Types of noise in the visual system

Leo Levi
Jerusalem College of Technology
Dept. of Physics/Electro-Optics
P. O. Box 16031
Jerusalem, Israel

Abstract. With the human observer serving as the final detector in many imaging systems, the functioning of the visual system becomes an important design determinant. Here the contrast threshold is investigated in terms of underlying noise sources. Rose has shown that the threshold found at intermediate brightness levels can be accounted for on the basis of the quantum noise of the radiation absorbed by the retinal detectors. Here it is shown how reasonable and simple assumptions concerning other sources of noise would affect the threshold contrast over the full range of observable brightness levels. The resulting model may prove useful in predicting performance of observer-device systems. Results obtained with the proposed model are compared with experimentally observed threshold data.

Keywords: vision; displays; spatial noise.

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CONTENTS

1. INTRODUCTION

In many imaging systems, a human observer is the final detector. Therefore, the performance of his visual system is likely to be a major design factor. Here we wish to interpret the observed contrast threshold curves in terms of noise sources in the visual system and hence extract quantitative parameters of these sources. The resulting model should be useful in predicting performance of image devices used in conjunction with an observer.

1.1. Basic concepts

There are essentially two types of threshold in detection processes:

(1) Absolute thresholds. In these a signal below a given level has no effect at all.

(2) Noise-limited thresholds. In these any signal does have an effect, but this is masked by random fluctuations in the output until the signal is large enough to rise above these fluctuations. Here there is no absolute level below which the signal is lost and above which it is detected. Instead, as the signal level is raised, the probability of detection rises.[1-3]

There is strong evidence that visual thresholds are of the second type.[4] Indeed, when presenting experimental threshold data, the investigator must state the detection probability at which the threshold was defined.

When considering the visual system as a radiation detector, it is evident that at least three types of noise must be considered.[5]

(1) *Dark light.* Occasionally the photosensitive element will provide an output even in the absence of illumination. This is analogous to dark current in photoelectric detectors and has, in fact, been observed in the visual system. It has been called *dark light.*[6]

(2) *Quantum noise.* The fact that the incoming flux is in the form of photons and is, therefore, quantized, implies random fluctuations, analogous to the shot noise in electrical signals.

(3) *Neural and sensation noise.* There must be some random stimulation due to neural activity not produced by action of the photosensitive material. This must occur both in the brain and at earlier stages. The fact that there must be some minimal contrast that the observer will fail to detect even under optimum conditions is evidence of such noise.

1.2. General formulation

We now seek an expression for the liminal contrast when all of the above noise sources are considered. For our discussion, we choose the threshold contrast required for detecting a circular patch on a uniform background of a different luminance because there are extensive experimental data available for that case.

Following Rose,[1-3] we assume that, at the threshold of detectability, the signal must be some multiple, say b, of the rms noise value. In our case, the signal is the luminance difference between patch and background.

On the other hand, the (modulation) contrast, C, is defined as the ratio of this luminance difference to the background luminance. Hence

$$C = \Delta I/I_b = b\, I_N/I_b\,, \tag{1}$$

where I_b is the background intensity, ΔI is the intensity difference between patch and background, and I_N is the noise-equivalent intensity (the intensity equal in magnitude to the noise level), all of these being measured in troland* units.

We are now ready to formulate, in general terms, the three noise factors discussed in Section 1.1 and their dependence on the background intensity level.

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*A troland (Td) is a unit of visual stimulus whose value is the product of the object luminance (in cd/m^2) and the pupil area (in mm^2). It is readily seen that it is essentially the intensity (in μcd) of the illuminated pupil aperture, as viewed from the retina. When investigating factors controlled by retinal illumination, the intensity (in Td) rather than the object luminance (in cd/m^2) is the appropriate variable.

The threshold contrast as required due to dark light intensity fluctuations, I_{Nd}, will have the form

$$C_d = b\,I_{Nd}/I_b = k_d/I_b\,, \tag{2}$$

where $k_d = bI_{Nd}$ is a constant.

