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**COMPUTATIONAL TESTS AS A MEANS OF ASSESSING THE
INACCURACIES OF INDEX NUMBERS**

by

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NOTE: This paper, without Appendices C and D, has been published in the IMA Journal of Mathematics Applied in Business and Industry, Vol. 3 No. 2 pp. 141-152 1991. There is a spelling error in the published paper, where we repeatedly wrote "Fischer" instead of "Fisher". Here, this has been corrected.

ABSTRACT

In this paper we propose using computational tests for appraising the inaccuracies of index numbers (for example, price and wage indices). This is done by programming a hypothetical situation where the true average index value is known and the variation of prices and quantities and the relation between them can be chosen. The indices are then calculated using the formulae of Laspeyres, Paasche, Fisher, the Unit index and Normalized Unit index formulae. By comparing the average values of the above formulae with the true average index value, we are able to obtain a quantitative indication of the errors in these formulae under various situations. The Normalized Unit and Fisher index formulae were found to be consistently accurate for all our tests whereas the formulae of Laspeyres and Paasche were found to have very poor accuracy in many of our tests. The Unit index was also consistently accurate in all our tests but its use is appropriate in the single item case thus making this formula suitable for measuring wages.

KEYWORDS

index number, error, computational test.

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1. INTRODUCTION

1.1 A Layman's Guide to Index Formulae

To enable the lay reader to understand this paper, we first explain by means of a simple example how price and wage indices are calculated. Let us say a family wishes to measure the change in the price of fruit and records details of the "fruit basket" it purchases as follows.

Fruit basket - initial purchase

3 kilos apples at 3 coins a kilo

5 kilos bananas at 2 coins a kilo

Fruit basket - most recent purchase

4 kilos apples at 2 coins a kilo

1 kilo bananas at 4 coin a kilo

There are several ways of measuring the change in the family's price index of fruit from the initial to the most recent purchase. We give five methods corresponding to five kinds of indices.

Method 1 - (Laspeyres' Index)

The fruit basket at the initial purchase consisted of 3 kilos apples and 5 kilos bananas. At the time of the initial purchase this basket costs $3 \times 3 + 5 \times 2 = 19$ coins. At the time of the most recent purchase this basket would have cost $3 \times 2 + 5 \times 4 = 26$ coins. The change expressed as a percentage ratio is $26/19 \times 100 = 136.8$. This is the value of Laspeyres' index at the most recent purchase.

Method 2 - (Paasche's Index)

The fruit basket at the most recent purchase consisted of 4 kilos apples and 1 kilo bananas. At the time of the initial purchase this basket would have cost $4 \times 3 + 1 \times 2 = 14$ coins. At the time of the most recent purchase this basket cost $4 \times 2 + 1 \times 4 = 12$ coins. The change expressed as a percentage ratio is $12/14 \times 100 = 85.7$. This is the value of Paasche's index at the most recent purchase.

Method 3 - (Fisher's Index)

Fisher proposed comparing the geometric mean (square root of the product) of the costs of the initial and most recent basket at initial prices with the geometric mean of the costs of the initial and most recent basket at most recent prices. That is the initial geometric mean $\sqrt{(14 \times 19)} = 16.30$ is compared with the most recent geometric mean $\sqrt{(26 \times 12)} = 17.66$. The change expressed as a percentage ratio is $(17.66/16.30) \times 100 = 108.3$ which is the value of Fisher's index at the most recent purchase. Fisher's index also equals to the geometric mean (square root of the product) of Laspeyres' and Paasche's indices, namely $\sqrt{(136.8 \times 85.7)} = \sqrt{11723.76} = 108.3$.

Method 4 - (Unit Index)

From the family's viewpoint the cost per kilo of fruit at each purchase is the total amount paid divided by the total kilos bought.

At the initial purchase this is :-

$$(3 \times 3 + 5 \times 2) / (3 + 5) = 19/8 = 2.375 \text{ coins per kilo.}$$

At the most recent purchase this is :-

$$(4 \times 2 + 1 \times 4) / (4 + 1) = 12/5 = 2.400 \text{ coins per kilo.}$$

The change expressed as a percentage ratio is $2.400/2.375 \times 100 = 101.1$. This is the value of the Unit index at the most recent purchase. (What we have done is calculate the cost per "unit" of fruit, the unit in this case being the kilo).

Method 5 - (Normalized Unit Index)

The Unit index is a natural way of measuring change if we restrict ourselves to one item or similar items which are sold in the same units. To use it in a more general situation where we wish to determine the change in price of different items such as food, clothing furniture etc. we have to in a certain sense equate all items in a natural way. One way of doing this is to say that one "normalized unit" of an item, is the amount of that item that can be purchased for one unit of currency based on its mean price for the purchases compared. We can now calculate as follows and will treat apples and bananas as if they were separate commodities.

The mean price of apples over the two purchases is the total amount paid for the apples purchased divided by the total amount of apples purchased that is:

$$(3 \times 3 + 4 \times 2) / (3 + 4) = 17/7 \text{ coins per kilo}$$

Similarly the mean price for bananas is:

$$(5 \times 2 + 1 \times 4) / (5 + 1) = 14/6 \text{ coins per kilo}$$

Therefore the quantity of apples which can be purchased for one coin is $7/17$ kilo and this is the normalized unit for apples.

Similarly the quantity of bananas which can be purchased for one coin is $6/14$ kilo and this is the normalized unit for apples.

We now calculate for each purchase the total amount paid divided by total units bought which gives the cost per normalized unit.

At the initial purchase we have :-

Total cost for 3 kilos apples and 5 kilos bananas is of course as before, namely 19 coins.

$$\text{Total units bought is } 3/(7/17) + 5/(6/14) = 3 \times (17/7) + 5 \times (14/6) = 18.952$$

The cost per unit $19 / 18.952 = 1.003$ coins per normalized unit.

At the most recent purchase we similarly have:-

Total cost for 4 kilos apples and 1 kilo bananas is as before, namely 12 coins.

$$\text{Total units bought is } 4/(7/17) + 1/(6/14) = 4 \times (17/7) + 1 \times (14/6) = 12.048$$

The cost per unit is $12 / 12.048 = 0.996$ coins per normalized unit.

The change in the cost per unit when expressed as a percentage ratio is $0.996 / 1.003 \times 100 = 99.3$. This is the value of the Normalized Unit index at the most recent purchase.

Some General Comments:

1) Mathematical formulae for various indices are given in appendix A.

2) Method 1 (Laspeyres' index) is used for prices. That is, we calculate monthly, the change in price of a fixed (or initial) basket of commodities.

3) Method 4 (the Unit index) is used for wages. The unit in this case is the employee post. The cost per employee post is the total wages paid divided by the number of employee posts filled, and this is calculated monthly.

4) Methods 2,3,5 (Paasche's Fisher's and the Normalized Unit index) are not used generally. In this paper they are used in conjunction with the other formulae, to understand the errors and uncertainties in indexation.

5) As can be seen from the above example different methods of calculation can give different index values, the greatest difference being between the indices of Laspeyres and Paasche.

6) We would mention that "chaining" [1,2,3] can be used to improve the characteristics of various indices and this is briefly discussed later on in the paper.

1.2 A Pragmatic Assessment of the Accuracy of Index Numbers

Table 1 presents a comparison of Laspeyres and Paasche Price indices (with and without chaining). These data are for the United Kingdom and covers the years 1958 to 1967 when inflation was running between 1% to 5% a year (or about 25% for ten years). The results show that for a large economy running at low inflation there is reasonable agreement between the formulae of Laspeyres and Paasche with about a 5% difference between these formulae after 25% inflation in ten years. However whether agreement between these formulae can be expected for a small economy running at high inflation is questionable to say the least. Bear in mind that some countries experienced inflation of 25% within one or two months not in ten years. Can a 5% difference between these formulae be expected every one or two months in these countries and what differences can be expected with an accumulated inflation rate of hundreds or a thousand percent ?

TABLE 1 - COMPARISON OF LASPEYRES & PAASCHE PRICE INDICES.
(U.K.)

Year	Expenditure per Household	Laspeyres	Paasche	Chain Laspeyres	Chain Paasche
1958	100.0	100.0	100.0	100.0	100.0
1959	104.31987	100.85823	100.54480	100.85823	100.54480
1960	109.14066	101.74241	101.11714	101.72658	101.12728
1961	115.07212	105.03774	103.76592	104.63828	103.85400
1962	117.48085	109.33042	107.86972	108.53627	107.63539
1963	126.63335	111.71473	108.43431	110.67846	109.39957
1964	127.79486	115.58460	112.41367	114.23699	112.62434
1965	143.59308	121.22315	116.13442	120.25972	117.84037
1966	147.08847	126.08628	120.82135	124.86262	122.30600
1967	153.92306	129.44595	123.40484	127.84142	125.42371

The above table is taken from [4].

1.3 Mathematical Tests for Qualitatively Assessing Index Numbers

Many formulae have been published in the literature for measuring inflation and we have illustrated five of them. The previous discussion shows there is uncertainty in determining the index value but it does not indicate which formula is to be preferred. How then do we compare the quality of the formulae to determine which are better? Mathematical tests have been proposed [1,2,3] and we mention some of them here. Let I_{bi} be any formula for the index in month i with respect to month b (a base month). We assume I_{bi} is expressed as a pure ratio not as a percentage ratio (i.e. regarding these tests, the index value in the base month is 1 and not 100 as is usual). With this in mind then regardless of the formula used the following properties should hold for months b, i, k .

- 1) $I_{ii} = 1$.
- 2) $I_{bi} = 1 / I_{ib}$ for $b \neq i$.
- 3) $I_{bi} = I_{bk} \times I_{ki}$ for $b < k < i$
- 4) The value of I_{bi} should be independent of the units in which quantities are expressed.

(The restrictions imposed on properties 2,3 above, make the first three conditions independent of each other. Properties 1,2 can in fact be derived from an unrestricted version of property 3. Similarly property 1 can be derived from an unrestricted version of property 2.)

Regarding these tests, the formulae of Laspeyres and Paasche satisfy properties 1 and 4, while the formula of Fisher satisfies properties 1,2, and 4. The unit index satisfies properties 1,2,3,4 when used to measure the change in price of one item sold from many shops but it only satisfies properties 1,2,3 if used to measure the change in price of many items. The normalized unit index satisfies properties 1,2,4. (It satisfies property 4 because normalized unit is defined in terms of average price of an item and it does not depend on the actual units used to sell an item.)

