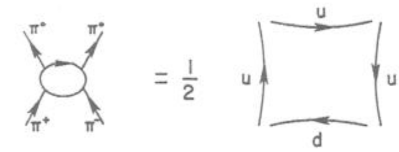

FIG. 6. Example of relation of hadron and quark diagrams.

ponding to the insertion into s-channel unitarity of an amplitude and its flip; if particles E and F are of the same type, then only one of the two diagrams in Fig. 7 is to be kept, since in that case the phase-space integration for E and F will automatically include the flip amplitude.

Another subtlety in assembling the cylinder contributions is illustrated in Fig. 8. Starting with Figs. 1(a) and 1(b), one might think that the cylinder corrections to Fig. 1(a) are given by Figs. 7(a) and 7(b) and those to Fig. 1(b) are given by Figs. 8(a) and 8(b). But this is incorrect—the diagrams of Figs. 8(a) and 8(b) are the *same* contribution as that of Figs. 7(a) and 7(b), since the same ordered amplitudes occur in both cases. In particular, Fig. 8(a) represents the same contribution as Fig. 7(b), while Fig. 8(b) is the same as Fig. 7(a).

In terms of quark diagrams, the remarks of the last paragraph mean that the cylinder contributions of Fig. 9(a) are identical to those of 9(b) so that only one of them is kept as a correction to Figs. 2(a) and 2(b).

Next we consider another double-counting question by examining the diagram in Fig. 10. This diagram looks like Fig. 9(a) except that the $u\bar{u}$ and $d\bar{d}$ intermediate states have been slipped past each other to exchange their relative positions. Clearly, Fig. 10 involves unitarity for the same ordered amplitudes as Fig. 9(a) so one wonders whether Fig. 10 is a separate contribution from Fig. 9(a). Certainly if one assumed no connection between particle order and longitudinal momentum or rapidity ordering, then integrating over the full

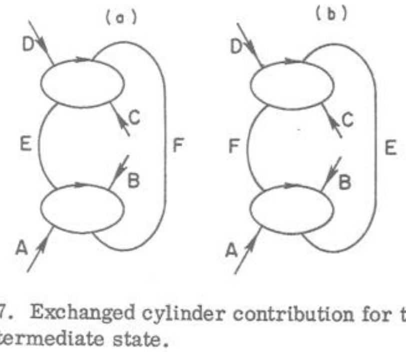

FIG. 7. Exchanged cylinder contribution for two-particle intermediate state.

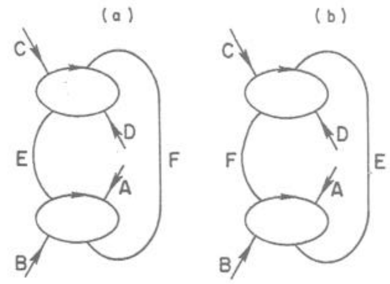

FIG. 8. Equivalent diagrams to Figs. 7(a) and 7(b).

phase space of Fig. 9(a) would include the configuration of Fig. 10. However, quark diagrams are usually assumed to specify not only particle order but also rapidity order. Thus in Fig. 9(a) the $u\bar{u}$ intermediate state has negative rapidity in the center of mass while the $d\bar{d}$ state has positive rapidity. On the other hand, Fig. 10 represents a region of phase space where $d\bar{d}$ has negative rapidity while $u\bar{u}$ has positive rapidity. Since Figs. 9(a) and 10 are interpreted as representing two different regions of phase space, both contribute to building up the cylinder. Note that such a doubling mechanism is absent on the planar level.

To further illustrate the different phase-space regions represented by Figs. 9(a) and 10, we show in Figs. 11(a) and 11(b) hadron diagrams. The contributions in Figs. 11(a) and 11(b) comprise the cylinder term Fig. 7(a). Note that the exchanged Regge poles in Fig. 11(a) may have different quantum numbers from those in Fig. 11(b), consistent with these two diagrams, each representing distinct contributions.

In Fig. 12 we show the hadron graphs that contribute to the two-particle intermediate states of the exchanged cylinder. The appropriate weighting in g^2 is given for each term including the phase-space factor of 2. The cylinder strength is thus $2g^4$, which is of the order g^2 since $g^2 \sim \frac{1}{2}$. This is to be compared with the planar singlet which is $\frac{1}{2}$ (i.e., $1/N$) of the planar amplitude Figs. 2(a) and 2(b)—that is, $\frac{1}{2}\times(2g^2)=g^2$. That the planar singlet and cylinder strengths be the same is a necessary condition for the Pomeron-f identity.

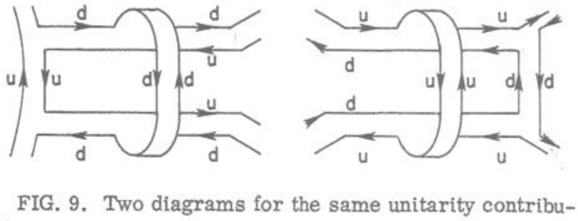

FIG. 9. Two diagrams for the same unitarity contribution.

