TECHNICAL NOTES

A NOTE ON SOCIALLY OPTIMAL R&D PROGRAMS AND THEIR INDUCEMENT

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This paper considers optimal public policies regarding R&D programs in a random environment. For a stochastic R&D decision model without rivalry, we investigate and derive the privately and the socially optimal policies. The study focuses on the socially optimal R&D program and its inducement by governmental incentives. The appropriate instruments that should be employed in supporting R&D projects are examined. Our proposed R&D model provides a theoretical economic justification for public intervention in support of private R&D activities. Furthermore, some of the results shed light on practical issues in designing a functional and efficient R&D project support system.

Posner and Zuckerman (1990a) examine an R&D decision model for a specific project without rivalry, assuming a stochastic relationship between the firm's expenditure strategy and the project's status. Furthermore, the termination time of the R&D project is incorporated into the model as a decision variable. The firm's optimal expenditure and termination strategy is analyzed. In this note, we consider spinoff effects to society at large and compare the socially optimal strategy and the privately optimal one. We refer to Posner and Zuckerman (1990a) for a review of related papers.

Our proposed R&D model provides a theoretical economic justification for public intervention in support of private R&D activities. Furthermore, some of the results shed light on practical issues involved in designing a functional and efficient R&D project support system.

The paper is organized as follows: In Section 1, we describe the R&D decision model. In Section 2, we formulate the optimal R&D programs of the two parties. The socially optimal R&D program and its inducement is examined. Finally, we discuss the realism of the proposed model.

1. THE R&D MODEL

We consider the R&D model without rivalry proposed by Posner and Zuckerman (1990a). Specifically, let $\{X(t); t \geq 0\}$, $(X(0) = 0)$, be a one-sided, nondecreasing stochastic process, interpreted as the monetary value of the technological knowledge accumulated by the research program up to time t. We assume that X is a jump process. Every jump in the proposed setting represents a scientific breakthrough or a new discovery. Progress is achieved via the expenditure of resources. Let $C(t, x)$ and $\lambda(t, x)$ be the expenditure rate and the jump rate, respectively, at time t given that $X(t) = x$. The relationship between research effort and the arrival of new discoveries is given by $\lambda(t, x) = R(C(t, x))$, where $R(\cdot)$ is a nonnegative, nondecreasing concave function, representing a diminishing return due to increased R&D effort. At any time t during the R&D program the project can be terminated and a return of $X(t)$ is then realized by the firm.

An R&D policy is composed of two ingredients: A stopping time T which determines when the R&D program should be terminated, and an expenditure

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strategy which specifies the expenditure rate throughout the R&D program. In what follows, we assume that the times between discoveries (jumps) are exponentially distributed, which is a common assumption in the R&D literature. Assuming an infinite planning horizon, we can restrict attention to time independent expenditure strategies of the form $C(x)$. Correspondingly, under the proposed setup the jump rate of new discoveries is time independent and will now be denoted by $\lambda(C(x))$. In what follows, we adopt the following reasonable assumptions regarding the arrival rate of new discoveries:

$\lambda(c)$ is defined for $c \geq c_1 > 0$, where $\lambda(c_1) = 0$. Expenditure rates below c_1 are insufficient to lead to new discoveries.

$\lambda(c)$ is monotonically increasing, differentiable, concave and bounded.

The jump magnitudes associated with the process $X(t)$ are assumed to be state dependent. More explicitly, if currently $X(t) = x$, then the next jump Y_x is a random variable which has a distribution function F_x and is independent of the research effort. Furthermore, the technological value associated with a new discovery, specified by F_x, *decreases stochastically* in x. The independence between the expenditure policy and the magnitude of new discoveries is a realistic assumption in some cases. For example, consider a development project in which an increase in R&D expenditures is reflected in an increase in the number of research groups working on the project, with each group employing a different scientific approach. In such a case, the expenditure policy affects the arrival rate of new discoveries, but not necessarily their magnitude.

To avoid an unrealistic situation in which smaller values of the project state are better than higher values, we assume that for any pair of states x_1 and x_2 with $x_1 < x_2$, $x_1 + Y_{x_1}$ is stochastically smaller than $x_2 + Y_{x_2}$.

An essential feature of R&D programs, from a social point of view, is the *spinoff* (or externality) flowing from the technology developed by the firm that benefits various applications outside the domain of the firm's activity. Specifically, let $S(x)$ be the spinoff effect from the project, measured in monetary units, assuming that a technological level of x has been reached by the R&D program. The social benefits of this externality are realized when the developed technology is introduced. Throughout the study, we restrict our attention to concave and increasing spinoff functions representing decreasing, marginal spinoff effects due to increased technological level.

