

# A Note on Capital Budgeting in the Purchase of Information

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A capital budgeting problem of purchasing perfect information is defined. The relationship between this problem and the Expected Value measure is shown and an illustrative example provided.

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## INTRODUCTION

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A company, when deciding about its participation in some proposed project, generally must make the decision under conditions of uncertainty about the outcome of the project. The firm should engage in research about the proposed project if the cost of the research is compensated for by an improved expected present worth of the project. Research will be defined as any activity such as marketing, R & D, etc., which spends money to reduce the uncertainties of the project.

Ideally, research should eliminate all of the uncertainty about the project before the company must make its decision. In reality, this is often impossible or else prohibitively expensive. Instead, a company will use its limited research funds to purchase partial information, that is, to reduce uncertainty but not necessarily eliminate it.

Here we will consider two types of companies – a neutral-risk company and a risk-averse company – faced with a portfolio of such projects. We will assume that for each project in the portfolio the company can institute research and eliminate all of the uncertainty. The research required for all the projects will perhaps be too expensive and thus we will assume a budgetary constraint. Also we will assume that the situation of each project does not allow for the purchase of partial information. The company will either do not research on the project or will spend the required sum to do complete research. An example of such a project may be where the research is paying another company not to enter a market in order to ensure its own position in the market.

Let us assume that the expected value of all the projects in the portfolio are increased by the company's decision as to which projects to research. Later, when the company is faced with the capital budgeting decisions about which project actually to carry out, their choice will be a better one. They will be choosing from a portfolio with better ex-

pected returns than the pre-researched portfolio. In the next section we formulate the problem assuming that the company is risk-neutral. In the third Section we will observe the decisions of a risk-averse company by means of an illustrative example.

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## THE MODEL FOR THE RISK-NEUTRAL COMPANY

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Let  $X_i$  be a random variable representing the discounted present value of project  $i$  in the portfolio of  $n$  projects. A company will choose the project unresearched only if  $E(X_i)$  is positive. Then the value of having the project in the company's portfolio is given by  $\max\{E(X_i), 0\}$ . This expression, called the 'Recourse Problem', was defined by Walkup and Wets (1967). If the company decides on researching the project, it will then only choose the project if  $X_i$  is positive. Thus, the expected value for having the project in the company's portfolio, given that it will be researched, is given by  $E\{\max(X_i, 0)\}$ . This term can be found in the literature and is called the 'Wait and See Problem', discussed by Madansky (1960). The difference between the two is called the Expected Value of Perfect Information ( $EVPI(X_i)$ ) and is the most that the company should be willing to pay for information about project  $i$ . The EVPI has been discussed by many authors in the literature, including Gunderson and Morris (1978), Huang *et al.* (1977), Ben-Tal and Hochman (1972) and Avriel and Williams (1977). Recently some aspects of the distributional properties of the EVPI were studied by Mehrez and Stulman (1982) with respect to the project-selection problem.

Let us define  $u_i$  to be an indicator variable, equal to one if research into project  $i$  is undertaken, and zero otherwise. Let  $v_i = 1 - u_i$ . Then the total expected present value of all the projects in the

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portfolio is given by

$$Z = \sum_{i=1}^n u_i E(\max\{X_i, 0\}) + \sum_{i=1}^n v_i \max\{E(X_i), 0\} \quad (1)$$

If  $c_i$  is the cost of researching project  $i$ , and  $c_0$  is budget constraint, then our problem becomes

max  $Z$

$$\text{s.t. } u_i + v_i = 1, \quad u_i = 0.1 \quad \sum_{i=1}^n u_i c_i \leq c_0 \quad (2)$$

Now we can write Eqn (1) as

$$Z = \sum_{i=1}^n u_i E(\max\{X_i, 0\}) + \sum_{i=1}^n (i - u_i) \max\{E(X_i), 0\} \quad (3)$$

or

$$Z = \sum_{i=1}^n u_i EVPI(X_i) + \sum_{i=1}^n \max\{E(X_i), 0\} \quad (4)$$

Since the second term in this last expression does not involve  $u_i$  in any way, then we need not consider it when choosing the  $u_i$  to maximize  $Z$ , subject to the constraints in Eqn (2). Thus our problem reduces to

$$\begin{aligned} \max Z &= \sum_{i=1}^n u_i EVPI(X_i) \\ \text{s.t. } u_i &= 0, 1 \quad \sum_{i=1}^n u_i c_i \leq c_0 \end{aligned} \quad (5)$$