We now proceed to the quantum noise. If the incident quanta are assumed to be Poisson-distributed, the quantum noise (I_{Nq}) will vary with the square root of the intensity, and the threshold contrast due to it is

$$C_q = b\,I_{Nq}/I_b = k_q\sqrt{I_b}/I_b = k_q/\sqrt{I_b}\,, \tag{3}$$

where k_q is a constant determined by the magnitude of the photons, the efficiency with which they are detected, and the space-time domain over which they are integrated.

The neurologically generated noise is far more difficult to formulate. It occurs at higher levels, after the detected signal has undergone a certain gain. To facilitate the evaluation of its effect on threshold, we refer it back to the retinal stage, where the contrast, C, is measured, i.e., we divide it by the gain and call it neural-noise-equivalent intensity ($I_{N\psi}$). It may be determined from

$$B_N = \tfrac{1}{2}[G(I_b + I_{N\psi}) - G(I_b - I_{N\psi})]$$

$$\approx I_{N\psi}\,G'(I_b) \tag{4}$$

where B_N is the rms value of the neural and sensation noise, measured in brightness units, G is the transfer characteristic of the visual system, and G′ is its derivative, the differential gain of the visual system. This relationship must be expressed in functional form since the visual transfer characteristic (called brightness function) is nonlinear. In the absence of any more specific data, we must lump all the neural noise components together; here I referred them to the psychological "brightness" stage.

The threshold contrast due to the neurological and sensation noise can now be written

$$C_\psi = b\,I_{N\psi}/I_b = b\,G'^{-1}(B_N)/I_b\,, \tag{5}$$

where G'^{-1} is the reciprocal of the differential gain G′.

To obtain the overall threshold contrast, we must combine the fluctuations due to these three factors. Since the variance of the sum of random variables equals the sum of their individual variances, we must combine the squares of the noise rms values. This is equivalent to combining their "specific noise" values[7] and this, in turn, is equivalent to combining the squares of their threshold contrast values. Hence the square of the observed threshold contrast is, on combining Eqs. (2), (3), and (5)

$$C^2 = \sum_i C_i^2 = k_d^2/I_b^2 + k_q^2/I_b + [b\,G'^{-1}(B_N)/I_b]^2\,. \tag{6}$$

2. DETERMINATION OF CONTRAST COEFFICIENTS

To compare the above theory with observed threshold data, we must now obtain estimates of the coefficients in Eq. (6) and of the visual transfer characteristic, G. Since many parameters of the visual system are not yet known accurately, I decided to limit myself to an approximate analysis and to test the theory in its simplest possible formulation. I therefore neglected the variations of efficiency and the gain exponent with background intensity, and even the differences between scotopic and photopic observations.

2.1. Dark light

The value of the dark light coefficient, k_d, can be obtained from absolute threshold data. At very low intensities, the other factors will be negligible and threshold will occur when the object intensity equals b times the dark light fluctuations. Hence

$$k_d = b\,I_{Nd} = I_o\,, \tag{7}$$

where I_o is the absolute threshold intensity. Here we use the absolute threshold data of Blackwell,[8] translating his luminance values into the equivalent intensities using the average pupil diameters as found by de Groot and Gebhard.[9] Blackwell's data, together with the equivalent intensity values, are shown in Table 1, for various patch diameters, α.

The expected dependence of I_o on area depends on the process involved. If we assume that the dark light is summed over the retinal area covered by the patch, its fluctuations will vary with the square root of that area, whereas the signal varies directly with it. Hence the ratio varies inversely with the root of the area. This is illustrated by the fact that ($I_o\sqrt{\Omega}$) is roughly constant for large patches, as shown in Table 1. Here Ω is the solid angle subtended by the patch. If, however, the dark light is summed over a fixed element area, the fluctuations are constant, as the signal varies with the area. This condition is illustrated by the sensible constancy of ($I_o\Omega$) at small angles.

