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A one-glass achromatic doublet

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Abstract. This note presents the design of a doublet, of positive power, whose focal points at two arbitrary wavelengths coincide, the two components being made of identical material. Achromatism is achieved by having the rear focal points of the first component (at the two wavelengths) straddle the second component.

1. Introduction

In work with I.R. lasers, it is often convenient to superimpose, upon the I.R. beam, the beam from a low-power laser operating in the visible portion of the spectrum. This may facilitate alignment and aiming of the I.R. laser. The effectiveness of this procedure depends on the achromatization of the optical system for the two wavelengths. Such achromatization may be very difficult in the mid-I.R., where available optical materials—transparent in the visible as well—are quite limited. In such instances, as well as in other applications, a one-glass achromatic doublet may be helpful. Indeed, it is possible to construct such a doublet with positive power by having the rear focal points of the first element, taken at the two wavelengths, straddle the second element. With such a doublet, the change in effective focal length can be compensated for exactly by the change in principal plane distance. This note presents the specifications for such a doublet, followed by an illustrative example and the proof of the results given. These cover a special case ; the derivation of the general solution is rather complex and is presented in the Appendix.

2. Specifications

The total curvature of the first and second components and their spacing are, respectively,

$$1/r_{11}+1/r_{12}=c_1=p/(n-1)(p+1)f, \tag{1}$$

$$1/r_{21}+1/r_{22}=c_2=c_1/p^2, \tag{2}$$

$$d=(1-p^2)f/p. \tag{3}$$

The resulting back focal length is

$$f_b=pf. \tag{4}$$

Here f is the effective focal length of the doublet, r_{ij} is the jth radius of the ith element, with convex surfaces taken as positive, and

$$p=(n'-n)/(n'+n-2), \tag{5}$$

where n and n' are the refractive indices at the two wavelengths.

3. Illustrations

By way of illustration, consider an arsenic trisulphide glass doublet to be achromatized for the $0{\cdot}6328\ \mu m$ line of the He–Ne laser and the $10{\cdot}6\ \mu m$ line of the CO_2 laser. The corresponding refractive indices are 2·6062 and 2·3770 respectively [1]. The corresponding value of p is then found to be 0·07683 and the required design parameters are

$$c_1 = 0{\cdot}5181 \times 10^{-3} = (1930)^{-1}$$

$$c_2 = 0{\cdot}08778 = (11{\cdot}39)^{-1},$$

$$d = 1294,$$

$$f_b = 7{\cdot}683,$$

all for an effective focal length of 100.

Finite object-to-image distances

The above is for an infinitely distant object. To deal with finite object and image distances, two doublets, with focal lengths equal to object and image distances, respectively, may be used in combination, with the doublet on the object side inverted. The second lens of the object doublet may then be combined with the first lens of the image doublet into a single lens having the combined power. In this way, a triplet results.

For instance, if it is desired to image an object at $2\times$ magnification, we may place before the above doublet an inverted doublet which is scaled down from the former by a factor of one-half. The resulting triplet would then have the following parameters :

$$1/c_1 = 11{\cdot}39/2 = 5{\cdot}696,$$

$$c_2 = 2/1930 + 1/1930 = 1/643,$$

$$1/c_3 = 11{\cdot}39,$$

$$d_{12} = 647, \quad d_{23} = 1294,$$

$$\text{distance of object to first lens} = 3{\cdot}842,$$

$$\text{distance of image to last lens} = 7{\cdot}683.$$

Note that object and image distances are small compared to the effective focal length and that the lens separation is large compared to it. These facts must be considered in weighing the practicability of this design.