Actually, from any index formula I_{bj} , we can derive a formula which satisfies conditions 1,2,3. To derive a formula to satisfy conditions 1,2 instead of I_{bj} , we use $G_{bj} = \sqrt{I_{bj}/I_{jb}}$. Intuitively, what we have done is take to take the geometric mean of two estimates of the index from month b to i, namely I_{bj} and $1/I_{jb}$. Incidentally, if we apply this transformation to the index of Laspeyres or to the index of Paasche, we derive in fact Fisher's index as can be easily verified.

Regarding deriving a formula satisfying property 3, chaining [1,2,3] may be used. Namely, instead of I_{bj} use $C_{bj} = I_{b,b+1} \times I_{b+1,b+2} \times \dots \times I_{i-1,i}$.

In short by using both these techniques we can always derive a formula satisfying properties 1,2,3.

2. QUANTATIVE ASSESSMENT OF THE INACCURACIES OF INDEX NUMBERS

There is a major problem of giving a quantitative indication of the accuracy of index numbers in that one would need to know in advance the true index value or inflation rate, which is impossible with real world data. However by using a computer to simulate a situation where the true average index value is known, we can get a clearer picture of the inaccuracies of these formulae. In this section we present this approach and return to using the accepted practice that the index value in the base month is 100.

2.1 The Use of Computational Tests

In Table 2 we present a comparison of Laspeyres, Paasche, Fisher, Unit and Normalized Unit Indices. These results are produced by a computer program. We simulate a hypothetical situation where there are a number of shops selling the same item. Prices, which are the independent variables, are programmed to vary randomly about a fixed midpoint with known random variation. The table gives data for five different kinds of relationship between quantity and price namely, (1) quantity inversely proportional to price squared, (2) quantity inversely proportional to price, (3) no correlation between quantity and price, (4) quantity directly proportional to price, (5) quantity directly proportional to price squared. Regarding the quantity variation shown in table 2, we would stress that this is the random variation in the quantity level which is independent of and not related to the price variation. Similarly, the price variation shown in table 2, is the random variation in the price level which is independent of and not related to the quantity variation. (Mathematical formulae, details of the simulation, as well as the program and its output are given in the appendices.)

Now, as prices are programmed to vary uniformly about a fixed midpoint, we can expect that on the average the index should remain constant at 100 which is therefore the true average index value. We use the formulae of Laspeyres, Paasche, Fisher, the Unit Index and the Normalized Unit index to calculate these indices and compare these calculated values with the pre-programmed value of 100*. The **mean deviation per measurement**# from this pre-programmed value is calculated and given in the table.

* There is no loss in generality of having a fixed programmed index value, as the errors of index measurement are caused by variations in the price and quantity levels of a commodity and these variations can be altered by the user of the program. (Just multiplying all price and quantity levels by a constant factor causes no error in the value of any of the formulae presented.)

The mean deviation per measurement is calculated from the geometric mean of the error factors (error ratios) of the calculated index with respect to the true average index value. This geometric mean of the error factors is then converted to a percentage error. The reason we make the calculation in this way and not using the arithmetic mean is that these errors typically accumulate multiplicatively and not additively. For example if we have an error of +50% in measuring the change from month 1 to month 2 followed by an error of -50% from month 2 to month 3, the combined error is not 50%-50% = 0% but -25%. This is clearly seen from the error factors which are 1.5 and 0.5. So the combined error factor is $1.5 \times 0.5 = 0.75$ which as a percentage is $100 \times (0.75 - 1) = -25\%$. The mean error per month should be calculated from the geometric mean of the error factors namely $\sqrt{1.5 \times 0.5} = \sqrt{0.75} = 0.866$ which as a percentage is -13%. We would further mention that in a previous version of this paper we used the arithmetic mean of the error deviations which in fact gave similar results to the results obtained using the geometric mean, probably due to the fact that the number of shops is large.

Now, regarding the data from Table 2, we firstly observe that in all the tests, the deviations from the expected value is not significantly changed by the independent quantity variation. We see that when there is no correlation between quantity and price, all five formulae agree with the pre-programmed index value to within about 0.1% per measurement regardless of the price variation. On the other hand when the quantity is inversely proportional to price (or to price squared), we see very poor accuracy in the formulae of Laspeyres (too high) and Paasche (too low) as the price variation increases. However the Unit index, and the Normalized Unit index agree with the pre-programmed index value to within about 0.1% per measurement regardless of the price variation and Fisher's index to within about 0.2%. Similarly, in the case when quantity is directly proportional to price (or to price squared), we see very poor accuracy in the formulae of Laspeyres (too low) and Paasche (too high) as the price variation increases. However Fisher's index, the Unit index, and the Normalized Unit index agree with the pre-programmed index value to within about 0.05% per measurement regardless of the price variation.

In addition as it is unusual that the elasticity of demand is positive, it is probable that in practice the index of Laspeyres will be higher than the true index value and the index of Paasche to be lower than the true index value. This deduction is consistent with the data for the U.K. presented in Table1 where Laspeyres' index is consistently higher than the index of Paasche.

In short, we see that, the Unit Index, the Normalized Unit index and the index of Fisher give consistently accurate measurements of the true index value under all our tests whereas the indices of Laspeyre and Paasche gave accurate results only in the case where there is no correlation between price and quantity.

2.2 Some Clarifying Remarks

Some words of clarification are in order regarding the direction we have taken and also what we have and what we have not simulated.

1) The prime aim of this paper is to give a quantitative indication of the errors of various index numbers and this is done by simulating a variety of situations where the true average index is known. The simulations do not

constitute an economic model but we believe we have included sufficient detail in the simulations to appraise the inaccuracies of index numbers (after all we are able to specify the relationship between price and quantity levels and the independent variation in price and quantity levels). We will draw an analogy from the physical sciences in support of using such a simplification. In order to test a scale for weighing human beings, one uses standard test weights; no one would suggest using "standard human beings". Obviously, the more test weights used the greater the confidence in the scale. Similarly to test indices for accuracy, test cases where the true average index is known seems to us a valid approach for gaining a quantitative indication of the potential errors of index formulae; the use of a realistic economic model not being an essential requirement.

2) The approach we have presented has a practical computational bias and the economic side is not discussed. In order to develop the economic side, it would be necessary to develop a realistic economic model whose true index is known. The paper also lacks an analytical mathematical and statistical approach. However further research is needed to fully develop these aspects.

TABLE 2 - COMPARISON OF LASPEYRES, PAASCHE, FISHER, UNIT,
AND NORMALIZED UNIT INDICES FROM COMPUTER GENERATED
DATA

Quantity inversely proportional to price squared (ELASTICITY = -2)

INDEPENDENT PRICE	VARIATION (%) QUANTITY	MEAN PERCENTAGE ERROR PER MEASUREMENT				
		LASPEYRES	PAASCHE	FISHER	UNIT	NOR. UNIT
0.0	0.0	0.0000	0.0000	0.0000	0.0000	0.0000
20.0	0.0	2.7615	-2.6800	0.0038	0.0044	0.0038
40.0	0.0	12.3871	-11.0060	0.0088	0.0119	0.0081
60.0	0.0	35.1909	-26.0042	0.0177	0.0254	0.0112
80.0	0.0	101.9879	-50.4549	0.0375	0.0573	0.0109

Quantity inversely proportional to price (ELASTICITY = -1)

INDEPENDENT PRICE	VARIATION (%) QUANTITY	MEAN PERCENTAGE ERROR PER MEASUREMENT				
		LASPEYRES	PAASCHE	FISHER	UNIT	NOR. UNIT
0.0	0.0	0.0000	0.0000	0.0000	0.0000	0.0000
20.0	0.0	1.3675	-1.3420	0.0035	0.0039	0.0036
40.0	0.0	5.9095	-5.5651	0.0078	0.0094	0.0078
60.0	0.0	15.5090	-13.4028	0.0138	0.0181	0.0128
80.0	0.0	37.2726	-27.1147	0.0258	0.0361	0.0183

No correlation between quantity and price (ELASTICITY = 0)

INDEPENDENT PRICE	VARIATION (%) QUANTITY	MEAN PERCENTAGE ERROR PER MEASUREMENT				
		LASPEYRES	PAASCHE	FISHER	UNIT	NOR. UNIT
0.0	0.0	0.0000	0.0000	0.0000	0.0000	0.0000
20.0	0.0	0.0033	0.0033	0.0033	0.0033	0.0033
40.0	0.0	0.0065	0.0065	0.0065	0.0065	0.0065
60.0	0.0	0.0098	0.0098	0.0098	0.0098	0.0098
80.0	0.0	0.0130	0.0130	0.0130	0.0130	0.0130

Quantity directly proportional to price (ELASTICITY = 1)

INDEPENDENT PRICE	VARIATION (%) QUANTITY	MEAN PERCENTAGE ERROR PER MEASUREMENT				
		LASPEYRES	PAASCHE	FISHER	UNIT	NOR. UNIT
0.0	0.0	0.0000	0.0000	0.0000	0.0000	0.0000
20.0	0.0	-1.3107	1.3341	0.0030	0.0027	0.0030
40.0	0.0	-5.0507	5.3303	0.0052	0.0038	0.0051
60.0	0.0	-10.6944	11.9898	0.0065	0.0033	0.0059
80.0	0.0	-17.5580	21.3147	0.0071	0.0012	0.0050

Quantity directly proportional to price squared (ELASTICITY = 2)

INDEPENDENT PRICE	VARIATION (%) QUANTITY	MEAN PERCENTAGE ERROR PER MEASUREMENT				
		LASPEYRES	PAASCHE	FISHER	UNIT	NOR. UNIT
0.0	0.0	0.0000	0.0000	0.0000	0.0000	0.0000
20.0	0.0	-2.5574	2.6299	0.0026	0.0021	0.0027
40.0	0.0	-9.1794	10.1154	0.0037	0.0016	0.0041
60.0	0.0	-17.6250	21.4049	0.0036	-0.0006	0.0045
80.0	0.0	-25.9931	35.1309	0.0031	-0.0029	0.0052

TABLE 2 CONTINUED - COMPARISON OF LASPEYRES, PAASCHE, FISHER, UNIT, AND NORMALIZED UNIT INDICES FROM COMPUTER GENERATED DATA

Quantity inversely proportional to price squared (ELASTICITY = -2)

