Use of interference methods to determine Poisson ratio and obtain conoscopic patterns

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To determine the Poisson coefficient, Cornu suggested using bands of equal thickness from the air gap between two optically plane plates, the lower plate being subjected to pure bending. The method is suitable only for optically perfect surfaces. In this article the authors describe a method in which one can use a lower plate of low quality. The developed lensless demonstration conoscope enables one to obtain, with the aid of a cine lamp, patterns of high clarity, measuring several meters, practically without limitation of the aperture.

MODIFICATION OF CORNU'S METHOD TO DETERMINE POISSON RATIO

One of the most elegant methods of determining Poisson's ratio is the method of Cornu,[1-3] used by him in 1869, then more thoroughly in 1899 by R. Straubel, and in 1921 by Jessop, but not achieving wide popularity. In Cornu's method aa prismatic rod or plate is deformed by pure bending. The surface then acquires a double curvature, since if the upper layer is elongated, then in the transverse direction it must be compressed, and vice versa below the neutral plane. Approximating the median curved lines of the upper surface by arcs of a circle, we find that the radii of curvature will be R and R/μ respectively, since the transverse compression is μ times less than the longitudinal elongation.

If on the undeformed test plate, having an optically plane surface, we lay a second plate with a good surface and bring them into contact, then after deformation of the lower plate, as a result of interference of light from the two surfaces, there will appear lines of equal thickness in the form of two families of hyperbolae with angle 2α between the asymptotes, where, as shown in Refs. 4 and 5,

$$\tan^2 \alpha = \mu. \quad (1)$$

A simple (without use of the theory of elasticity) conclusion from (1) may be as follows: we approximate the saddle-shaped surface of the bent plate by two crossed cylindrical surfaces. Using the expression, known from the theory of Newton's rings,

$$h = r^2/2R, \quad (2)$$

where r is the radius of a ring and h is the distance between the periphery and the tangent at its lowest point, we consider two crossed cylinders touching at one point and lying one upon the other. We find the distance between them by determining how far a point on each of them is distant from the common tangential plane passing through the point of contact. We introduce axes x and y, mutually perpendicular and lying in this tangential plane along the generators of the cylinders. Using expression (2), we find that $h_1 = x^2/2R_1$, and the

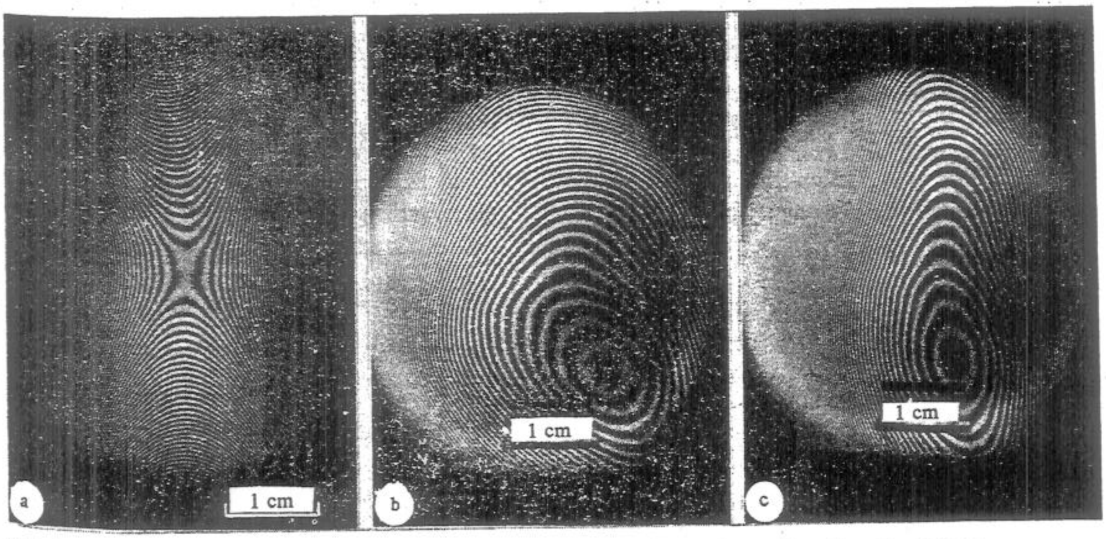

FIG. 1. Fringes of equal thickness: a) from air gap between upper and lower plates of good quality; b) between upper and lower undeformed ordinary photographic plate; c) between upper and deformed photographic plates.