4. Proof of formula

We now show that the formulae (1)–(3) do, indeed, yield a system which exhibits a common focal point for refractive indices n and n'. For a combination of two thin lenses, the basic formulae for effective focal length (f) and rear principal plane distance (d_{p2}) are

$$f = f_1 f_2/(f_1 + f_2 - d) \tag{6}$$

and

$$d_{p2} = df/f_1, \tag{7}$$

where f_1 and f_2 are the focal lengths of the two components and d is their spacing. For the thin lenses,

$$1/f_i=(n-1)c_i, \quad c_i=1/r_{i1}+1/r_{i2}, \quad i=1, 2, \tag{8}$$

where n is the refractive index, c is the total curvature, and r_{i1} and r_{i2} are the radii of curvature of the ith thin lens. Formulae (6)–(8) are found in standard texts on geometrical optics under the heading ' Gaussian optics ' (see, for example, [2]). From equations (7) and (6) we find the back focal distance :

$$f_b=f-d_{p2}=f(1-d/f_1)=(f_1f_2-df_2)/(f_1+f_2-d). \tag{9}$$

On substituting (1) and (2) into (8), we find

$$f_1=1/(n-1)c_1=(p+1)f/p, \tag{10}$$

$$f_2=p(p+1)f. \tag{11}$$

On substituting these, together with (3), into (9), we find

$$f_b=pf,$$

confirming equation (4).

We now calculate the back focal length at the other wavelength, denoting the corresponding focal length by a prime. This f'_b must also equal pf. We first note that, from (5),

$$(1-p)/(1+p)=\frac{(n'+n-2)-(n'-n)}{(n'+n-2)+(n'-n)}=(n-1)/(n'-1). \tag{12}$$

Now, from (8) and (1),

$$f'_1=1/(n'-1)c_1=\frac{n-1}{n'-1}\frac{p+1}{p}f. \tag{13}$$

On substituting (12) for the first factor of this, we obtain

$$f'_1=(1-p)f/p. \tag{14}$$

From (8) we note that f_i is inversely proportional to c_i. Hence from (2), it is evident that

$$f'_2=p^2f'_1=p(1-p)f. \tag{15}$$

By analogy from (9), we now write the expression for f'_b as

$$f'_b=f'_2(f'_1-d)/(f'_1+f'_2-d). \tag{16}$$

On substituting into this the values of f'_1, f'_2 and d from equations (14), (15) and (3), respectively, we find after some algebraic simplification that

$$f'_b=pf=f_b, \tag{4'}$$

confirming the coincidence of the two foci.

Appendix

Derivation of specifications for achromaticity

1. *Condition for achromatism*

On substituting equation (8) into equation (6), we find

$$f=f_2/(1+f_2/f_1-d/f_1)=f_2/(R+A), \tag{A 1}$$

where we have introduced the arbitrary variables

$$R=f_2/f_1, \tag{A 2}$$

$$A=f_b/f=1-d/f_1 \tag{A 3}$$

(see equation (9)). Similarly, we define

$$A'=f'_b/f'=1-d/f'=1-\frac{(n'-1)d}{(n-1)f_1}. \tag{A 4}$$

(For the last step see equation (A 7).) From (A 3), upon substitution of (A 1), we find that

$$1/f_b-1/Af=(1+R/A)/f_2. \tag{A 5}$$

Hence

$$1/f'_b-1/f_b=(1+R/A')/f'_2-(1+R/A)/f_2. \tag{A 6}$$

Multiplying through by f_2 and noting that, from (8),

$$f_2/f'_2=(n'-1)/(n-1), \tag{A 7}$$

we find

$$\begin{aligned} f_2(1/f'_b-1/f_b)&=(1+R/A')(n'-1)/(n-1)-(1+R/A)\\ &=[(n'-1)/(n-1)]-1+R[(n'-1)/(n-1)A'-1/A]. \end{aligned} \tag{A 8}$$

On simplifying and factoring out $(n-1)^{-1}$, this becomes

$$f_2(1/f'_b-1/f_b)=\{(n'-n)+R[(n'-1)A-(n-1)A']/AA'\}/(n-1).$$

Substituting for A and A' (within the brackets) from (A 3) and (A 4), and simplifying, gives

$$f_2\,(1/f'_b-1/f_b)=(1+R/AA')(n'-n)/(n-1). \tag{A 9}$$

For the two back foci to coincide, this expression must vanish, i.e.

$$R=-AA'. \tag{A 10}$$

This is the condition yielding achromatism.

