Simplistic explanations of Brewster's law

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(Received 28 July 1983; accepted for publication 1 September 1983)

When light is incident upon a surface separating two media with indices of refraction n_1 and n_2, it is partly reflected. The amplitude of the electric field of the reflected beam depends on the angle of incidence ϕ, polarization, and indices of refraction, according to Fresnel's equations. In particular, no light is reflected from that component of the electric field of the incident beam which is linearly polarized in the plane of incidence when the angle of incidence equals Brewster's angle, which satisfies:

$$\tan\phi = n_2/n_1 = \sin\phi/\sin\phi'. \tag{1}$$

At this angle of incidence the directions of reflection and refraction are mutually perpendicular. Brewster's law is properly derived by substitution of the zero reflection requirement in the Fresnel's equations.[1] However, Fresnel's equations are derived from Maxwell's equations of electromagnetism; thus a complete proof of Brewster's law is lengthy and complicated. The possibility of "explaining" Brewster's law by simple qualitative assumptions seems therefore attractive. Some textbooks[2–5] make use of an oscillating dipole model, according to which the reflected light is radiated from vibrating dipoles, stimulated by the electric field of the refracted beam. These dipoles vibrate in a plane perpendicular to the refracted ray. At Brewster's angle, for the in-plane component, the direction of vibration is that in which the reflected beam propagates, according to the law of reflection (see Fig. 1). Since a dipole oscillator does not radiate in the direction of vibration, there is no reflected beam for that component.

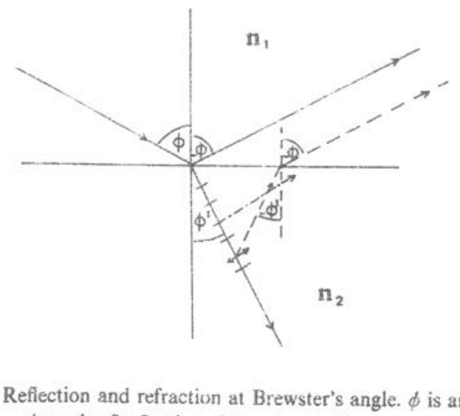

Fig. 1. Reflection and refraction at Brewster's angle. ϕ is angle of incidence; $-\phi$, angle of reflection; ϕ', angle of refraction.

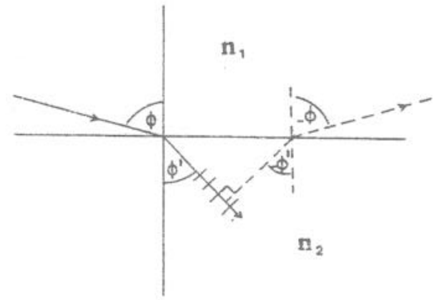

Fig. 2. Reflection and refraction with oscillating dipoles radiating reflected beam perpendicular to refracted beam.

This explanation ignores the fact that light radiated from dipoles inside the second medium will be refracted when it re-enters the first medium. (The dipoles have to be *inside* the second medium if they are stimulated by the *refracted* beam.) In order to have a reflected beam propagating at an angle $-\phi$, light must be emitted from the dipole at an angle $-\phi'$ to the normal and this direction is *not* the direction of vibration of the dipoles (see Fig. 1). Therefore this explanation of Brewster's law is not adequate.

It should be noted that if the oscillating dipole model were valid, no light would be reflected when the angle of refraction is $\phi' = 45°$. In this case, light reflected at an angle $-\phi$ $[= (n_2/n_1)\sin 45°]$ would have to be radiated from these dipoles in their direction of vibration, as can be seen from Fig. 2. This situation exists, for example, when light passes from air (say $n_1 = 1$) to water (say $n_2 = 1.33$) at an angle of incidence ϕ for which

$$\sin\phi = 1.33 \sin 45° = 0.94, \tag{2}$$

hence ϕ is 70°, whereas Brewster's angle is approximately 53°. On the other hand, at Brewster's angle, the angle of refraction is $\phi' = 37°$, whereas the dipoles oscillate in a direction approximately 53° to the surface normal.

[1] B. Rossi, *Optics* (Addison-Wesley, Reading, MA, 1965), p. 378.
[2] Reference 1, p. 401.
[3] F. A. Jenkins and H. E. White, *Fundamentals of Optics*, 4th ed. (McGraw–Hill, Kogakusha, Tokyo, 1976), p. 501.
[4] F. S. Crawford, Jr., *Waves, Berkeley Physics Course*, Vol. 3 (McGraw-Hill, New York, 1968), p. 416.
[5] M. V. Klein, *Optics* (Wiley, New York, 1970), p. 494.