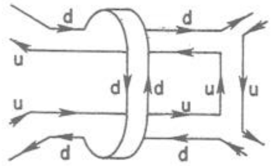

FIG. 10. A different region of phase space for the diagram in Fig. 9(a).

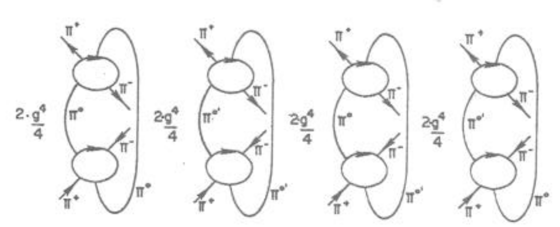

FIG. 12. Cylinder contribution in terms of hadron diagrams.

V. MANY-PARTICLE INTERMEDIATE STATES IN THE CYLINDER

We now consider the many-particle (or many-cluster) contributions to the cylinder having at least two pairs of twists. The simplest example is in Fig. 13(a) where momentum transfers AE, BG, ED, CG are all taken to be small so that Regge exchanges can be used in these channels. The intermediate states in Fig. 13 are planar clusters of particles or single particles. We shall now show that if the number of clusters separated by small momentum transfers is odd [as in Fig. 13(a)] there is a new doubling mechanism unlike that discussed in the previous section.

If in Fig. 13(a) the momentum transfers AE, BG, ED, CG are small and the energy is asymptotic, then one has a cylinder diagram with two pairs of twisted Reggeons. Let us now ask if a mechanism for doubling this cylinder contribution [Fig. 13(a)] exists in analogy with that of Figs. 11(a) and 11(b). This would involve in Fig. 13(a), e.g., slipping the lines E and F past each other [as in Figs. 11(a) and 11(b)]. However, at high energy this means a Regge region where the momentum transfers DF, CG, AF, BG are small and one has only *one* pair of twisted Reggeons instead of two. In this case E and G can be combined into a single planar cluster and this contribution has already been counted in, e.g., Fig. 9(a) which has only one pair of twists between clusters.

Fig. 13(b) is the diagram which does lead to the numerical doubling of Fig. 13(a). In Fig. 13(b) the ordered amplitudes at the top and bottom of the diagram are the flip amplitudes of those occurring in Fig. 13(a). One might think that the relation of the contribution of Fig. 13(b) to Fig. 13(a) is the same as that of Fig. 7(a) to Fig. 7(b) where flip amplitudes also appear in Fig. 7(b). There is, however, an important difference between the two situations. The contribution of Fig. 7(b) can be thought of as already included in the unitarity sum of Fig. 7(a) if we agree in Fig. 7(a) to sum over particle types as one normally would. In fact, the quark diagram Fig. 9(a) includes both Figs. 7(a) and 7(b), for in quark diagram language there is only one intermediate two-cluster state, since there are no closed quark loops [in terms of the hadron diagrams of Fig. 12, of course, Fig. 9(a) is broken down into four pieces as shown].

By contrast, no sum over particle types in Fig. 13(a) will include the contribution of Fig. 13(b). In terms of quark diagram language both Figs. 14(a) and 14(b) must be separately included. The point here is that the cylinder term with three intermediate clusters gets a factor of 2 because Figs. 14(a) and 14(b) are topologically inequivalent but numerically equal, whereas in the case of the two-cluster intermediate state a factor of 2 arises from a different mechanism: Figures 9(a) and 10 are equivalent topologically but they represent two different (but numerically equal) regions of phase space. This latter doubling mechanism does not operate at the level of the three-particle intermediate clusters of Figs. 14(a) and 14(b) because, as already pointed out, sliding quark lines past

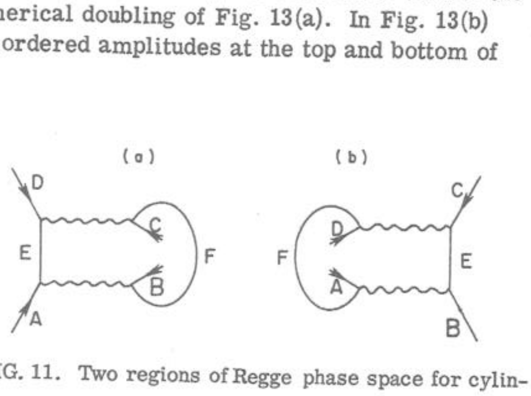

FIG. 11. Two regions of Regge phase space for cylinder.

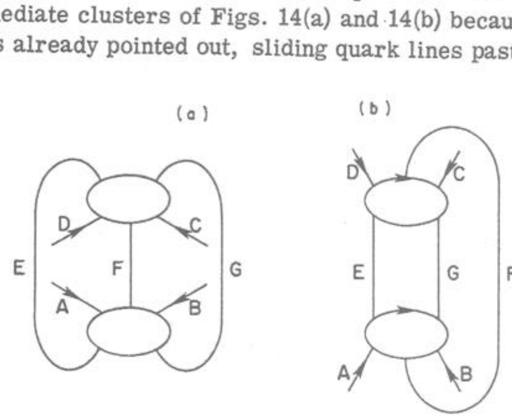

FIG. 13. (a) Cylinder term with double twists. (b) An additional double-twist team.