2. *Conditions for reality of image*

Assuming that the material characteristics and the desired system focal length are known, equations (A 1) and (A 10) are a system of two equations with three unknown quantities (f_1, f_2, d). This would permit us to choose one of these arbitrarily. There are, however, some restrictions we must observe in order to obtain a real image, viz. the back focal length of the system must be positive :

$$f_b > 0. \tag{A 11}$$

In addition, the distance d between the first and second elements is positive, by definition :

$$d > 0. \tag{A 12}$$

Hence, substituting R from (A 10) into (A 5) and applying (A 12), we find that

$$1/f_b = (1-A')/f_2 = (n'-1)d/(n-1)f_1 f_2 > 0, \tag{A 13}$$

with the last step following from (A 4). Since d and the parenthetical factor are positive, f_1 and f_2 must have the same sign. Therefore, from (A 2),

$$R > 0 \tag{A 14}$$

and, hence, from (A 10), A and A' must have opposite signs. Note that from (A 3)–(A 4)

$$\begin{aligned} A - A' &= [(n'-1)/(n-1)-1]d/f_1 \\ &= d(n'-n)/(n-1)f_1. \end{aligned} \tag{A 15}$$

Arbitrarily denoting the greater of the two indices by n',

$$A > A'. \tag{A 16}$$

Then it follows from (A 15) that

$$0 < A < d(n'-n)/(n-1)f_1. \tag{A 17}$$

Hence, we write for the arbitrary factor, k,

$$0 < k < 1, \tag{A 18}$$

$$A = kd(n'-n)/(n-1)f_1 \tag{A 19}$$

and

$$A' = (k-1)d(n'-n)/(n-1)f_1. \tag{A 20}$$

The choice of k, together with (A 1) and (A 10), now fixes all the unknown variables.

3. *Explicit solutions for the unknown variables*

Equating (A 3) to (A 19), we find

$$1 - d/f_1 = kd(n'-n)/(n-1)f_1,$$

which yields, on solving for f_1,

$$\begin{aligned} f_1 &= d[1+k(n'-n)/(n-1)] = d[n-1+k(n'-n)]/(n-1) \\ &= dg/(n-1), \end{aligned} \tag{A 21}$$

where we have used the abbreviation

$$g = n-1+k(n'-n). \tag{A 22}$$

From (A 10), and on substitution from (A 19)–(A 20),

$$f_1f_2=f_1^2\,R=-f_1^2\,AA'=k(1-k)(n'-n)^2\,d^2/(n-1)^2. \qquad \text{(A 23)}$$

The third equation required is obtained by solving (6) for d, yielding

$$d=f_1+f_2-f_1f_2/f=f_1f_2(1/f_1+1/f_2-1/f). \qquad \text{(A 24)}$$

We now proceed to solve equations (A 21), (A 23) and (A 24) for the unknown quantities. First we solve (A 21) for d and equate the result to (A 24) :

$$(n-1)f_1/g=f_1+f_2-f_1f_2/f\,; \qquad \text{(A 25)}$$

on dividing through by f_1, this becomes

$$1+f_2/f_1-f_2/f=(n-1)/g. \qquad \text{(A 26)}$$

From (A 23) we find

$$d^2=f_1f_2(n-1)^2/k(1-k)(n'-n)^2. \qquad \text{(A 27)}$$

Equating this to the square of the first member of (A 25) gives

$$(n-1)^2\,f_1^2/g^2=f_1f_2(n-1)^2/k(1-k)(n'-n)^2. \qquad \text{(A 28)}$$

Multiplying through by $k(1-k)/(n-1)^2\,f_1^2$ and solving the result for f_2/f_1 gives