TABLE 1. Threshold Intensity Values for Circular Discs of Various Sizes

Disc Diam.	Solid Angle	Absolute Threshold			Some Constants			
α	Ω	L_o	L_o	I_o	$I_o\sqrt{\Omega}$	$I_o\Omega$	k_q	k_ψ
min	µ sr	µft-L	µcd/m²	mTd	µTd.rd	nTd.sr	$Td^{1/2}$	
3.6	.86	540	1850	74	69	64	.63	.019
9.7	6.23	65.6	224	9.0	22	56	.23	.0055
18.2	22.0	25.5	87.3	3.5	16	77	.12	.0028
55.2	202	3.6	12.3	.49	7	100	.041	.0012
121	973	1.46	5.0	.20	6	195	.019	.0012

*Data in ft-L from Blackwell.[8]

2.2. Quantum fluctuations

The expected quantum fluctuations in visual luminance discrimination were first calculated by de Vries[10,11] and Rose.[1] The calculation is quite straightforward, if Poisson statistics are assumed. For a given signal patch, the signal-to-noise power ratio is then simply equal to the number, n, of effective quanta in the patch. When the signal is in the form of a difference between two luminance values, the noise power values add, while the signal equals the luminance difference. Hence, here the signal-to-noise power ratio is[12]*

$$R^2 = (Cn)^2/[n + (1 + C)n] + C^2n/(2 + C)\,, \tag{8}$$

where $C = (I_s - I_b)/I_b$ is the modulation contrast. At the threshold, R must equal b. Hence the threshold contrast due to quantum noise is, if we assume $C \ll 2$

$$C_q = b/\sqrt{n/2}\,, \tag{9}$$

and the coefficient in Eq. (3) is

$$k_q = b\sqrt{2\,I_b/n}\,. \tag{10}$$

The effective photon number, n, is given by the photon irradiation (E_p) multiplied by the space-time extent of the element (δA δt) and the quantum efficiency (η) of the photon detection process, which includes, for our purposes, the transmission losses in the

*This is based on the assumption that the background flux is integrated over an are equal to that of the patch. This may not be correct, especially when small areas are t be detected. In accordance with our general philosophy here, we neglect the effect c the intensity on the integration area.

ocular media. The photon irradiation corresponding to a given intensity, I, in trolands is

$$E_p = \frac{I \times 10^{-6}}{f_e^2} \Big/ Q_p K_o = 10^{-6}\, I \lambda_o / f_e^2\, hc\, K_o$$

$$= 3.7 \times 10^9\, I/f_e^2\,, \tag{11}$$

where $f_e \approx 17$ mm is the eye's effective focal length or, more precisely, the distance from the pupil to the retina divided by the refractive index of the ocular media,

Q_p = hc/λ_o is the energy of a photon at wavelength λ_o,
K_o = 680 lm/W is the peak visual spectral efficiency, and
$Q_p K_o$ is the luminous efficacy in lumens/photon as derived in the appendix.

Hence the effective number of photons is

$$n = \eta\, \delta A\, \delta t\, E_p = 3.7 \times 10^9 \eta \Omega\, \delta t\, I\,, \tag{12}$$

where we have substituted

$$\Omega = \delta A / f_e^2 \tag{13}$$

for the solid angle subtended by the luminous patch.

On substituting this into Eq. (10) we find

$$k_q = 2.3 \times 10^{-5}\, b/\sqrt{\eta \delta t \Omega}\,. \tag{14}$$

For the purposes of our calculation, we assume the following reasonable values* for the variables: $b = 3$, $\eta = 0.07$, $\delta t = 0.2$ s. This yields

$$k_q = .58 \times 10^{-3}/\sqrt{\Omega}\,. \tag{14a}$$

Note that there are, *a priori*, good reasons to assume that the quantum efficiency is higher in scotopic than in the photopic vision. However, in accordance with our basic aims here, we assume this value constant for all regions.

The values of k_q used are listed in Table 1.