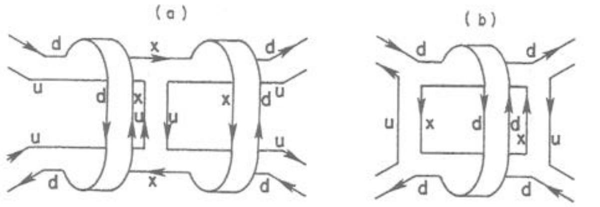

FIG. 14. (a) Quark-line model example of Fig. 13(a). The x quark line is summed over. (b) Quark-line model example of Fig. 13(b). The x quark line is summed over.

each other in this case reduces the number of twist pairs to one and this contribution has already been included in Figs. 9(a) and 10.

One final remark about the three-cluster intermediate-state contribution to the cylinder which points up to a difference in working with hadron diagrams versus quark diagrams. We note that the quark amplitudes at the top and bottom of Fig. 14(b) are not flips of those in Fig. 14(a). In fact, the lower amplitude in Fig. 14(a) has no flip amplitude as it violates the rule discussed earlier that a quark line amplitude only has a nonzero flip amplitude if every other quark line is of the same type. Of course, if the quark diagrams in Figs. 14(a) and 14(b) were expanded in terms of their hadron content, the terms would take the form of Figs. 13(a) and 13(b).

Proceeding beyond three-cluster intermediate states, one may show straightforwardly that all even numbers of cluster configurations have a doubling mechanism similar to the two-cluster case and that all odd numbers of cluster configurations have a doubling mechanism similar to the three cluster case. For example, the doubling that is shown in Figs. 11(a) and 11(b) has an analog in the case of the four clusters shown in Figs. 15(a) and 15(b).

VI. SUMMING THE CYLINDER DIAGRAMS AND POMERON-f IDENTITY

In this section we sum up the various cylinder contributions to generate a cylinder integral equation of the type proposed by Chan *et al.*[2] Despite the appropriate factors of 2 that are a consequence of the doubling mechanisms discussed earlier, the phenomenon of the Pomeron-f identity is unaltered.

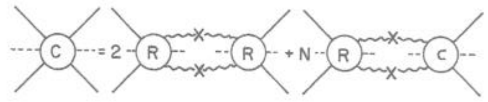

FIG. 16. Integral equation for the cylinder.

The cylinder summation may be summarized by Fig. 16 where R represents the ordered (ring) amplitude, N represents the number of quarks, and the exchanged Reggeons are twisted. It is straightforward to check that the factors of 2 and N are indeed the correct ones to account for closed quark loops and to generate the various doublings discussed in the preceding sections.

To study the Pomeron-f identity issue it is convenient to focus on the sum of the cylinder and the singlet planar projection, as suggested by Chew and Rosenzweig.[3,4] The planar amplitude is built from an ordered amplitude R and its flip—that is, both Figs. 1(a) and 1(b) [or Figs. 2(a) and 2(b)]. The planar singlet is thus $2/N$ times the R amplitude.

Figure 17 shows the integral equation for $C' \equiv (2/N)R + C$. The Pomeron-f identity follows immediately from Fig. 17 since C' cannot have a pole at the Regge poles of R without generating a double pole. In the case where a planar quark diagram has no flip amplitude, the Pomeron-f identity still holds. Although the flip amplitude is zero, the singlet projection does not vanish and is still equal to the singlet projection of the unflipped amplitude. There are, of course, amplitudes for which no singlet projection exists at all either in the quark diagram or its flip.

Finally we point out that the integral equations

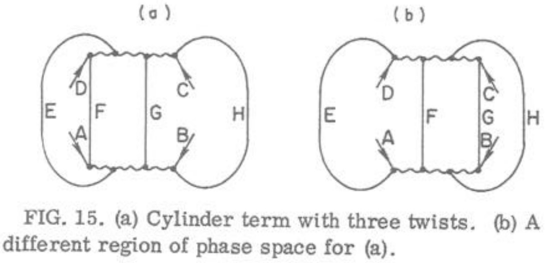

FIG. 15. (a) Cylinder term with three twists. (b) A different region of phase space for (a).

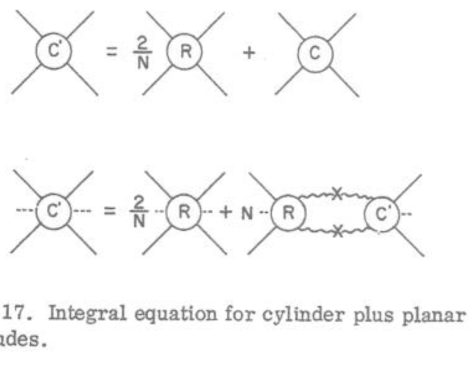

FIG. 17. Integral equation for cylinder plus planar amplitudes.

of Figs. 16 and 17 ignore those regions of phase space when clusters overlap to the extent that small-momentum-transfer Regge exchanges cannot describe the process. The appropriate equations that include cluster overlap will be discussed elsewhere.

ACKNOWLEDGMENTS

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