The meaning of this is that a company will maximize the total value of their portfolio subject to the budgetary constraint if they choose those projects that give them the largest total expected value of perfect information. Note that Eqn (5) is just a knapsack problem, easily solvable by the standard methods of integer programming.

### An Example

Let us consider the following problem. The portfolio of the firm is given by four projects, each with a different symmetric distribution  $X_i$ , having a zero mean, equal variance  $\sigma_i^2 = \sigma^2$  and equal costs  $c_i = c_1$ . The data of these projects are given in Table 1.

Thus Eqn (5) is reduced to

$$\begin{aligned} \max Z &= \sigma \sum_{i=1}^4 u_i K_i \\ \text{s.t. } \sum_{i=1}^4 u_i &\leq c_0/c_1 \quad u_i = 0, 1 \end{aligned} \quad (6)$$

**Table 1**

$i$ Distribution	(K) Constant multiplier associated with EVPI	Kurtosis $\mu_4/\mu_2^2$
(1) Uniform	$0.433\sigma$	1.8
(2) Triangular	$0.4082\sigma$	2.4
(3) Normal	$0.3989\sigma$	3
(4) Double exponential	$0.3635\sigma$	6

Observation of Table 1 and Eqn (6) suggests several interesting points for our specific example:

- (1) The objective function is a function of the standard deviation.
- (2) The kurtosis of a distribution and the constant associated with the EVPI are inversely related.
- (3) The research cost on project  $i$  is compensated by the improved expected present worth of the project if  $\sigma > c_i/K_i$ .
- (4) The relative sensitivity of Eqn (5) to the shape of the underlying distribution function of  $X$ .

In light of the relationship between the kurtosis of the distribution function and the constant associated with the EVPI, the question arises whether under different conditions where the mean is not equal to zero, the same order of projects will be preserved. To examine this question, we consider the distributions that are given in Table 2. The standard deviation and expected values were taken to be the following:

$$\sigma = 0.5, 1, 1.5, 2, 2.5$$

$$E(X) = -1.5, -0.5, 0.5, 1.5, 2.5, 3.5, 4.5, 5.5$$

**Table 2. Computations of  $E(\max\{X_i, 0\})$**

$i$ Distribution	$\sigma$ $E(X)$	0.5	1.0	1.5	2.0	2.5
1		0.000001	0.0078	0.1160	0.2784	0.4624
2	-1.5	0.0002	0.0293	0.1261	0.2625	0.4219
3		0.00006	0.0238	0.1269	0.2727	0.4394
4		0.0025	0.0424	0.1269	0.2448	0.3783
1		0.0387	0.2191	0.4236	0.6341	0.8470
2	-0.5	0.0417	0.1979	0.3815	0.5729	0.7675
3		0.0423	0.2058	0.3948	0.5911	0.7905
4		0.0430	0.1743	0.3310	0.4965	0.6661
1		0.5387	0.7191	0.9236	1.1341	1.3470
2	0.5	0.5417	0.6969	0.8815	1.0729	1.2645
3		0.5423	0.7058	0.8948	1.0911	1.2905
4		0.5430	0.6743	0.8310	0.9465	1.1661
1		1.5000	1.5078	1.6160	1.7784	1.9624
2	1.5	1.5002	1.5293	1.6261	1.7625	1.9219
3		1.5000	1.5238	1.6269	1.7727	1.9394
4		1.5025	1.5424	1.6269	1.7448	1.8783
1		2.5000	2.5000	2.5009	2.5671	2.6934
2	2.5	2.5000	2.5020	2.5298	2.6013	2.7084
3		2.5000	2.5000	2.5200	2.5959	2.7115
4		2.5002	2.5103	2.5002	2.6207	2.7149
1		3.5000	3.5000	3.5000	3.5000	3.5398
2	3.5	3.5000	3.5001	3.5050	3.5324	3.5918
3		3.5000	3.5000	3.5001	3.5190	3.5803
4		3.5000	3.5025	3.5196	3.5595	3.6220
1		4.5000	4.5000	4.5000	4.5000	4.5000
2	4.5	4.5000	4.5000	4.5006	4.5085	4.5357
3		4.5000	4.5000	4.5000	4.5004	4.5190
4		4.5000	4.5006	4.5078	4.5293	4.5693
1		5.5000	5.5000	5.5000	5.5000	5.5000
2	5.5	5.5000	5.5000	5.5000	5.5018	5.5125
3		5.5000	5.5000	5.5000	5.5000	5.5011
4		5.5000	5.5001	5.5030	5.5145	5.5394