2.3. Neural fluctuations

Let us now estimate the effects due to the phenomena which we have lumped under the heading "neural noise." To do this, we must find the transfer characteristic, G(I), of the visual system. The required experimental data are given by Stevens and Stevens.[13] They found the brightness, B, to be given as a function of object luminance, L, in the form:**

$$B = k' L^{\beta'}\,, \tag{15}$$

where both k' and β' are functions of the adaptation luminance, L_a. Their data permit the calculation of the brightness at the adaptation luminance. The characteristic may there be expressed in terms of the intensity, I(in Td). In these terms of I, we now assume a relationship of the form:

$$B = k\, I^{\beta}\,, \tag{16}$$

with β assumed constant. On the basis of certain theoretical considerations,[14] we assume for k a form:

$$k = B_\infty / (I_a^{\beta} + K)\,. \tag{17}$$

This leads to

$$B = B_\infty\, I^{\beta} / (I_a^{\beta} + K) \tag{18}$$

and, for the adaptation brightness ($I = I_a$):

$$B_a = B_\infty / (1 + K\, I_a^{-\beta})\,. \tag{19}$$

A function of this form was fitted to the points calculated from the data of Ref. 13. The results are compared in Table 2. The values used for B_∞, K, and β were 37.5 bril,+ 19, and .32, respectively.

TABLE 2. Adaptation Brightness Function and Its Approximation

Adapt. Lum.	Pupil Diam.	Intensity	Adaptation Brightness*	
			Meas.	Calc.
mL	mm	Td	bril	bril
10^{-3}	6.4	0.10	0.92	0.92
10^{-2}	5.8	0.84	1.8	1.8
10^{-1}	5.1	6.5	3.3	3.3
1.0	4.4	48.4	5.5	5.8
10	3.7	337	8.9	9.5
10^2	2.8	1960	14.5	14.0
10^3	2.1	11,240	20.9	19.1
10^4	2.0	10^5	23.7	25.4

*The measured values are from Stevens and Stevens.[13] The calculated values are based on the approximation: $B_a = B_\infty/(1 + K\, I^{-\beta})$ with $B_\infty = 37.5$, $K = 19$, $\beta = 0.32$.

From Eq. (16) the differential gain is:

$$G' = dB/dI = \beta\, k\, I^{\beta}/I = \beta B/I\,. \tag{20}$$

At the adaptation level its reciprocal is:

$$G'^{-1} = I/\beta B_a = I_a (I_a^{\beta} + K)/\beta B_\infty I_a^{\beta} \tag{21}$$

and hence, from Eq. (5), upon substituting $I_a = I_b$:

$$C_\psi = (b\, B_N/\beta B_\infty)(1 + K\, I_a^{-\beta}) = k_\psi (1 + K\, I_a^{-\beta})\,. \tag{22}$$

The value of k_ψ can be obtained from Blackwell's contrast threshold data at high intensity levels, where the other noise effects are negligible. At these levels we may take

$$C_\psi = C\,,$$

*The factor b was first introduced by Rose.[1] It signifies the multiple by which the signal must exceed the noise standard deviation in order to be identified as a signal by the observer. For Gaussian-distributed noise, 32% of the background area would be identified as signal if b were unity; half of these identifications would be positive and half would appear as darker patches. If b = 2, 4.6% of the background would be so identified. By taking b = 3, we remain with one out of about 400 falsely identified elements. The quantum efficiency was taken as 7%, considering reflection and absorption losses in the ocular media, limitations in the efficiency with which the retinal detectors absorb radiation, and the actual quantum efficiency of the detection process. It is estimated that about 10% of the radiation incident on the cornea is absorbed by the retinal detectors[6,17] and that the quantum efficiency is about 70%.[18] These efficiency values are for light at the peak of the visual sensitivity curve. (The spectral distribution of the radiation has no effect on this figure, provided luminous values of intensity are used. See the appendix.) In contrast to Rose we assumed this efficiency constant over the full seven decades of the intensity range.