FIG. 2. Moiré fringes obtained by superposing patterns of Figs. 1b and c in a single exposure.

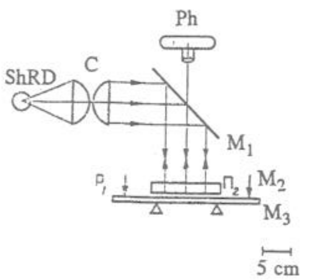

FIG. 3. Scheme of apparatus for obtaining interference patterns.

distance of the points of the second cylinder to the same plane is $h_2 = y^2/2R_2$. According to the principle of superposition, the interval between the cylinders is $h = h_1 + h_2 + x^2/2R_1 + y^2/2R_2$. Therefore, the lines of equal thickness are determined by the equation $x^2/2R_1 + y^2/2R_2 = \text{const}$ and take the form of ellipses.

In the case of the deformed plate, taking the form of a saddle-shaped surface, for small values of x and y we find that the resulting gap is expressed as $x^2/2R_1 - y^2/2R_2 = \text{const}$, i.e., the lines of equal thickness between the deformed plate and the tangential plane are hyperbolae. In the usual setting of Cornu's method, in the deformed plate there is laid an optically plane upper plate, and the lines of equal thickness from the air gap are observed. Since $\tan\varphi = y/x$, for half the angle between the asymptotes, for which $h = 0$, we get $\tan^2\alpha = R_2/R_1 = \mu$.

The photographs are taken in a vertical direction through a filter which passes the mercury green line at 546 nm. Usually the surfaces of plates of low quality have an initial curvature, which leads to an initial intricate pattern of lines of equal thickness and to rough distortion of the pattern of hyperbolae after deformation. Evidently this has led to the low popularity of the method.

In order to get rid of the influence of the poor quality of the surface of the lower plate, we can use the method of double-exposure holography, obtaining first the hologram from the undeformed plate, and then that from the deformed one. In recovering the hologram of the air gap between the plates we obtain an interference pattern in the form of the above-mentioned families of hyperbolae. Our method was simpler: We photograph the interference pattern from the air layer between the undeformed lower and the upper plates. Then on the same negative we photograph the pattern with deformed lower plate. After the photograph is processed, superposition of the patterns leads to the appearance of moiré fringes,[6,7] caused only by change in the thickness of the air gap, i.e., we get the required hyperbolae. In Fig. 1a we give the pattern from a comparatively good lower plane plate, in Fig. 1b we give the pattern of fringes from an underformed ordinary photographic plate, and in Fig. 1c that from a deformed one; Figure 2 shows the moiré pattern obtained by superposing Figs. 1b and 1c. We see that the moiré pattern enables us to find μ even from low-planarity lower plates, provided that with an unloaded plate the interference fringes, even of irregular form, cover the working section.

Our experiment was as follows: A test plate was obtained by cutting up an exposed and developed (to avoid reflections from the second surface) photographic plate measuring $180 \times 240 \times 1.5$ mm into four longitudinal strips, so that the length-to-width ratio exceeded three. On the test plate we laid an optically flat PI-80 glass plate, in which the local deviations from planarity did not exceed 0.02 μm. With the aid of a sheet of mirror glass (300×300 mm), lying at 45° to the horizontal, we illuminated the gap between the plates with an ShRD-250 mercury lamp sited at the focus of the condenser (Fig. 3). The test glass strip was laid on two prisms with rubber packings. On the ends of the span projecting beyond the prisms we hung or laid a load not exceeding 0.5 kg. On account of deformation, between the plates there appeared a system of interference fringes of equal thickness.

Let us make a more detailed examination of the surface of the deformed plate. In the longitudinal direction the downward curvature was a maximum, and therefore the interference fringes were close together. The upward curvature in the transverse direction was $1/\mu$ times less, and therefore the interval between the fringes was greater. The asymptotes pass along the places where the curvature is zero.