2. THE PRIVATELY AND SOCIALLY OPTIMAL R&D PROGRAMS

In this section, we explore the privately and socially optimal R&D programs. The following optimality criteria are considered:

a. maximum expected net return;
b. maximum expected discounted net return.

Assuming that an expenditure strategy $C(x)$ and a stopping rule T are employed, the firm's expected discounted net return is given by:

$$f_\alpha(C, T) = E\left[e^{-\alpha T} X_C(T) - \int_0^T e^{-\alpha t} C(X_C(t))\, dt\right], \quad (1)$$

where $X_C(t)$, for $t \in [0, T]$, is the project state at time t under policy (C, T), and α is a continuous discount factor. On the other hand, the social objective function associated with the proposed R&D model includes the spinoff effects and can be expressed as:

$$g_\alpha(C, T) = E\left[e^{-\alpha T}(X_C(T) + S(X_C(T))) - \int_0^T e^{-\alpha t} C(X_C(t))\, dt\right]. \quad (2)$$

Note that when α approaches 0, (1) and (2) represent the objective functions of the two parties in the undiscounted case.

In Theorems 1–6 we define and compare the optimal R&D policies of the two parties. In addition, we provide the characteristics of governmental subsidy policies which induce the private firm to behave optimally from a social point of view. The underlying mathematics of the proofs are a straightforward extension of the Posner and Zuckerman (1990a) results, and therefore all the proofs are omitted. For more details regarding the mathematical derivation we refer to Posner and Zuckerman (1990b). Theorems 1 and 2 characterize the privately and socially optimal R&D programs.

Theorem 1. *For a constant expenditure rate $C(x) = c$, the firm's optimal stopping strategy $T^*_{f,\alpha}(c)$ is a control limit policy. That is, we stop the project whenever the technological value exceeds a fixed critical value $\xi_{f,\alpha}(c)$ given by*

$$\xi_{f,\alpha}(c) = \inf\{x;\ \alpha x \geq \lambda(c)\mu_x - c\},\ \alpha > 0, \quad (3)$$

and $\mu_x = \int u\, dF_x(u)$.

In the undiscounted case, ($\alpha = 0$), the firm's optimal expenditure rate $C^*(x)$ remains constant throughout *the R&D program at a rate of* $c^*_{f,0}$, *which minimizes* $c/\lambda(c)$ *over the range* (c_1, ∞).

Theorem 2. *For a constant expenditure rate* $C(x) = c$, *the socially optimal stopping strategy* $T^*_{g,\alpha}(c)$ *is a control limit policy with an optimal control level*:

$$\xi_{g,\alpha}(c) = \inf\{x; \alpha x \geq \lambda(c)\mu_x - c + \lambda(c) \cdot [E(S(x+Y_x)) - S(x)] - \alpha S(x)\}, \alpha \geq 0. \quad (4)$$

In the undiscounted case ($\alpha = 0$), *the socially optimal expenditure rate remains constant throughout the R&D program at a rate of* $c^*_{g,0}$, *which minimizes* $c/\lambda(c)$ *over the range* (c_1, ∞).

The above results provide a formal theoretical comparison between the optimal private and social policies in the *undiscounted* case. Specifically, it turns out that the optimal R&D efforts of the two parties are identical. On the other hand, the desired technological level, from the firm's point of view, is smaller than is socially desirable. Furthermore, using (3) and (4) we find that the optimal control levels of the two parties $\xi_{f,\alpha}(c)$ and $\xi_{g,\alpha}(c)$ under a constant expenditure strategy $C(x) = c$ are monotonically decreasing in α, and they converge to $\xi_{f,0}(c) = \xi_{g,0}(c)$, respectively, when α approaches zero.

Theorem 3 specifies the characteristics of a governmental subsidy policy which induces the private firm to behave optimally from a social point of view in the undiscounted case ($\alpha = 0$).

Theorem 3. *Let* $\xi_{g,0} = \xi_{g,0}(c^*_{g,0})$ *and*

$$\gamma = c^*_{g,0}/[\lambda(c^*_{g,0})\mu_{\xi_{g,0}}] - 1. \quad (5)$$

Then, a governmental subsidy of $\gamma/(\gamma + 1)$ *dollars for each dollar directed by the firm to the project will induce the firm to behave optimally from a social point of view.*

For a constant expenditure rate, $C(x) = c$, the relationship between the privately and the socially desired technological levels is given in Theorem 4.

Theorem 4. *If for* $x = \xi_{f,\alpha}(c)$,

$$S(x) \geq [\lambda(c)/(\lambda(c) + \alpha)]E[S(x + Y_x)], \quad (6)$$

then

$$\xi_{g,\alpha}(c) \leq \xi_{f,\alpha}(c).$$

Otherwise,

$$\xi_{g,\alpha}(c) \geq \xi_{f,\alpha}(c).$$

Next we propose for the discounted case a governmental subsidy policy which induces the firm to terminate the R&D program at a socially optimal point in time.