**We neglect here the constant L_0 which is significant only for objects much darker than the adaptation level.

+A bril is the brightness sensation due to a micro-Lambert under certain standard conditions.[13]

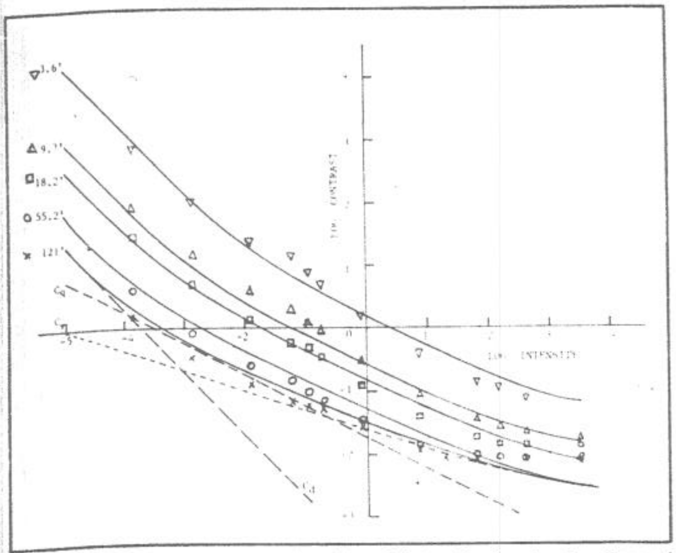

Fig. 1. Threshold contrast as a function of intensity for circular discs of various diameters, as indicated (in minutes of arc). Calculated data (solid lines) are compared to experimental data (points). The broken curves refer to the effects of the individual noise sources as explained in the text.

permitting Eq. (22) to be solved for k_ψ. For reasons explained in the footnote to Section 3, the best estimates are obtained at the point where k_ψ has its minimum value. The values found are listed in Table 1. The values of k_ψ found for the two largest patches were almost equal and therefore their mean value was taken for both.

2.4. Summary

The estimated threshold contrast, Eq. (6), can now be written in the form:

$$C^2 = I_o^2/I_b^2 + k_q^2/I_b + k_\psi^2 (1 + K\, I_b^{-\beta})^2 \,. \tag{23}$$

At very low luminance levels, the first term controls, and the contrast varies inversely with I_b. At intermediate levels, the second term tends to dominate, and the contrast varies inversely with $\sqrt{I_b}$; this is the region investigated by Rose. At high levels the last term controls, and the contrast tends towards constancy. The hypothetical contrast thresholds due to each of these terms, if it were acting in isolation, are shown in Fig. 1 by the broken lines. These refer to the largest patch. The solid lines show the total effects.

3. RESULTS

In Fig. 1 the results of the proposed model (solid lines) are compared to the data points found by Blackwell.

As noted earlier, the purpose of this investigation was more qualitative than quantitative; sufficient data for a precise model are simply not available. In view of this limitation, the agreement between the model and the experimental data seems quite good. Over the full range of over seven decades in intensity, five decades in contrast, and three decades in object size, the discrepancies do not exceed 0.2 logarithmic units, except in two regions, where discrepancies were to be expected because of the simplifications used.

1. At *low intensities,* the *small-area* patches attain high contrast values, so that the approximation of Eq. (9) is no longer valid. Hence in the mesopic region ($-2 < \log I < -1$) the observed contrasts are about 0.3 logarithmic units higher than the calculated values in the two smallest patches. At still lower values, the dark current component becomes dominant, and this effect vanishes.

2. In the *high-intensity* region, the retinal area over which the exposure is integrated shrinks.[15] The effect of this shrinkage is most pronounced in the *large-area* patches, where it raises the observed threshold contrast above the value calculated on the basis of our model, which ignores this shrinkage.*

Indeed, the observed threshold contrast curves for the large-area patches do stabilize above 100 Td, in contrast to the present simple model which predicts a continued decline, albeit at a slower rate. N.B. In view of the discomfort associated with high intensities and the resulting reduction in visual performance, we would expect the neural noise to rise there. This region is, however, not covered by Blackwell's data.

A third effect to be expected is an increase in threshold contrast to be observed in the smallest patch. The spread function of the human visual system has a diameter of several minutes of arc,[16] roughly equal to that of the smallest patch. Hence the retinal image of that patch would be blurred significantly, lowering the signal (intensity contrast) reaching the retina. This effect was allowed for in plotting Fig. 1 by adding 0.3 to all the values calculated for the 3.6′-diameter patch before plotting its curve. This corresponds roughly to a spread function with a diameter equal to that of the patch.