INDEPENDENT PRICE	VARIATION (%) QUANTITY	MEAN PERCENTAGE			ERROR PER MEASUREMENT	
		LASPEYRES	PAASCHE	FISHER	UNIT	NOR. UNIT
0.0	80.0	0.0000	0.0000	0.0000	0.0000	0.0000
20.0	80.0	2.7735	-2.6534	0.0233	0.0055	0.0191
40.0	80.0	12.4260	-10.9466	0.0596	0.0139	0.0397
60.0	80.0	35.2873	-25.9080	0.1185	0.0273	0.0606
80.0	80.0	102.1881	-50.3198	0.2235	0.0534	0.0724

Quantity inversely proportional to price (ELASTICITY = -1)

INDEPENDENT PRICE	VARIATION (%) QUANTITY	MEAN PERCENTAGE			ERROR PER MEASUREMENT	
		LASPEYRES	PAASCHE	FISHER	UNIT	NOR. UNIT
0.0	80.0	0.0000	0.0000	0.0000	0.0000	0.0000
20.0	80.0	1.3770	-1.3176	0.0206	0.0050	0.0190
40.0	80.0	5.9366	-5.5142	0.0475	0.0116	0.0394
60.0	80.0	15.5702	-13.3260	0.0846	0.0214	0.0627
80.0	80.0	37.4116	-27.0142	0.1454	0.0394	0.0920

No correlation between quantity and price (ELASTICITY = 0)

INDEPENDENT PRICE	VARIATION (%) QUANTITY	MEAN PERCENTAGE			ERROR PER MEASUREMENT	
		LASPEYRES	PAASCHE	FISHER	UNIT	NOR. UNIT
0.0	80.0	0.0000	0.0000	0.0000	0.0000	0.0000
20.0	80.0	0.0101	0.0258	0.0180	0.0044	0.0188
40.0	80.0	0.0205	0.0514	0.0359	0.0088	0.0375
60.0	80.0	0.0310	0.0766	0.0538	0.0131	0.0561
80.0	80.0	0.0416	0.1016	0.0716	0.0174	0.0747

Quantity directly proportional to price (ELASTICITY = 1)

INDEPENDENT PRICE	VARIATION (%) QUANTITY	MEAN PERCENTAGE			ERROR PER MEASUREMENT	
		LASPEYRES	PAASCHE	FISHER	UNIT	NOR. UNIT
0.0	80.0	0.0000	0.0000	0.0000	0.0000	0.0000
20.0	80.0	-1.3065	1.3550	0.0154	0.0038	0.0181
40.0	80.0	-5.0487	5.3705	0.0254	0.0059	0.0326
60.0	80.0	-10.7012	12.0501	0.0297	0.0060	0.0393
80.0	80.0	-17.5788	21.3968	0.0283	0.0043	0.0338

Quantity directly proportional to price squared (ELASTICITY = 2)

INDEPENDENT PRICE	VARIATION (%) QUANTITY	MEAN PERCENTAGE			ERROR PER MEASUREMENT	
		LASPEYRES	PAASCHE	FISHER	UNIT	NOR. UNIT
0.0	80.0	0.0000	0.0000	0.0000	0.0000	0.0000
20.0	80.0	-2.5559	2.6493	0.0129	0.0031	0.0171
40.0	80.0	-9.1866	10.1509	0.0159	0.0034	0.0256
60.0	80.0	-17.6457	21.4540	0.0113	0.0016	0.0226
80.0	80.0	-26.0260	35.1883	0.0021	-0.0007	0.0161

The results in table 2 are abstracted from the runs giving summary output in appendix D which appertains to 1000 shops being simulated for 1000 cycles. The cumulative results for 1000 cycles are presented here. Only results where the independent quantity variation is zero or eighty are shown. Appendix D contains similar results for other values of the independent quantity variation. As can be seen from this table or from Appendix D, there is only a slight change in the values of the mean deviations of the various indices for different values of the independent quantity variation. In other words, these deviations depend primarily on the independent price variation, the independent variable of the simulation

3 CONCLUSION

In view of the results presented, we conclude that the values of the Laspeyres and Paasche Indices are likely to give serious errors in times of high instability. However, we expect that the Normalized Unit index and the index of Fisher would give reliable index values in times of high instability, in view of their consistently accurate measurements under all our tests; in particular the tests with high price and quantity variation. The Unit index was also consistently accurate in all our tests but its use is appropriate in the single item case thus making this formula suitable for measuring wages.

It is a pity that in practice, price indices are measured using the formulae of Laspeyres (and Paasche), formulae which we have shown to have a potential for serious error. Great strides have been made in the fields of computation and data collection since the formulae of Laspeyres and Paasche were proposed, making it much easier to use the more accurate formulae in practice. The use of the more accurate formulae is strongly recommended.

4 ACKNOWLEDGMENT

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INTRODUCTION TO THE APPENDICES

In order that the paper should be understandable to as wide a readership as possible we have minimized mathematical details or a description of the computer programs and algorithms in the body of the paper. In the appendices, we give these details as well as discuss some auxiliary issues.

The appendices give the following information.

- A) Various index formulae (including the Normalized Unit Index).
- B) Mathematical and computational details of the simulation.
- C) Listing of the computer program.
- D) Output from the computer program. Seven runs are given. Two runs gives full output on a month by month basis as well as summary tables which give results cycle by cycle cumulatively. The full output feature was used for testing and debugging the program and we include this output to facilitate others to check our work. Table 2 is summarized from these long runs which give the average deviations of the various indices from their programmed value 100.0 after the final cycle. The deviation of the mean value from the expected value 1.0 of the random numbers used is also reported so that the quality of the random numbers can be assessed.

APPENDIX A - Mathematical Formulae

i - month number (base month is month b)

j - item number

p_{ij} - price in month i of item j

q_{ij} - quantity sold in month i of item j

L_{bi} - Laspeyres' price index in month i with respect to base month b

P_{bi} - Paasche's price index in month i with respect to base month b

F_{bi} - Fisher's price index in month i with respect to base month b

U_{bi} - Unit (Wages) index in month i with respect to base month b

N_{bi} - The Normalized Unit index in month i with respect to base month b

$$L_{bi} = \frac{\sum_j p_{ij} q_{bj}}{\sum_j p_{bj} q_{bj}} \times 100$$

$$P_{bi} = \frac{\sum_j p_{ij} q_{ij}}{\sum_j p_{bj} q_{ij}} \times 100$$

$$F_{bi} = \frac{\sqrt{(\sum_j p_{ij} q_{ij} \times \sum_j p_{ij} q_{bj})}}{\sqrt{(\sum_j p_{bj} q_{ij} \times \sum_j p_{bj} q_{bj})}} \times 100 = \sqrt{(L_i \times P_i)}$$

The previous formulae for L_{bi} , P_{bi} and F_{bi} can be used to measure price change of a set of different items. The next formula U_{bi} the unit index is only useful for measuring price change of a single item, being sold for example, from many shops. It has the same form as the wage measurement formula.

$$U_{bi} = \frac{\sum_j p_{ij} q_{ij} / \sum_j q_{ij}}{\sum_j p_{bj} q_{bj} / \sum_j q_{bj}} \times 100$$

The Normalized Unit Index

As written, the unit index formula can only be used in the case of a single item, being sold for example from various shops. To extend it to the case of several items we have to in a certain sense equate all items in a natural way. One way of doing this is to say that one "normalized unit" of the j-th item, is the amount of that item that can be purchased for one unit of currency based on its mean price M_j over several months (or over a year). The cost of one "normalized unit" of item j in month i is therefore p_{ij}/M_j and the quantity consumed is $q_{ij}M_j$. Thus the value of the normalized unit index in month i now becomes after a little simplification :-

$$N_{bi} = \frac{\sum_j p_{ij} q_{ij} / \sum_j q_{ij} M_j}{\sum_j p_{bj} q_{bj} / \sum_j q_{bj} M_j} \times 100$$

In words the above formula says the normalized unit index N_{bi} is the change in price expressed as a percentage ratio of one "normalized unit" of each and every item surveyed where "normalized unit" was defined above.

Regarding defining M_j the mean price of the jth item it is natural to define it as the ratio of the total value of j'th item sold over the desired months to total quantity sold over these months. For example if we only use the data from base and current months we get:-

$$M_j = (p_{ij} q_{ij} + p_{bj} q_{bj}) / (q_{ij} + q_{bj})$$

The above formula for M_j is particularly appropriate for chained indices where the previous month is used as the base and we calculate the change in price level from previous to current month. It is also the formula used in the simulation.

If we forget for a moment the interpretation we gave to M_j and allow ourselves to substitute freely on it we can in fact derive both the Paasche and Laspeyres formula from the Normalized Unit Index formula . The substitution $M_j = p_{bj}$ causes that formula to reduce to the Paasche formula. This was pointed out to the author independently by Miriam Tsadiq and by Michael Daly. This caused the author to notice that the substitution $M_j = p_{ij}$ causes that formula to reduce to the Laspeyres formula.

APPENDIX B - Details of the Simulation

a) We program a hypothetical situation where there are a number of shops selling the same item. The price of this item in month i in shop j is p_{ij} and the quantity sold in month i in shop j is q_{ij} .

b) The main parameters of the simulation of the simulation are:-

elasticity - the elasticity of the relationship between price and quantity

pricevariation - the independent variation (%) of the price level

quantityvariation - the independent variation (%) of the quantity level

c) Independent random number values randomp , randomq are used for causing the independent variation of price and quantity level.

The random numbers "randomp" have mean 1.0 and are uniformly distributed in the range $0 < \text{randomp} < 2$. They are generated using a linear congruence formula. See the procedure nextrandomp which is given in the next appendix.

Similarly, the random numbers "randomq" have mean 1.0 and are uniformly distributed in the range $0 < \text{randomq} < 2$. They are generated using a linear congruence formula. See the procedure nextrandomq which is given in the next appendix.

d) The program determines p_{ij} as follows.

$p_{ij} := \text{pricemidpoint} \times (1 + (\text{pricevariation} / 100.0) \times (\text{randomp} - 1));$

where pricemidpoint is a constant set to 100.0. Since randomp varies in the range 0 to 2, p_{ij} will vary in the range:

$\text{pricemidpoint} \times (1 \pm (\text{pricevariation} / 100.0)).$

e) The program then determines q_{ij} by firstly determining a quantity midpoint from p_{ij} and the elasticity, and then q_{ij} is determined from this midpoint as follows.