Let us try to approximate the cross section of the surface along the directions of the bisectrices of the angles between the asymptotes in the form of an arc of a circle. The path difference between the plane and the sphere is $\Delta = 2h - (\lambda/2)$, where h is the camber. For the interference maximum we have $2h - (\lambda/2) = p\lambda$, $2h = (2p + 1)\lambda/2$, where p is the order of the maximum. But $x^2 = 2Rh$, i.e., $x_p{}^2 \sim (2p + 1)$. Taking the directions of the bisectrices as the x and y axes, we find that (x_p/x_1) and (y_p/y_1) must be proportional to the odd numbers $(2p + 1)$. Measuring the photograph, projected in enlarged form, of the moiré fringes of the interval between the vertices of the hyperbolae, we see that this dependence is satisfied along the x axis, and somewhat less accurately along

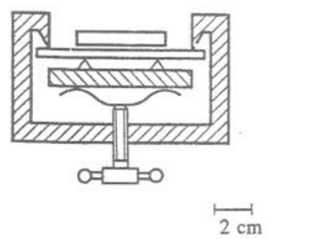

FIG. 4. Scheme of apparatus for smooth deformation of plate of tested material.

the y axis:

p	x_p/x_1	$\sqrt{2p+1}$	y_p/y_1
0	1	1	1
1	1.67	1.73	1.6
2	2.1	2.24	2.07
3	2.5	2.64	2.4
4	2.93	3	2.7
5	3.1	3.3	3
6	3.5	3.6	3.2
7	3.7	3.87	3.5
8	4	4.12	3.7.

Obtaining the scale on the photograph (1 cm), we can also find the radii of curvature along the axes of coordinates, since $R_1 = x_p^2/(2p + 1)\lambda$ and $R_2 = y_p^2/(2p + 1)\lambda$. The advantage over the usual methods of obtaining Newton's rings is that here the "lenses" have radii of curvature of tens and hundreds of meters. We got $R_1 \approx 34$ m, $R_2 \approx 144$ m, and $\mu = 0.25$.

The saddle-shaped surface is similar to a hyperbolic paraboloid, the equation of which is obtained by replacing h by z:

$$z = x^2/2R_1 - y^2/2R_2. \quad (3)$$

If this approximation is satisfactory, then the cross sections made by the plates z = const must be hyperbolae satisfying the equation $x^2/a^2 - y^2/b^2 = 1$, where a and b are determined from the photograph. This is satisfied only for small x and y. Figure 4 shows the scheme of the apparatus permitting smooth deformation of the plate.

DEMONSTRATION CONOSCOPE

The literature describes methods of exhibition of conoscopic patterns with the aid of hand-made optical stages, reproducing in essence the scheme of a polarization microscope or of special devices of the Dubosq type. But their fairly complex construction with a large number of parts sharply limits the aperture of the beams; this does not enable us to indicate sufficiently complete patterns with a marked angle between the optic axes, reduces the illumination, and complicates the work of the demonstrators. A very simple method, which yields large, bright, colored conoscopic patterns with large aperture, was realized by us: It comprises the following. Next to each other there are: a cine lamp (17 V, 300 W), a polaroid polarizer 80 mm in diameter, then the crystal plate and a polaroid analyzer. The polaroids and crystal plate are mounted on rotating holders. At a distance of several tens of centimeters there is a mat screen, on which, without any lenses, we obtain a conoscopic pattern. Its illumination and size are sufficient for display to a lecture audience. The device is shown in Fig. 5.

By this method we demonstrated patterns of a family of lemniscates of muscovite mica (50 × 50 × 0.25 mm, Fig. 6a), isochromes and isobars from Iceland spar (plates, measuring 20 × 30 × 3.5 mm, cut perpendicular to the optic axis), isochromes from a quartz plate of the same orientation (20 × 30 × 4 mm), Airy's spirals from two quartz plates with left- and right-hand rotation (15 × 15 × 5 mm), an interference pattern from Savart's plates (9 × 9 × 3.5 mm) cut from a single crystal of quartz at an angle of 45° to the axis, and so forth.

To demonstrate the conoscopic patterns from plates cut

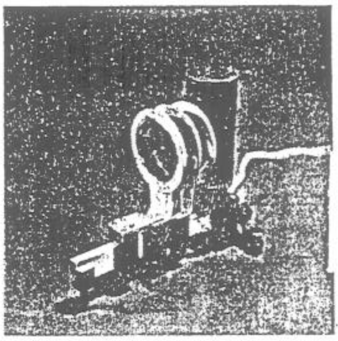

FIG. 5. Demonstration lensless conoscope.