All the curves of Blackwell's data show a pronounced break at about 0.03 Td. This is presumably due to the changeover from rod to cone vision. In terms of our model, this changeover is accompanied by an increase in the effective dark noise, which is, presumably, much higher in the cones than in the rods. Due to lack of data, this hypothesis was not tested quantitatively.

It should be noted that the neurological noise factor was calculated from one point of the same data with which it is subsequently compared. This could guarantee agreement near the high intensity end of the curves. However, its course at the lower intensity is then prescribed by the independently established visual gain characteristic. The other factors, too, were independently established.

Comparing our present results with those of Rose, we note that, by including the two additional noise sources, we have been able to extend the region of qualitative fit from about half a decade to the full range of contrasts—even while maintaining the quantum efficiency of the detection process constant. As a result of this inclusion, we have also been able to accommodate the expected higher quantum efficiencies of the visual process.

Comparing these results with Weber's law, which postulates a constant threshold contrast, independent of intensity, we note that this implies horizontal lines in our figure. We can see that this is approached at high intensities, at least for the larger patches, but certainly not at lower intensities.

4. CONCLUSIONS

Observed data are found to be consistent with the assumption of three independent noise sources in the visual system. One is the quantum noise inherent in the photon nature of light. The second one is analogous to dark current and is located in the retinal detectors. A third noise source is located in the neural system and may be partially psychological. It limits the detectability of small intensity contrasts even under optimum viewing conditions.

5. APPENDIX. THE SPECTRAL DEPENDENCE OF THE EFFECTIVE VISUAL QUANTUM EFFICIENCY

Here we seek the spectral dependence of the quantum efficiency of the visual system.

We assume that the brightness sensation is controlled strictly by the number of photons detected, i.e., by the number of photons received, each one weighted by the quantum efficiency, $\eta_q(\lambda)$, corresponding to its wavelength. Once a photon is detected, its effect is independent of its wavelength.

The effective luminous photon flux is, then,

*This effect makes the estimation of the coefficient k_ψ for large-area patches difficult According to our model, the apparent increase of k_ψ at high values of intensity is fic titious and due to the shrinkage of the integration region. Here we obtain a reasonabl estimate of k_ψ at that intensity level which yields the lowest value for this coefficient

$$\Phi_{pv} = \int \eta_q \Phi_{p\lambda}\, d\lambda = \int \eta_q (\Phi_\lambda / Q_p) d\lambda \,, \tag{A1}$$

where $\Phi_{p\lambda}$ is the spectral photon flux, Φ_λ is the spectral radiant flux, and Q_p is the energy of a photon. Note that all of these are functions of λ.

We must now find η_q in terms of the standard visibility function, V. At any given wavelength, this is given by:

$$V = \Phi_v / K_o \Phi = c' \eta_q N / K_o N Q_p \,, \tag{A2}$$

where N is the number of photons incident, $K_o = 680$ lm/W is the peak luminous efficacy, and c′ is a proportionality constant. This constant can be determined at the wavelength (λ_o) of peak sensitivity ($\lambda_o = 0.555\ \mu$m, V = 1). There

$$c' = K_o Q_{po} / \eta_o \,, \tag{A3}$$

where $Q_{po} = hc/\lambda_o$ is the energy of a photon at λ_o and η_o is the quantum efficiency there. On substituting this value of c′ into Eq. (A2), we find:

$$\eta_q = (Q_p / Q_{po}) \eta_o V [= \lambda_o \eta_o V / \lambda] \,. \tag{A4}$$

This may now be substituted into Eq. (A1), yielding:

$$\Phi_{pv} = (\eta_o / Q_{po}) \int \Phi_\lambda V\, d\lambda = \eta_o \Phi_v / Q_{po} \,. \tag{A5}$$

This implies that the visually effective photon flux is directly proportional to the luminous flux and is independent of the spectral distribution of the illumination. It must merely be understood that the quantum efficiency, η_o, is the actual quantum efficiency only at the peak of the visibility curve. Elsewhere it is as given by the bracketed number of Eq. (A4).

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