$\text{quantitymidpoint} := \text{nominalquantity} \times ((p_{ij} / \text{pricemidpoint}) ** \text{elasticity});$

$q_{ij} := \text{quantitymidpoint} \times (1 + (\text{quantityvariation} / 100.0) \times (\text{randomq} - 1));$

where nominalquantity is a constant set to 100.0 and $**$ denotes exponentiation. Similarly since randomq varies in the range 0 to 2, q_{ij} varies in the range:

$\text{quantitymidpoint} \times (1 \pm (\text{quantityvariation} / 100.0)).$

f) In each simulation cycle, two sets of price and quantity levels generated (that is for base month and current month) and one set of index values calculated using the various index formulae. The program then calculates the errors of the various index formulae and outputs these results.

APPENDIX C - Program Listing

```
PROGRAM index(input,output);

  {WRITTEN BY: R.B.YEHEZKAEL;

  ADDRESS: JERUSALEM COLLEGE OF TECHNOLOGY
  HAWAAD HALEUMI 21
  JERUSALEM
  ISRAEL.

  DATE: ELUL 5751 - AUGUST 1991 }

CONST
  numberofshops=1000; numberofcycles=1000;
  monthspcycle=2; {BASE MONTH AND CURRENT MONTH}
  evrandoms=1.0; {EXPECTED VALUE OF RANDOM NUMBERS}
  sdrandoms=0.81649658092773; {STANDARD DEVIATION OF THE RANDOM NUMBERS
    WHICH EQUALS THE SQUARE ROOT OF 2/3}
  programmedindex=100.0; {PROGRAMMED VALUE OF THE INDEX}
  maximumvariation=99; {NOTE - PROBLEMS OF NUMERICAL ACCURACY OCCUR WHEN THE
    PRICE VARIATION IS 100 AND THE ELASTICITY IS
    NEGATIVE CAUSED BY DIVISION BY NUMBERS
    WHICH ARE ALMOST ZERO. HENCE THIS LIMIT IS NEEDED}
  fulloutputcycles=10; {CYCLES FOR WHICH OUTPUT IS NEEDED IN FULL
    OUTPUT MODE}
  twotothe15=32768 { 2**15 }; twotothe16=65536 {2**16};

TYPE
  real=double {DOUBLE PRECISION REALS VAX/VMS FEATURE
    GIVING ABOUT 15 SIGNIFICANT DIGITS };

VAR
  seedp,seedq:unsigned; {SEEDS OF RANDOM NUMBER GENERATORS.
    NOTE unsigned MEANS 32 BIT UNSIGNED INTEGERS
    THIS IS A VAX/VMS FEATURE}
  randomp,randomq:real; {PRICE RANDOMS AND QUANTITY RANDOMS}
  prandomstoskip,qrandomstoskip:integer; {FOR INITIALIZING RANDOM NUMBERS}
  pricevariation,quantityvariation:real; {THE INDEPENDENT VARIATION IN THE
    PRICE LEVEL AND QUANTITY LEVEL}
  percentagetest:integer;
  totalrandomp,totalrandomq:real; {FOR SKEWNESS OF RANDOM NUMBERS}
  randomsusedpercycle, randomsusedinall:real; {THE NUMBER OF PRICE RANDOMS
    OR QUANTITY RANDOMS USED}
  cycle,cyclefactor:integer;
  basetotalvalue,basetotalquantity,basetotalnormalizedquantity:real;
  totalvalue,totalquantity,totalnormalizedquantity:real;
  laspeyressum,paaschesum:real;
  laspeyres,paasche,fisher,unitindex,normalizedunitindex:real;
  {THE VALUES OF THE VARIOUS INDICES}
  lerrortotal,perrortotal,ferrortotal,uerrortotal,nerrortotal:real;
  {ERROR TOTALS OF THE LOGARITHMS OF THE ERROR FACTORS (OR RATIOS)}
  summary:ARRAY [1..numberofcycles] OF
  RECORD {SUMMARY INFORMATION ABOUT EACH CYCLE}
    lerror,perror,ferror,uerror,nerror:real;
    {LOGARITHM OF THE ERROR FACTORS (OR RATIOS) OF THE VARIOUS INDICES}
    subtotalrandomp,subtotalrandomq:real;
    {FOR SKEWNESS OF RANDOM NUMBERS}
  END;
  e:real {ELASTICITY};
  p,q:ARRAY [1..numberofshops] OF real
  {CURRENT PRICE AND QUANTITY VECTORS};
  baseprice,basequantity:ARRAY[1..numberofshops]OF real; {BASE PRICES AND QUANTITIES}
  full:boolean; {TRUE FOR FULL LISTING, FALSE FOR SUMMARY LISTING}

PROCEDURE skipblanks;

BEGIN
  WHILE (input^ = ' ') AND (NOT eoln) DO get(input);
END;

PROCEDURE nextrandomp;

BEGIN
  {GENERATE RANDOM NUMBERS WITH EXPECTED VALUE 1.0 UNIFORMLY DISTRIBUTED IN
  THE RANGE 0 < randomp < 2 . THIS ROUTINE IS MACHINE DEPENDENT AND RELIES
```

```

ON 32 BIT UNSIGNED ARITHMETIC OF VAX/VMS.}

REPEAT
  seedp:=(seedp*13077+6925); {MOD 2**32}
  {BY USING THE LOWEST 16 BITS OF THE 32 BIT SEED, WE ENSURE
   THAT THE SEQUENCE OF PSEUDO RANDOM NUMBERS GENERATED
   MAY CONTAIN REPETITIONS}
  randomp:=(seedp MOD twotothe16)/twotothe15;
UNTIL randomp <> 0.0;
END;

PROCEDURE nextrandomp;

BEGIN
  {GENERATE RANDOM NUMBERS WITH EXPECTED VALUE 1.0 UNIFORMLY DISTRIBUTED IN
   THE RANGE 0 < randomp < 2 . THIS ROUTINE IS MACHINE DEPENDENT AND RELIES
   ON 32 BIT UNSIGNED ARITHMETIC OF VAX/VMS.}

  REPEAT
    seedq:=(seedq*16125+3077); {MOD 2**32}
    {BY USING THE LOWEST 16 BITS OF THE 32 BIT SEED, WE ENSURE
     THAT THE SEQUENCE OF PSEUDO RANDOM NUMBERS GENERATED
     MAY CONTAIN REPETITIONS}
    randomq:=(seedq MOD twotothe16)/twotothe15;
  UNTIL randomq <> 0;
  END;

PROCEDURE simulatepriceandquantitychanges;

CONST
  pricemidpoint=100.0; nominalquantity=100.0;
VAR
  j:integer; {THE SHOP NUMBER}
  quantitymidpoint:real; {THE VALUE OF quantitymidpoint DEPENDS ON p[j] AND e}

BEGIN
  WITH summary[cycle] DO
    BEGIN
      FOR j:=1 TO numberofshops
        DO
          BEGIN
            {USE RANDOM NUMBER GENERATOR nextrandomp TO VARY PRICES ABOUT pricemidpoint}
            nextrandomp; subtotalrandomp:=subtotalrandomp+randomp;
            p[j]:=pricemidpoint*( 1.0 + (pricevariation/100.0)*(randomp-1.0) );

            {DETERMINE quantitymidpoint FROM THE PRICE p[j] AND THE ELASTICITY e}
            quantitymidpoint:=nominalquantity*(p[j]/pricemidpoint)**e;
            {USE RANDOM NUMBER GENERATOR nextrandomp TO VARY QUANTITIES ABOUT
            quantitymidpoint}
            nextrandomp; subtotalrandomq:=subtotalrandomq+randomq;
            q[j]:=quantitymidpoint*( 1.0 + (quantityvariation/100.0)*(randomq-1.0)
            );

          END;
        END;
      END;
    END;

PROCEDURE calculateerrors;

  {CALCULATE ERROR FACTORS (OR RATIOS) OF THE VARIOUS INDICES
   FROM programmedindex AND UPDATE CORRESPONDING ERROR PRODUCTS}

BEGIN
  WITH summary[cycle] DO
    BEGIN
      lerror:=ln(laspeyres/programmedindex);
      perror:=ln(paasche/programmedindex);
      ferror:=ln(fisher/programmedindex);
      uerror:=ln(unitindex/programmedindex);
      nerror:=ln(normalizedunitindex/programmedindex);
    END;
  END;

```

```

END;

PROCEDURE calculateonesetofindices;

VAR
  j:integer; {THE SHOP NUMBER}
  mj:real; {MEAN VALUE OF PRICE IN SHOP NUMBER j BASED ON VALUES
           OF PRICE AND QUANTITY IN BASE AND CURRENT MONTH}

BEGIN

  {CALCULATE SUMS}
  basetotalvalue:=0.0; basetotalquantity:=0.0;
  totalvalue:=0.0; totalquantity:=0.0;
  basetotalnormalizedquantity:=0; totalnormalizedquantity:=0;
  laspeyressum:=0.0; paaschesum:=0.0;
  FOR j:=1 TO numberofshops DO
    BEGIN
      basetotalvalue:=basetotalvalue+baseprice[j]*basequantity[j];
      basetotalquantity:=basetotalquantity+basequantity[j];
      laspeyressum:=laspeyressum+p[j]*basequantity[j];
      paaschesum:=paaschesum+baseprice[j]*q[j];
      totalvalue:=totalvalue+p[j]*q[j];
      totalquantity:=totalquantity+q[j];
      mj:=(baseprice[j]*basequantity[j]+p[j]*q[j]) / (basequantity[j]+q[j]);
      basetotalnormalizedquantity:=basetotalnormalizedquantity + mj*basequantity[j];
      totalnormalizedquantity:=totalnormalizedquantity + mj*q[j];
    END;
  laspeyres:=100.0*laspeyressum/basetotalvalue;
  paasche:=100.0*totalvalue/paaschesum;
  fisher:=sqrt(laspeyres*paasche);
  unitindex:=100.0*(totalvalue/totalquantity) / (basetotalvalue/basetotalquantity);
  normalizedunitindex:=100.0*(totalvalue/totalnormalizedquantity) /
  (basetotalvalue/basetotalnormalizedquantity);

  calculateerrors;

  {FOR FULL LISTING WRITE INDEX VALUES AND ERRORS FOR THE CYCLE}
  IF full AND (cycle <= fulloutputcycles)
    THEN writeln(cycle:4,' ',
                 laspeyres:11:4,paasche:11:4,fisher:11:4,
                 unitindex:11:4,normalizedunitindex:11:4);

END;

PROCEDURE simulateonecycle;

VAR
  j:integer; {THE SHOP NUMBER}

BEGIN

  {INITIALIZE SUBTOTALS AND SUMS AND COUNTERS}
  WITH summary[cycle] DO
    BEGIN
      subtotalrandomp:=0.0; subtotalrandomq:=0.0;
    END;

  {GET PRICE AND QUANTITY VALUES FOR BASE MONTH}
  simulatepriceandquantitychanges;

  FOR j:=1 TO numberofshops
    DO BEGIN baseprice[j]:=p[j]; basequantity[j]:=q[j];
    END;

  {GET PRICE AND QUANTITY VALUES FOR CURRENT MONTH}
  simulatepriceandquantitychanges;

  calculateonesetofindices;

END;

PROCEDURE setupfixedvalues;

VAR
  c:char;{FOR SWITCH}