Theorem 5. *For a constant expenditure rate,* $C(x) = c$,

if $\xi_{g,\alpha}(c) \geq \xi_{f,\alpha}(c)$, *then a state dependent subsidy payment of* $\gamma(x)$ *dollars for each dollar directed by the firm to the R&D program will induce the firm to select a socially optimal stopping strategy, where* $\gamma(x)$ *is defined by*

$$\gamma(x) = \xi_{g,\alpha}(c)\alpha - [\lambda(c)\mu_x - c]. \quad (7)$$

If $\xi_{g,\alpha}(c) < \xi_{f,\alpha}(c)$, *then a lump sum subsidy of* $V_{f,\alpha}(\xi_{g,\alpha}(c) \mid c) - \xi_{g,\alpha}(c)$ *provided to the firm at time* $T^*_{g,\alpha}(c)$ *will induce the firm to stop the R&D program at time* $T^*_{g,\alpha}(c)$, *where* $V_{f,\alpha}(\mid c)$ *is the firm's maximum value function assuming a constant expenditure rate of* c.

Regarding the incentive scheme proposed in Theorem 5, note that if the government does not have information throughout the development stage concerning the project state, then the firm will have an incentive to lie and assert that the project's current state is higher than it really is (since subsidy increases with the state). The source of this moral hazard or incentive problem is an asymmetry of information between the two parties. The issue of moral hazard and observability in a principal-agent relationship has been investigated by several authors (see, for example, Holmstrom 1979). Our suggested incentive scheme can be employed only if the project level of technology reported by the firm can be verified costlessly by the government.

Finally, we characterize qualitatively the privately and the socially optimal expenditure strategies. The relationship between the optimal research effort of the two parties is also provided.

Theorem 6. *Let* $C^*_{f,\alpha}(x)$ *and* $C^*_{g,\alpha}(x)$ *be the optimal expenditure strategies of the two parties. Then,* $C^*_{f,\alpha}(x)$ *and* $C^*_{g,\alpha}(x)$ *are monotonically increasing in* x. *For every* $\alpha > 0$ *and* x *contained in the state space of* $X = \{X(t); t \geq 0\}$,

$$C^*_{g,\alpha}(x) \geq C^*_{f,\alpha}(x). \quad (8)$$

3. CONCLUDING COMMENTS

1. Besides the positive social aspects associated with R&D activities, in certain industries new technological

advancement can lead to significant social and environmental hazards. These negative aspects should be included in the social objective function.

2. There is limited knowledge regarding the behavior of the spinoff effects from new technology in the various industries.

3. Usually, the desired technological level from the firm's point of view is smaller than is socially desirable. But, as demonstrated in Theorem 4, this is not always the case. For example, suppose that in a given monopolistic industry an important scientific breakthrough with a significant spinoff effect has been achieved. In this case, if the demand for the firm's existing line of products is high and the new products which can be developed with the aid of the new discovery are substitutes for the existing ones, then it might be preferable for the firm to achieve a higher technological level than is socially optimal and to delay the introduction time of the new technology.

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ALLOCATION OF WAITING TIME BY TRADING IN POSITION ON A G/M/S QUEUE

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Customers on an S-server queue with exponential service times face losses due to waiting that are proportional to waiting time, with different loss rates. Customers are otherwise identical. They have complete knowledge of each other's loss rates. Instead of bribing a queue manager for priority assignment, they buy and sell queue positions among themselves. It is shown that the resulting market in queue positions optimally allocates waiting time. The transactions that can occur are completely characterized, including balking and reneging rules.

Past research on decentralized priority assignment in queues has considered either priority prices set by the queue manager, with customers choosing which price to pay and, hence, their own priorities, or customer bribing of the server. Examples of the former are Marchand (1968, 1974), Levhari and Sheshinski (1974), Ghanem (1975), Babad and Modiano (1976), Rose-Ackerman (1978), and Rashid (1981), with related work on priority prices in Balachandran (1972), Adiri and Yechiali (1974), Balachandran and Lukens (1976), Tilt and Balachandran (1979), and Alperstein (1988). Dolan (1978) shows how to sort customers by waiting cost by using state-dependent prices that motivate customers to reveal their true loss rates due to waiting. Bribing is discussed in Kleinrock (1967), Levhari and Sheshinski (1974), Boe (1974), Lui (1985), and Glazer and Hassin (1986).

Further decentralization of customer choice of service order is possible. Since queueing is a negative consumption externality imposed by each customer on later arrivals, the recipients of compensation for that externality should be other customers, rather than

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