- 1 = Uniform.  
2 = Normal.  
3 = Triangular.  
4 = Double exponential.

Clearly, the optimal order of project selection depends on  $E(\max\{X_i, 0\})$ . In Table 2 these values are computed to the fourth decimal point.

The following computations can be verified from the results of Table 2. The optimal selection of the projects that was found for the case where  $E(X) = 0$  holds as long as  $\sigma > |E(X)|$ . This order is inversely related to the kurtosis. Where  $\sigma < |E(X)|$ , the order is reversed. Thus, it is demonstrated that the relationship between the kurtosis and the EVPI is not clear-cut.

In addition, we observe that for a given standard deviation, the overall value of the project increases as the absolute value of the expected value of the project increases. This result has been verified in a more general structure by Mehrez and Stulman (1982). Finally, we observe that the overall value of

the project increases as the standard deviation increases.

## A MODEL FOR A RISK-AVERSE COMPANY

The framework suggested for the project selection problem of a risk averse company is a zero-one non-linear model. Such a company that maximizes its expected utility must solve the following problem:

$$Z = Eu \left[ \sum_{i=1}^n u_i (\max\{X_i, 0\} - c_i) + \sum_{i=1}^n v_i X_i \right] \quad (7)$$

s.t.  $u_i + v_i \leq 1, \quad u_i = 0, 1 \quad \sum_{i=1}^n u_i c_i \leq c_0$

**Table 3. Decisions of a Risk-averse Company**

Distribution	$\sigma$	$\sigma=0.5 \quad v=0.2$										$\sigma=1 \quad v=2$									
		0.5		1		1.5		2		2.5		0.5		1		1.5		2		2.5	
	$E(x)$	A	B	A	B	A	B	A	B	A	B	A	B	A	B	A	B	A	B	A	B
1		0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
2	-1.5	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
3		0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
4		0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
		0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
1		0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0
2	-0.5	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
3		0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
4		0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0
		0.00	0.00	0.00	0.00	0.00	0.00	0.03	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
1		0	1	0	1	0	1	0	1	0	0	1	0	1	0	1	0	0	0	0	0
2	0.5	0	1	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0
3		0	1	0	1	1	0	1	0	1	0	1	0	1	0	1	0	0	0	0	0
4		0	1	0	1	1	0	1	0	1	0	0	1	0	1	0	0	0	0	0	0
		0.57	0.31	0.22	0.30	0.36	0.57	0.31	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
1		0	1	0	1	0	1	1	0	1	0	0	1	0	1	0	1	0	1	1	0
2	1.5	0	1	0	1	0	1	1	0	0	0	0	1	0	1	0	1	0	0	0	0
3		0	1	0	1	0	1	1	0	1	0	0	1	0	1	0	1	1	0	1	0
4		0	1	0	1	0	1	1	0	1	0	0	1	0	1	0	1	0	1	1	0
		0.94	0.91	0.79	0.69	0.67	0.94	0.91	0.79	0.51	0.51	0.30									
1		0	1	0	1	0	1	0	1	1	0	0	1	0	1	0	1	0	1	0	1
2	2.5	0	1	0	1	0	1	1	0	1	0	0	1	0	1	0	1	0	1	0	0
3		0	1	0	1	0	1	1	0	1	0	0	1	0	1	0	1	0	1	1	0
4		0	1	0	1	0	1	0	1	1	0	0	1	0	1	0	1	0	1	0	1
		0.99	0.99	0.97	0.93	0.87	0.99	0.99	0.97	0.91	0.79										
1		0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1
2	3.5	0	1	0	1	0	1	0	1	1	0	0	1	0	1	0	1	0	1	0	1
3		0	1	0	1	0	1	1	0	1	0	0	1	0	1	0	1	0	1	1	0
4		0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1
		1.0	1.0	1.0	0.99	0.97	1.0	1.0	1.0	1.0	0.99	0.95									
1		0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1
2	4.5	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1
3		0	1	0	1	0	1	1	0	1	0	0	1	0	1	0	1	0	1	1	0
4		0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1
		1.0	1.0	1.0	1.0	0.99	1.0	1.0	1.0	1.0	0.99	0.95									
1		0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1
2	5.5	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1
3		0	1	0	1	0	1	0	1	1	0	0	1	0	1	0	1	0	1	1	0
4		0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	1
		1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0