```

```

BEGIN

{READ FIXED VALUES}
writeln;

REPEAT
  writeln('SUMMARY OR FULL LISTING (S/F) ? ');
  skipblanks;
  readln(c);
  UNTIL (c='S') OR (c='s') OR (c='F') OR (c='f');
  full:=(c='F') OR (c='f');

REPEAT
  writeln('ELASTICITY (2>=...>=-2) ? ');
  readln(e);
  UNTIL (2.0 >= e) AND (e >= -2.0);

IF full
  THEN
  BEGIN
  REPEAT
    writeln('INDEPENDENT PERCENTAGE VARIATION OF PRICES ( ',
      maximumvariation:1, '>=...>=0 ) ? ');
    readln(pricevariation);
    UNTIL (maximumvariation >= pricevariation) AND (pricevariation >= 0);

    REPEAT
      writeln('INDEPENDENT PERCENTAGE VARIATION OF QUANTITIES ( ',
        maximumvariation:1, '>=...>=0 ) ? ');
      readln(quantityvariation);
      UNTIL (maximumvariation >= quantityvariation) AND (quantityvariation >= 0);
    END
  ELSE
  BEGIN
  REPEAT
    writeln('PERCENTAGE STEP ( ',
      maximumvariation:1, '>=...>=1 ) ? ');
    readln(percentagestep);
    UNTIL (maximumvariation >= percentagestep) AND (percentagestep >= 1);
  END;

REPEAT
  writeln('NUMBER OF PRICE RANDOMS TO SKIP ( >=0 ) ? ');
  readln(prandomstoskip);
  UNTIL prandomstoskip >= 0;

REPEAT
  writeln('NUMBER OF QUANTITY RANDOMS TO SKIP ( >=0 ) ? ');
  readln(qrandomstoskip);
  UNTIL qrandomstoskip >= 0;

{OUTPUT DATA FOR CYCLES WHICH ARE MULTIPLES OF cyclefactor}
IF full
  THEN cyclefactor:=numberofcycles DIV 10
  ELSE cyclefactor:=numberofcycles; {LAST CYCLE ONLY}

writeln;
writeln('SIMULATING ',numberofshops:1,' SHOPS FOR ',
  numberofcycles:1,' CYCLES AND GENERATING ONE SET OF VALUES OF THE');
writeln('LASPEYRES, PAASCHE, FISHER, UNIT, AND NORMALIZED UNIT INDICES IN EACH
CYCLE. ');

END;

PROCEDURE initializerandomnumbers;

VAR
  n:integer;

BEGIN
  seedp:=0; seedq:=0;
  FOR n:=1 TO prandomstoskip DO nextrandomp;
  FOR n:=1 TO qrandomstoskip DO nextrandomq;
END;

PROCEDURE writesummaries;

```



```

BEGIN

lerrortotal:=0.0; perrortotal:=0.0; ferrortotal:=0.0;
uerrortotal:=0.0; nerrortotal:=0.0;
FOR cycle:=1 TO numberofcycles DO
  WITH summary[cycle] DO
    BEGIN

      {CALCULATE ERROR TOTALS}
      lerrortotal:=lerrortotal+lerror;
      perrortotal:=perrortotal+perror;
      ferrortotal:=ferrortotal+ferror;
      uerrortotal:=uerrortotal+uerror;
      nerrortotal:=nerrortotal+nerror;

      {CALCULATE MEAN OF LOGARITHMIC SUM, GEOMETRIC MEAN OF ERROR FACTORS,
      CONVERT GEOMETRIC MEAN OF ERROR FACTOR TO A PERCENTAGE AND
      WRITE PERCENTAGE ERROR}

      IF full AND ((cycle MOD cyclefactor = 0) OR (cycle <= fulloutputcycles))
      THEN
        BEGIN
          writeln(cycle:4, ' ',
            100.0*(exp(lerrortotal/cycle)-1.0):11:4,
            100.0*(exp(perrortotal/cycle)-1.0):11:4,
            100.0*(exp(ferrortotal/cycle)-1.0):11:4,
            100.0*(exp(uerrortotal/cycle)-1.0):11:4,
            100.0*(exp(nerrortotal/cycle)-1.0):11:4);
          END;
        IF (NOT full) AND (cycle MOD cyclefactor = 0)
        THEN
          BEGIN
            writeln(pricevariation:7:1, ' ',
              quantityvariation:7:1, ' ',
              100.0*(exp(lerrortotal/cycle)-1.0):11:4,
              100.0*(exp(perrortotal/cycle)-1.0):11:4,
              100.0*(exp(ferrortotal/cycle)-1.0):11:4,
              100.0*(exp(uerrortotal/cycle)-1.0):11:4,
              100.0*(exp(nerrortotal/cycle)-1.0):11:4);
            END;
          END;

        END;

      PROCEDURE writerandomnumberstatistics;

      BEGIN

        randomsusedpercycle:=monthspercycle*numberofshops;
        randomsusedinall:=0.0;
        totalrandomp:=0.0; totalrandomq:=0.0;
        writeln; writeln;
        IF full
        THEN
          BEGIN {WRITE HEADINGS}
            writeln('THE DEVIATIONS OF THE MEAN OF THE RANDOM NUMBERS, UP TO THE CURRENT
CYCLE,');
            writeln('FROM THEIR EXPECTED VALUE 1.0, IS GIVEN IN THE FOLLOWING TABLE. ');
            writeln('THESE RANDOM NUMBERS ARE USED TO CAUSE THE INDEPENDENT VARIATION');
            writeln('IN PRICE AND QUANTITY LEVELS. ');
            writeln;

            writeln('          NUMBER OF STANDARD DEVIATIONS');
            writeln('CYCLE  PRICE RANDOMS      QUANTITY RANDOMS');
            writeln;

            END;
          FOR cycle:=1 TO numberofcycles DO
            WITH summary[cycle] DO
              BEGIN

                randomsusedinall:=randomsusedinall+randomsusedpercycle;
                {CALCULATE TOTALS}
                totalrandomp:=totalrandomp+subtotalrandomp;
                totalrandomq:=totalrandomq+subtotalrandomq;
                IF full AND ((cycle MOD cyclefactor = 0) OR (cycle <= fulloutputcycles))
                THEN writeln(cycle:4,

```

```

                {MEAN DEVIATIONS OF RANDOM NUMBERS}
                (totalrandomp/randomsusedinall-evrandoms)/sdrandoms:13:4,
                ' ',
                (totalrandomq/randomsusedinall-evrandoms)/sdrandoms:13:4);
        END;

    IF NOT full
    THEN
        BEGIN
            writeln('THE SAME SEQUENCES OF RANDOM NUMBERS (APPROPRIATELY SCALED)');
            writeln('ARE USED TO GENERATE THE RESULTS IN EACH ROW OF THE ABOVE TABLE. ');
            writeln('THESE RANDOM NUMBERS ARE USED TO CAUSE THE INDEPENDENT VARIATION');
            writeln('IN PRICE AND QUANTITY LEVELS.  THE DEVIATIONS OF THE MEAN OF THE
UNSCALED');
            writeln('RANDOM NUMBERS, UP TO THE FINAL CYCLE, FROM THEIR EXPECTED VALUE
1.0, ');
            writeln('ARE AS FOLLOWS. ');
            writeln;
            writeln('PRICE RANDOMS:  ',
                (totalrandomp/randomsusedinall-evrandoms)/sdrandoms:11:4,
                ' STANDARD DEVIATIONS');
            writeln('QUANTITY RANDOMS:',
                (totalrandomq/randomsusedinall-evrandoms)/sdrandoms:11:4,
                ' STANDARD DEVIATIONS');
            writeln;

        END;

    END;

PROCEDURE simulatenumberofcycles;

BEGIN

    initializerandomnumbers;

    FOR cycle:=1 TO numberofcycles DO
        BEGIN
            simulateonecycle;
        END;

    END;

PROCEDURE producefulloutput;

BEGIN;

    {HEADINGS FOR TABLE OF INDEX VALUES}
    writeln; writeln;
    writeln('          INDEX VALUES FOR CURRENT CYCLE (PROGRAMMED VALUE IS 100)');
    writeln('CYCLE      LASPEYRES    PAASCHE      FISHER      UNIT      NOR.UNIT');
    writeln;

    simulatenumberofcycles;

    {HEADINGS FOR SUMMARY TABLE IN FULL OUTPUT MODE}
    writeln; writeln;
    writeln('THE FOLLOWING TABLE GIVES THE MEAN ERROR PER MEASUREMENT OF THE VARIOUS
INDICES');
    writeln('WITH RESPECT TO THE INDEX VALUE 100, UP TO THE CURRENT CYCLE. ');
    writeln('THE MEAN ERROR PER MEASUREMENT IS THE GEOMETRIC MEAN OF THE ERROR
FACTORS');
    writeln('WHICH IS CONVERTED TO A PERCENTAGE ERROR. ');

    writeln;

    writeln('          MEAN PERCENTAGE ERROR PER MEASUREMENT');
    writeln('CYCLE      LASPEYRES    PAASCHE      FISHER      UNIT      NOR.UNIT');
    writeln;

    writesummaries;

    writerandomnumberstatistics;

    writeln;

END;

```

```

PROCEDURE producesummaryoutput;

BEGIN
  {HEADINGS FOR SUMMARY TABLE IN SUMMARY OUTPUT MODE}
  writeln; writeln;
  writeln('THE FOLLOWING TABLE GIVES THE MEAN ERROR PER MEASUREMENT OF THE VARIOUS
INDICES');
  writeln('WITH RESPECT TO THE INDEX VALUE 100, UP TO THE FINAL CYCLE. ');
  writeln('THE MEAN ERROR PER MEASUREMENT IS THE GEOMETRIC MEAN OF THE ERROR
FACTORS');
  writeln('WHICH IS CONVERTED TO A PERCENTAGE ERROR. ');

  writeln;

  writeln('INDEPENDENT VARIATION(%)           MEAN PERCENTAGE ERROR PER
MEASUREMENT');
  writeln('  PRICE      QUANTITY      LASPEYRES    PAASCHE      FISHER      UNIT
NOR.UNIT');
  {OUTPUT TABLE AT STEP LENGTH}
  pricevariation:=0.0;
  REPEAT
    writeln;
    quantityvariation:=0.0;
    REPEAT
      simulatenumberofcycles;

      writesummaries;

      quantityvariation:=quantityvariation+percentagestep;
    UNTIL quantityvariation > maximumvariation;
    pricevariation:=pricevariation+percentagestep;
  UNTIL pricevariation > maximumvariation;

  writerrandomnumberstatistics;

END;

BEGIN {MAIN PROGRAM}
  setupfixedvalues;

  IF full
  THEN producefulloutput
  ELSE producesummaryoutput;

END.