$$f_2/f_1=(n'-n)^2\,k(1-k)/g^2. \qquad \text{(A 29)}$$

Solving (A 26) for f_2/f_1 and equating the result to (A 29) yields

$$(n-1)/g+f_2/f-1=(n'-n)^2\,k(1-k)g^2. \qquad \text{(A 30)}$$

Solving this for f_2/f, we find

$$f_2/f=[g-(n-1)]/g+(n'-n)^2\,k(1-k)/g^2.$$

On substituting for g (in the numerator only) from (A 22), we obtain

$$f_2/f=k(n'-n)/g+(n'-n)^2\,k(1-k)/g^2.$$

Combining the fractions, and again substituting for g in the numerator, we finally find

$$\boxed{f_2/f=k(n'-n)(n'-1)/g^2.} \qquad \text{(A 31)}$$

This determines f_2 from the given constants. To find f_1, we invert (A 29) and multiply through by f_2/f_1, yielding

$$f_1/f=(f_2/f)g^2/(n'-n)^2\,k(1-k)$$

and, on substituting for f_2/f from (A 31),

$$\boxed{f_1/f=(n'-1)/(n'-n)(1-k).} \qquad \text{(A 32)}$$

We find d by dividing (A 27) by f^2, substituting for f_1/f and f_2/f from (A 31) and (A 32), and taking the root :

$$\boxed{d/f=(n'-1)(n-1)/(1-k)(n'-n)g.} \qquad \text{(A 33)}$$

Formulae (A 31)–(A 33) are the required solutions. In conclusion we note from (A 3) that

$$f_b/f = A = kd(n' - n)/(n-1)f_1 = k(n'-n)/g, \quad \text{(A 34)}$$

where we have used (A 19) and substituted for d and f from (A 33) and (A 32), respectively.

4. *The choice of k*

As mentioned in the body of the paper, a major limitation of these achromatic systems lies in the short back focal lengths obtained. We would therefore optimize the system by maximizing f_b/f and f_b/d. It can readily be shown that, as k approaches zero, the former decreases and the latter increases, and *vice versa* as k approaches unity.

The formulae given in the paper were derived by arbitrarily choosing $k = \frac{1}{2}$. When this is done, (A 22) becomes

$$g = \tfrac{1}{2}(n + n' - 2) \quad \text{(A 35)}$$

and equations (A 31)–(A 34) become

$$f_1/f = 2(n'-1)/(n'-n) \quad \text{(A 36)}$$

$$f_2/f = 2(n'-n)(n'-1)/(n+n'-2)^2 \quad \text{(A 37)}$$

$$d^2/f = 4(n'-1)(n-1)/(n'-n)(n+n'-2) \quad \text{(A 38)}$$

$$f_b/f = (n'-n)/(n+n'-2). \quad \text{(A 39)}$$

To obtain formulae (1)–(4) from these we note that, for (equation (5))

$$p = (n'-n)/(n'+n-2),$$

$$1 + 1/p = 2(n'-1)/(n'-n), \quad \text{(A 40)}$$

$$p^2 + p = (1 + 1/p)p^2 = 2(n'-n)(n'-1)/(n+n'-2)^2, \quad \text{(A 41)}$$

$$1/p - p = 4(n-1)(n'-1)/(n'-n)(n+n'-2), \quad \text{(A 42)}$$

results readily confirmable from equation (5). Accordingly (A 40) yields

$$f_1/f = 1 + 1/p, \quad \text{(A 43)}$$

(A 41) yields

$$f_2/f = p^2 + p = p^2(f_1/f), \quad \text{(A 44)}$$

(A 42) yields

$$d/f = 1/p - p \quad \text{(A 45)}$$

and (5) yields

$$f_b/f = p. \quad \text{(A 46)}$$

The above are equivalent to equations (1)–(4).

Acknowledgment

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References

[1] Levi, L., 1980, *Applied Optics*, Vol. 2 (New York : Wiley), table 73.
[2] Levi, L., 1968, *Applied Optics*, Vol. 1 (New York : Wiley), table 62.