- 1= Uniform.
- 2= Normal.
- 3= Double exponential.
- 4= Triangular.

where all the parameters of Eqn (7) were defined in Eqns (1) and (2). There are different methods of solving Eqn (7). For illustrative purposes, we enumerate the solution of Eqn (7) for specific examples in Table 3. The utility function that was chosen satisfies the property of constant risk-aversion. The function is of the form  $u = 1 - e^{-x/v}$ , where  $1/v$  is a constant that determines the degree of risk-aversion.

In Table 3, we define two columns, A and B. Column A refers to the decision to purchase information. Column B refers to the decision to accept or reject the project without purchasing information. The value 1 denotes a decision to purchase information or to accept the project. The value 0 denotes the decision not to purchase information or to reject the project. From Table 3 we observe that when  $E(X) \geq \sigma$  ( $\sigma = 0.5, 1, 1.5$ ), we accept all the projects without purchasing information. The rank of the projects is

- (1) Uniform
- (2) Triangular
- (3) Double exponential
- (4) Normal

For large expected values and large standard deviations ( $\sigma = 2, 2.5$ ) we will purchase information

in the following order:

- (1) Double exponential
- (2) Triangular
- (3) Uniform
- (4) Normal

Clearly, the less expensive is the cost of information, the more it is worthwhile to purchase it.

Finally, we observe that for a given expected value, the larger the standard deviation, the smaller is the value of the project.

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## CONCLUSION

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This paper discusses an important problem of purchasing perfect information. We do agree that the analysis which was provided suffers from some limitations. First, it deals only with perfect information and not with more realistic structures of imperfect information. Second, it illustrates by examples and the results are not generalized. However, the data that are reported in this paper show that the solution is very sensitive to the statistical parameters, and it is shown that each problem has its own characteristics and solution properties.

## REFERENCES

- N. Avriel and A. C. Williams (1970). The value of information and stochastic programming. *Ops. Res.* **18**, 947-54.
- A. Ben-Tal, and E. Hochmar (1972). More bounds on the expectation of a convex function of a random variable. *J. Appl. Prob.* **9**, 803-12.
- H. S. Gunderson and J. G. Morris (1978). Stochastic programming with resource:  $\epsilon$  modification from an application viewpoint. *J. Opl. Res. Soc.* **29**, 8, 769-78.
- C. C. Huang, I. Vertinsky and W. T. Ziemba (1977). Sharp

- bounds on the value of perfect information. *Ops. Res.* **25**, 1, 128-39.
- A. Madansky (1960). Inequalities for stochastic linear programming problems. *Mgmt. Sci.* **6**, 197-204.
- A. Mehrez and A. Stulman (1982). Some aspects of the distributional properties of the expected value of perfect information (EVPI). *J. Opl. Res. Soc.* **33**, 827-36.
- D. W. Walkup and R. J.-B. Wets (1967). Stochastic programming with resource, SIAM, *J. Appl. Math.* **15**, 1299-1314.