```

APPENDIX D - Program Output

```

$ RUN INDEX                ! FULL OUTPUT

SUMMARY OR FULL LISTING (S/F) ?
F
ELASTICITY (2>=...>=-2) ?
-2
INDEPENDENT PERCENTAGE VARIATION OF PRICES ( 99>=...>=0 ) ?
80
INDEPENDENT PERCENTAGE VARIATION OF QUANTITIES ( 99>=...>=0 ) ?
80
NUMBER OF PRICE RANDOMS TO SKIP ( >=0 ) ?
0
NUMBER OF QUANTITY RANDOMS TO SKIP ( >=0 ) ?
0

```

SIMULATING 1000 SHOPS FOR 1000 CYCLES AND GENERATING ONE SET OF VALUES OF THE LASPEYRES, PAASCHE, FISHER, UNIT, AND NORMALIZED UNIT INDICES IN EACH CYCLE.

CYCLE	INDEX VALUES FOR CURRENT CYCLE (PROGRAMMED VALUE IS 100)				
	LASPEYRES	PAASCHE	FISHER	UNIT	NOR.UNIT
1	210.3599	51.2408	103.8220	106.5939	100.7663
2	196.8308	50.4352	99.6353	101.9645	98.8895
3	213.5799	54.4652	107.8549	107.7995	102.8689
4	202.1844	50.8539	101.3995	106.3425	100.8069
5	198.0985	50.8661	100.3818	101.7637	100.5987
6	195.7832	49.2271	98.1725	96.7540	97.9335
7	206.5940	48.4341	100.0310	100.8903	99.4965
8	195.3666	50.9008	99.7212	97.7605	100.7009
9	189.5493	48.0458	95.4309	91.2796	97.9218
10	203.2589	49.7997	100.6093	101.5776	100.4801

THE FOLLOWING TABLE GIVES THE MEAN ERROR PER MEASUREMENT OF THE VARIOUS INDICES WITH RESPECT TO THE INDEX VALUE 100, UP TO THE CURRENT CYCLE. THE MEAN ERROR PER MEASUREMENT IS THE GEOMETRIC MEAN OF THE ERROR FACTORS WHICH IS CONVERTED TO A PERCENTAGE ERROR.

CYCLE	MEAN PERCENTAGE ERROR PER MEASUREMENT				
	LASPEYRES	PAASCHE	FISHER	UNIT	NOR.UNIT
1	110.3599	-48.7592	3.8220	6.5939	0.7663
2	103.4830	-49.1636	1.7071	4.2535	-0.1765
3	106.7944	-47.9817	3.7164	5.4224	0.8285
4	105.6322	-48.2753	3.1323	5.6517	0.8231
5	104.1029	-48.4481	2.5762	4.8624	0.7781
6	102.6921	-48.8431	1.8288	3.4653	0.2984
7	103.2450	-49.2412	1.5700	3.0934	0.1834
8	102.2431	-49.2235	1.3370	2.4112	0.2480
9	100.7917	-49.5344	0.6631	1.1102	-0.0132
10	101.0370	-49.6014	0.6578	1.1568	0.0360
100	102.5571	-50.2060	0.4297	0.5119	0.1952
200	102.4146	-50.2205	0.3797	0.2711	0.1591
300	102.3089	-50.2826	0.2910	0.1747	0.0975
400	102.1304	-50.3115	0.2176	0.0954	0.0927
500	102.2238	-50.2566	0.2961	0.1549	0.1231
600	102.2895	-50.2725	0.2963	0.1291	0.1007
700	102.2486	-50.2920	0.2665	0.1221	0.1006
800	102.2059	-50.2939	0.2541	0.0907	0.0936
900	102.2829	-50.2800	0.2871	0.1157	0.0916
1000	102.1881	-50.3198	0.2235	0.0534	0.0724

THE DEVIATIONS OF THE MEAN OF THE RANDOM NUMBERS, UP TO THE CURRENT CYCLE, FROM THEIR EXPECTED VALUE 1.0, IS GIVEN IN THE FOLLOWING TABLE. THESE RANDOM NUMBERS ARE USED TO CAUSE THE INDEPENDENT VARIATION IN PRICE AND QUANTITY LEVELS.

CYCLE	NUMBER OF STANDARD DEVIATIONS	
	PRICE RANDOMS	QUANTITY RANDOMS
1	0.0239	-0.0148
2	0.0168	-0.0172
3	0.0178	-0.0118
4	0.0158	-0.0086
5	0.0123	-0.0090
6	0.0082	-0.0076
7	0.0053	-0.0054
8	0.0068	-0.0047
9	0.0081	-0.0048
10	0.0093	-0.0046
100	0.0002	-0.0002
200	0.0003	-0.0001
300	0.0002	-0.0002
400	0.0001	-0.0001
500	0.0001	-0.0001
600	0.0002	-0.0001
700	0.0001	-0.0001
800	0.0001	-0.0001
900	0.0001	-0.0001
1000	0.0001	-0.0001

\$ RUN INDEX ! FULL OUTPUT

SUMMARY OR FULL LISTING (S/F) ?

F

ELASTICITY (2>=...>=-2) ?

-2

INDEPENDENT PERCENTAGE VARIATION OF PRICES (99>=...>=0) ?

80

INDEPENDENT PERCENTAGE VARIATION OF QUANTITIES (99>=...>=0) ?

80

NUMBER OF PRICE RANDOMS TO SKIP (>=0) ?

10000

NUMBER OF QUANTITY RANDOMS TO SKIP (>=0) ?

30000

SIMULATING 1000 SHOPS FOR 1000 CYCLES AND GENERATING ONE SET OF VALUES OF THE LASPEYRES, PAASCHE, FISHER, UNIT, AND NORMALIZED UNIT INDICES IN EACH CYCLE.

INDEX VALUES FOR CURRENT CYCLE (PROGRAMMED VALUE IS 100)					
CYCLE	LASPEYRES	PAASCHE	FISHER	UNIT	NOR.UNIT
1	201.2038	50.7410	101.0410	97.1878	99.9621
2	205.3495	49.1657	100.4796	98.3131	100.0336
3	199.0162	50.1623	99.9155	100.9485	98.8388
4	204.0704	48.3997	99.3828	96.2987	97.7060
5	197.9135	46.6735	96.1110	96.4166	100.0918
6	190.4018	49.1749	96.7625	97.9502	100.1316
7	192.4754	49.3403	97.4515	97.9446	100.1693
8	208.4674	49.4356	101.5170	104.0574	103.1175
9	201.8735	47.4341	97.8554	98.3684	101.8596
10	195.5372	44.6470	93.4353	91.8547	98.6310

THE FOLLOWING TABLE GIVES THE MEAN ERROR PER MEASUREMENT OF THE VARIOUS INDICES WITH RESPECT TO THE INDEX VALUE 100, UP TO THE CURRENT CYCLE. THE MEAN ERROR PER MEASUREMENT IS THE GEOMETRIC MEAN OF THE ERROR FACTORS WHICH IS CONVERTED TO A PERCENTAGE ERROR.

MEAN PERCENTAGE ERROR PER MEASUREMENT					
CYCLE	LASPEYRES	PAASCHE	FISHER	UNIT	NOR.UNIT
1	101.2038	-49.2590	1.0410	-2.8122	-0.0379
2	103.2661	-50.0528	0.7599	-2.2511	-0.0021
3	101.8395	-49.9812	0.4777	-1.1960	-0.3900
4	102.3949	-50.3910	0.2028	-1.8284	-0.8694
5	101.4906	-50.9925	-0.6292	-2.1819	-0.6779
6	99.5986	-50.9647	-1.0688	-2.1599	-0.5435
7	98.5651	-50.9212	-1.2815	-2.1450	-0.4420
8	99.7767	-50.8767	-0.9360	-1.3904	-0.0038
9	100.0085	-51.0674	-1.0710	-1.4172	0.2015
10	99.5568	-51.5138	-1.6346	-2.1116	0.0434
100	102.0746	-50.5112	0.0021	0.2340	0.0926
200	102.0894	-50.4962	0.0210	0.1373	0.0473
300	102.0840	-50.5320	-0.0166	0.0059	0.0078
400	101.9494	-50.5061	-0.0237	0.0498	0.0452
500	102.0565	-50.4856	0.0235	0.0790	0.0564
600	102.1315	-50.4990	0.0286	0.0656	0.0448
700	102.0633	-50.5106	-0.0001	0.0481	0.0448
800	102.1198	-50.4936	0.0311	0.0850	0.0431
900	102.1312	-50.4975	0.0299	0.0704	0.0386
1000	102.0595	-50.5148	-0.0052	0.0258	0.0286

THE DEVIATIONS OF THE MEAN OF THE RANDOM NUMBERS, UP TO THE CURRENT CYCLE, FROM THEIR EXPECTED VALUE 1.0, IS GIVEN IN THE FOLLOWING TABLE. THESE RANDOM NUMBERS ARE USED TO CAUSE THE INDEPENDENT VARIATION IN PRICE AND QUANTITY LEVELS.

CYCLE	NUMBER OF STANDARD DEVIATIONS	
	PRICE RANDOMS	QUANTITY RANDOMS
1	-0.0122	-0.0205
2	-0.0121	-0.0121
3	-0.0023	-0.0007
4	0.0028	0.0047
5	0.0062	0.0042
6	0.0057	0.0034
7	0.0051	0.0027
8	0.0017	0.0020
9	0.0020	0.0013
10	0.0026	0.0016
100	-0.0002	-0.0003
200	0.0000	0.0000
300	0.0001	0.0001
400	0.0001	0.0000
500	0.0000	0.0000
600	0.0001	0.0000
700	0.0000	0.0001
800	0.0000	0.0001
900	-0.0001	0.0001
1000	-0.0001	0.0001

\$ RUN INDEX ! SUMMARY OUTPUT

SUMMARY OR FULL LISTING (S/F) ?

S

ELASTICITY (2>=...>=-2) ?

-2

PERCENTAGE STEP (99>=...>=1) ?

20

NUMBER OF PRICE RANDOMS TO SKIP (>=0) ?

0

NUMBER OF QUANTITY RANDOMS TO SKIP (>=0) ?

0

SIMULATING 1000 SHOPS FOR 1000 CYCLES AND GENERATING ONE SET OF VALUES OF THE LASPEYRES, PAASCHE, FISHER, UNIT, AND NORMALIZED UNIT INDICES IN EACH CYCLE.

THE FOLLOWING TABLE GIVES THE MEAN ERROR PER MEASUREMENT OF THE VARIOUS INDICES WITH RESPECT TO THE INDEX VALUE 100, UP TO THE FINAL CYCLE.

THE MEAN ERROR PER MEASUREMENT IS THE GEOMETRIC MEAN OF THE ERROR FACTORS WHICH IS CONVERTED TO A PERCENTAGE ERROR.

INDEPENDENT VARIATION(%)		MEAN PERCENTAGE ERROR PER MEASUREMENT				
PRICE	QUANTITY	LASPEYRES	PAASCHE	FISHER	UNIT	NOR.UNIT
0.0	0.0	0.0000	0.0000	0.0000	0.0000	0.0000
0.0	20.0	0.0000	0.0000	0.0000	0.0000	0.0000
0.0	40.0	0.0000	0.0000	0.0000	0.0000	0.0000
0.0	60.0	0.0000	0.0000	0.0000	0.0000	0.0000
0.0	80.0	0.0000	0.0000	0.0000	0.0000	0.0000
20.0	0.0	2.7615	-2.6800	0.0038	0.0044	0.0038
20.0	20.0	2.7647	-2.6735	0.0086	0.0047	0.0072
20.0	40.0	2.7677	-2.6669	0.0135	0.0050	0.0109
20.0	60.0	2.7707	-2.6602	0.0184	0.0052	0.0148
20.0	80.0	2.7735	-2.6534	0.0233	0.0055	0.0191
40.0	0.0	12.3871	-11.0060	0.0088	0.0119	0.0081
40.0	20.0	12.3975	-10.9917	0.0216	0.0124	0.0149
40.0	40.0	12.4075	-10.9770	0.0342	0.0129	0.0223
40.0	60.0	12.4170	-10.9620	0.0469	0.0134	0.0305
40.0	80.0	12.4260	-10.9466	0.0596	0.0139	0.0397
60.0	0.0	35.1909	-26.0042	0.0177	0.0254	0.0112
60.0	20.0	35.2174	-25.9814	0.0430	0.0260	0.0224
60.0	40.0	35.2423	-25.9577	0.0682	0.0265	0.0343
60.0	60.0	35.2656	-25.9332	0.0934	0.0269	0.0469
60.0	80.0	35.2873	-25.9080	0.1185	0.0273	0.0606
80.0	0.0	101.9879	-50.4549	0.0375	0.0573	0.0109
80.0	20.0	102.0502	-50.4237	0.0845	0.0565	0.0275
80.0	40.0	102.1043	-50.3908	0.1311	0.0556	0.0436
80.0	60.0	102.1502	-50.3561	0.1774	0.0545	0.0586
80.0	80.0	102.1881	-50.3198	0.2235	0.0534	0.0724

THE SAME SEQUENCES OF RANDOM NUMBERS (APPROPRIATELY SCALED) ARE USED TO GENERATE THE RESULTS IN EACH ROW OF THE ABOVE TABLE. THESE RANDOM NUMBERS ARE USED TO CAUSE THE INDEPENDENT VARIATION IN PRICE AND QUANTITY LEVELS. THE DEVIATIONS OF THE MEAN OF THE UNSCALED RANDOM NUMBERS, UP TO THE FINAL CYCLE, FROM THEIR EXPECTED VALUE 1.0, ARE AS FOLLOWS.

PRICE RANDOMS: 0.0001 STANDARD DEVIATIONS
 QUANTITY RANDOMS: -0.0001 STANDARD DEVIATIONS

\$ RUN INDEX ! SUMMARY OUTPUT

SUMMARY OR FULL LISTING (S/F) ?

S

ELASTICITY (2>=...>=-2) ?

-1

PERCENTAGE STEP (99>=...>=1) ?

20

NUMBER OF PRICE RANDOMS TO SKIP (>=0) ?

0

NUMBER OF QUANTITY RANDOMS TO SKIP (>=0) ?

0

SIMULATING 1000 SHOPS FOR 1000 CYCLES AND GENERATING ONE SET OF VALUES OF THE LASPEYRES, PAASCHE, FISHER, UNIT, AND NORMALIZED UNIT INDICES IN EACH CYCLE.

THE FOLLOWING TABLE GIVES THE MEAN ERROR PER MEASUREMENT OF THE VARIOUS INDICES WITH RESPECT TO THE INDEX VALUE 100, UP TO THE FINAL CYCLE.

THE MEAN ERROR PER MEASUREMENT IS THE GEOMETRIC MEAN OF THE ERROR FACTORS WHICH IS CONVERTED TO A PERCENTAGE ERROR.

INDEPENDENT VARIATION(%)		MEAN PERCENTAGE ERROR PER MEASUREMENT				
PRICE	QUANTITY	LASPEYRES	PAASCHE	FISHER	UNIT	NOR.UNIT
0.0	0.0	0.0000	0.0000	0.0000	0.0000	0.0000
0.0	20.0	0.0000	0.0000	0.0000	0.0000	0.0000
0.0	40.0	0.0000	0.0000	0.0000	0.0000	0.0000
0.0	60.0	0.0000	0.0000	0.0000	0.0000	0.0000
0.0	80.0	0.0000	0.0000	0.0000	0.0000	0.0000
20.0	0.0	1.3675	-1.3420	0.0035	0.0039	0.0036
20.0	20.0	1.3699	-1.3360	0.0078	0.0041	0.0072
20.0	40.0	1.3724	-1.3299	0.0121	0.0044	0.0109
20.0	60.0	1.3747	-1.3238	0.0164	0.0047	0.0149
20.0	80.0	1.3770	-1.3176	0.0206	0.0050	0.0190
40.0	0.0	5.9095	-5.5651	0.0078	0.0094	0.0078
40.0	20.0	5.9166	-5.5527	0.0177	0.0099	0.0148
40.0	40.0	5.9235	-5.5400	0.0276	0.0105	0.0224
40.0	60.0	5.9302	-5.5272	0.0376	0.0110	0.0306
40.0	80.0	5.9366	-5.5142	0.0475	0.0116	0.0394
60.0	0.0	15.5090	-13.4028	0.0138	0.0181	0.0128
60.0	20.0	15.5252	-13.3843	0.0315	0.0189	0.0238
60.0	40.0	15.5408	-13.3653	0.0492	0.0198	0.0356
60.0	60.0	15.5558	-13.3459	0.0669	0.0206	0.0485
60.0	80.0	15.5702	-13.3260	0.0846	0.0214	0.0627
80.0	0.0	37.2726	-27.1147	0.0258	0.0361	0.0183
80.0	20.0	37.3099	-27.0908	0.0558	0.0370	0.0352
80.0	40.0	37.3455	-27.0661	0.0857	0.0378	0.0530
80.0	60.0	37.3794	-27.0405	0.1156	0.0386	0.0717
80.0	80.0	37.4116	-27.0142	0.1454	0.0394	0.0920

THE SAME SEQUENCES OF RANDOM NUMBERS (APPROPRIATELY SCALED) ARE USED TO GENERATE THE RESULTS IN EACH ROW OF THE ABOVE TABLE. THESE RANDOM NUMBERS ARE USED TO CAUSE THE INDEPENDENT VARIATION IN PRICE AND QUANTITY LEVELS. THE DEVIATIONS OF THE MEAN OF THE UNSCALED RANDOM NUMBERS, UP TO THE FINAL CYCLE, FROM THEIR EXPECTED VALUE 1.0, ARE AS FOLLOWS.

PRICE RANDOMS: 0.0001 STANDARD DEVIATIONS
 QUANTITY RANDOMS: -0.0001 STANDARD DEVIATIONS

\$ RUN INDEX ! SUMMARY OUTPUT

SUMMARY OR FULL LISTING (S/F) ?

S

ELASTICITY (2>=...>=-2) ?

0

PERCENTAGE STEP (99>=...>=1) ?

20

NUMBER OF PRICE RANDOMS TO SKIP (>=0) ?

0

NUMBER OF QUANTITY RANDOMS TO SKIP (>=0) ?

0

SIMULATING 1000 SHOPS FOR 1000 CYCLES AND GENERATING ONE SET OF VALUES OF THE LASPEYRES, PAASCHE, FISHER, UNIT, AND NORMALIZED UNIT INDICES IN EACH CYCLE.

THE FOLLOWING TABLE GIVES THE MEAN ERROR PER MEASUREMENT OF THE VARIOUS INDICES WITH RESPECT TO THE INDEX VALUE 100, UP TO THE FINAL CYCLE.

THE MEAN ERROR PER MEASUREMENT IS THE GEOMETRIC MEAN OF THE ERROR FACTORS WHICH IS CONVERTED TO A PERCENTAGE ERROR.

INDEPENDENT VARIATION(%)		MEAN PERCENTAGE ERROR PER MEASUREMENT				
PRICE	QUANTITY	LASPEYRES	PAASCHE	FISHER	UNIT	NOR.UNIT
0.0	0.0	0.0000	0.0000	0.0000	0.0000	0.0000
0.0	20.0	0.0000	0.0000	0.0000	0.0000	0.0000
0.0	40.0	0.0000	0.0000	0.0000	0.0000	0.0000
0.0	60.0	0.0000	0.0000	0.0000	0.0000	0.0000
0.0	80.0	0.0000	0.0000	0.0000	0.0000	0.0000
20.0	0.0	0.0033	0.0033	0.0033	0.0033	0.0033
20.0	20.0	0.0050	0.0089	0.0070	0.0036	0.0070
20.0	40.0	0.0068	0.0145	0.0106	0.0038	0.0109
20.0	60.0	0.0085	0.0202	0.0143	0.0041	0.0148
20.0	80.0	0.0101	0.0258	0.0180	0.0044	0.0188
40.0	0.0	0.0065	0.0065	0.0065	0.0065	0.0065
40.0	20.0	0.0101	0.0177	0.0139	0.0071	0.0140
40.0	40.0	0.0136	0.0288	0.0212	0.0076	0.0217
40.0	60.0	0.0171	0.0401	0.0286	0.0082	0.0295
40.0	80.0	0.0205	0.0514	0.0359	0.0088	0.0375
60.0	0.0	0.0098	0.0098	0.0098	0.0098	0.0098
60.0	20.0	0.0152	0.0264	0.0208	0.0106	0.0210
60.0	40.0	0.0205	0.0430	0.0318	0.0114	0.0325
60.0	60.0	0.0258	0.0598	0.0428	0.0123	0.0443
60.0	80.0	0.0310	0.0766	0.0538	0.0131	0.0561
80.0	0.0	0.0130	0.0130	0.0130	0.0130	0.0130
80.0	20.0	0.0203	0.0350	0.0276	0.0141	0.0280
80.0	40.0	0.0275	0.0571	0.0423	0.0152	0.0433
80.0	60.0	0.0346	0.0793	0.0570	0.0163	0.0589
80.0	80.0	0.0416	0.1016	0.0716	0.0174	0.0747

THE SAME SEQUENCES OF RANDOM NUMBERS (APPROPRIATELY SCALED) ARE USED TO GENERATE THE RESULTS IN EACH ROW OF THE ABOVE TABLE. THESE RANDOM NUMBERS ARE USED TO CAUSE THE INDEPENDENT VARIATION IN PRICE AND QUANTITY LEVELS. THE DEVIATIONS OF THE MEAN OF THE UNSCALED RANDOM NUMBERS, UP TO THE FINAL CYCLE, FROM THEIR EXPECTED VALUE 1.0, ARE AS FOLLOWS.

PRICE RANDOMS: 0.0001 STANDARD DEVIATIONS
 QUANTITY RANDOMS: -0.0001 STANDARD DEVIATIONS

\$ RUN INDEX ! SUMMARY OUTPUT

SUMMARY OR FULL LISTING (S/F) ?

S

ELASTICITY (2>=...>=-2) ?

1

PERCENTAGE STEP (99>=...>=1) ?

20

NUMBER OF PRICE RANDOMS TO SKIP (>=0) ?

0

NUMBER OF QUANTITY RANDOMS TO SKIP (>=0) ?

0

SIMULATING 1000 SHOPS FOR 1000 CYCLES AND GENERATING ONE SET OF VALUES OF THE LASPEYRES, PAASCHE, FISHER, UNIT, AND NORMALIZED UNIT INDICES IN EACH CYCLE.

THE FOLLOWING TABLE GIVES THE MEAN ERROR PER MEASUREMENT OF THE VARIOUS INDICES WITH RESPECT TO THE INDEX VALUE 100, UP TO THE FINAL CYCLE.

THE MEAN ERROR PER MEASUREMENT IS THE GEOMETRIC MEAN OF THE ERROR FACTORS WHICH IS CONVERTED TO A PERCENTAGE ERROR.

INDEPENDENT VARIATION(%)		MEAN PERCENTAGE ERROR PER MEASUREMENT				
PRICE	QUANTITY	LASPEYRES	PAASCHE	FISHER	UNIT	NOR.UNIT
0.0	0.0	0.0000	0.0000	0.0000	0.0000	0.0000
0.0	20.0	0.0000	0.0000	0.0000	0.0000	0.0000
0.0	40.0	0.0000	0.0000	0.0000	0.0000	0.0000
0.0	60.0	0.0000	0.0000	0.0000	0.0000	0.0000
0.0	80.0	0.0000	0.0000	0.0000	0.0000	0.0000
20.0	0.0	-1.3107	1.3341	0.0030	0.0027	0.0030
20.0	20.0	-1.3096	1.3393	0.0061	0.0029	0.0068
20.0	40.0	-1.3086	1.3446	0.0092	0.0032	0.0107
20.0	60.0	-1.3075	1.3498	0.0123	0.0035	0.0145
20.0	80.0	-1.3065	1.3550	0.0154	0.0038	0.0181
40.0	0.0	-5.0507	5.3303	0.0052	0.0038	0.0051
40.0	20.0	-5.0503	5.3404	0.0102	0.0043	0.0123
40.0	40.0	-5.0498	5.3505	0.0153	0.0049	0.0194
40.0	60.0	-5.0493	5.3606	0.0203	0.0054	0.0263
40.0	80.0	-5.0487	5.3705	0.0254	0.0059	0.0326
60.0	0.0	-10.6944	11.9898	0.0065	0.0033	0.0059
60.0	20.0	-10.6963	12.0050	0.0123	0.0040	0.0150
60.0	40.0	-10.6981	12.0202	0.0181	0.0047	0.0238
60.0	60.0	-10.6997	12.0352	0.0239	0.0054	0.0320
60.0	80.0	-10.7012	12.0501	0.0297	0.0060	0.0393
80.0	0.0	-17.5580	21.3147	0.0071	0.0012	0.0050
80.0	20.0	-17.5634	21.3353	0.0124	0.0020	0.0133
80.0	40.0	-17.5686	21.3559	0.0177	0.0027	0.0211
80.0	60.0	-17.5738	21.3764	0.0230	0.0035	0.0281
80.0	80.0	-17.5788	21.3968	0.0283	0.0043	0.0338

THE SAME SEQUENCES OF RANDOM NUMBERS (APPROPRIATELY SCALED) ARE USED TO GENERATE THE RESULTS IN EACH ROW OF THE ABOVE TABLE. THESE RANDOM NUMBERS ARE USED TO CAUSE THE INDEPENDENT VARIATION IN PRICE AND QUANTITY LEVELS. THE DEVIATIONS OF THE MEAN OF THE UNSCALED RANDOM NUMBERS, UP TO THE FINAL CYCLE, FROM THEIR EXPECTED VALUE 1.0, ARE AS FOLLOWS.

PRICE RANDOMS: 0.0001 STANDARD DEVIATIONS
 QUANTITY RANDOMS: -0.0001 STANDARD DEVIATIONS

\$ RUN INDEX ! SUMMARY OUTPUT

SUMMARY OR FULL LISTING (S/F) ?
 S
 ELASTICITY (2>=...>=-2) ?
 2
 PERCENTAGE STEP (99>=...>=1) ?
 20
 NUMBER OF PRICE RANDOMS TO SKIP (>=0) ?
 0
 NUMBER OF QUANTITY RANDOMS TO SKIP (>=0) ?
 0

SIMULATING 1000 SHOPS FOR 1000 CYCLES AND GENERATING ONE SET OF VALUES OF THE LASPEYRES, PAASCHE, FISHER, UNIT, AND NORMALIZED UNIT INDICES IN EACH CYCLE.

THE FOLLOWING TABLE GIVES THE MEAN ERROR PER MEASUREMENT OF THE VARIOUS INDICES WITH RESPECT TO THE INDEX VALUE 100, UP TO THE FINAL CYCLE. THE MEAN ERROR PER MEASUREMENT IS THE GEOMETRIC MEAN OF THE ERROR FACTORS WHICH IS CONVERTED TO A PERCENTAGE ERROR.

INDEPENDENT VARIATION(%)		MEAN PERCENTAGE ERROR PER MEASUREMENT				
PRICE	QUANTITY	LASPEYRES	PAASCHE	FISHER	UNIT	NOR.UNIT
0.0	0.0	0.0000	0.0000	0.0000	0.0000	0.0000
0.0	20.0	0.0000	0.0000	0.0000	0.0000	0.0000
0.0	40.0	0.0000	0.0000	0.0000	0.0000	0.0000
0.0	60.0	0.0000	0.0000	0.0000	0.0000	0.0000
0.0	80.0	0.0000	0.0000	0.0000	0.0000	0.0000
20.0	0.0	-2.5574	2.6299	0.0026	0.0021	0.0027
20.0	20.0	-2.5571	2.6348	0.0052	0.0023	0.0065
20.0	40.0	-2.5567	2.6397	0.0077	0.0026	0.0102
20.0	60.0	-2.5563	2.6445	0.0103	0.0029	0.0138
20.0	80.0	-2.5559	2.6493	0.0129	0.0031	0.0171
40.0	0.0	-9.1794	10.1154	0.0037	0.0016	0.0041
40.0	20.0	-9.1814	10.1245	0.0068	0.0020	0.0100
40.0	40.0	-9.1833	10.1335	0.0098	0.0025	0.0157
40.0	60.0	-9.1850	10.1423	0.0128	0.0029	0.0210
40.0	80.0	-9.1866	10.1509	0.0159	0.0034	0.0256
60.0	0.0	-17.6250	21.4049	0.0036	-0.0006	0.0045
60.0	20.0	-17.6304	21.4174	0.0055	0.0000	0.0099
60.0	40.0	-17.6357	21.4298	0.0074	0.0005	0.0149
60.0	60.0	-17.6408	21.4420	0.0093	0.0011	0.0192
60.0	80.0	-17.6457	21.4540	0.0113	0.0016	0.0226
80.0	0.0	-25.9931	35.1309	0.0031	-0.0029	0.0052
80.0	20.0	-26.0015	35.1453	0.0028	-0.0024	0.0090
80.0	40.0	-26.0097	35.1596	0.0025	-0.0018	0.0124
80.0	60.0	-26.0179	35.1740	0.0023	-0.0012	0.0149
80.0	80.0	-26.0260	35.1883	0.0021	-0.0007	0.0161

THE SAME SEQUENCES OF RANDOM NUMBERS (APPROPRIATELY SCALED) ARE USED TO GENERATE THE RESULTS IN EACH ROW OF THE ABOVE TABLE. THESE RANDOM NUMBERS ARE USED TO CAUSE THE INDEPENDENT VARIATION IN PRICE AND QUANTITY LEVELS. THE DEVIATIONS OF THE MEAN OF THE UNSCALED RANDOM NUMBERS, UP TO THE FINAL CYCLE, FROM THEIR EXPECTED VALUE 1.0, ARE AS FOLLOWS.

PRICE RANDOMS: 0.0001 STANDARD DEVIATIONS
 QUANTITY RANDOMS: -0.0001 STANDARD DEVIATIONS