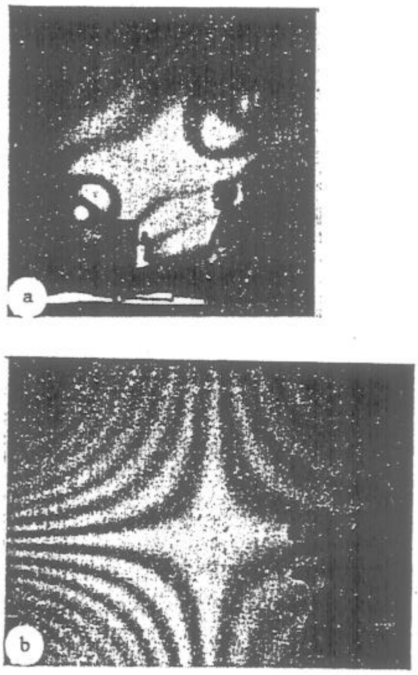

FIG. 6. Conoscopic patterns: a) from mica (2 × 2 m²) – diagonal position; b) from crossed gypsum plates (1.5 × 1.5 m²).

parallel to the axis of a uniaxial crystal or parallel to the axial plane of a biaxial one, we must use monochromatic light, e.g., from a laser, the beam of which is much widened by the objective of a microscope. The point is that in this cross section at the plates the path difference is a maximum. With illumination of the above device by laser light on the mat screen as before with no lenses we obtain a pattern of four families of hyperbolae. This experiment is conveniently performed with a gypsum plate, the cleavage plane of which is parallel to the plane of both optic axes. If from the gypsum we cut a plate perpendicular to the acute bisectrix, then by carefully heating the copper guide in which it is placed, we can show that the angle between the axes falls to zero, then again increases, but in a direction perpendicular to the original one.

The lensless demonstration conoscope enabled us to obtain a very large bright pattern in white light and from crystal plates oriented parallel to the optic axes (or the axis of a uniaxial crystal). Since in this case the path difference of the ordinary and extraordinary rays is a maximum, we must superpose two crossed identical plates, so that the order of interference is reduced to a minimum. Most convenient of all is gypsum, in which the perfect cleavage and both axes lies in the cleavage plane. The gypsum must be plane. So as to obtain optically good planes in a scratched natural specimen, we laid insulating tape on the surface of the band, then carefully removed it. This gives a bright specular surface without steps. The plane-parallel plate was cut with a hacksaw into two equal parts, laid crosswise upon one another and glued round the edge with paper. The dimensions of both plates were $6 \times 40 \times 40$ mm^2. Placing this "sandwich" between the polaroids close to the cine lamp of the conoscope, we obtain on the screen without any projection optics a large (1×1 m^2) a bright pattern of four families of hyperbolae (Fig. 6b). We can also dispense with the conoscope and smaller plates, but then it is necessary to use an arc source and place the polaroids and plate at the focus of a condenser, leading to marked heating.

If plates of quartz or iceland spar are used,[8] they must be specially cut, ground, and polished.

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Dynamic domain structure of the ferroelectric semiconductor $Sn_2P_2S_6$

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By means of nematic liquid crystals, the domain structure in an $Sn_2P_2S_6$ crystal has been observed. It is shown that the domains take the shape of rods orientated at 12–15° to the crystallographic [100] direction. An internal electric field in the crystals is due to the formation, in the process of screening, of spontaneous polarization of the bulk charge by the capture of carriers by traps with an activation energy of about 0.60 eV.

At $T_c = 337$ K, the crystals of tin thiohypodiphosphate $Sn_2P_2S_6$ undergo a ferroelectric phase transition of the second order with change of symmetry $P2_1/c \rightarrow Pc$ without change of the number of formula units in the unit cell. Consequently, they are proper uniaxial ferroelectrics, having domains with antiparallel polarization. The spontaneous polarization vector P_s lies in the (010) symmetry plane near the [100] direction. The value of P_s is about 15 $\mu C/cm^2$.

The compound $Sn_2P_2S_6$ – a ferroelectric semiconductor ($E_g = 2.3$ eV) with good photosensitivity – can serve as a model material for studying the influence of charge carriers on the domain structure and the associated processes. Investigation of the domain structure and its stability in $Sn_2P_2S_6$ is also important from the practical viewpoint, because this material, which has high pyroelectric and piezoelectric parameters,[1] can be used as the active element in pyroelectric and piezoelectric devices.

The aim of this article is to determine the form of the domain structure in $Sn_2P_2S_6$ by visual observation, investigation of the aging process and the influence of illumination on it, and the study of the nature of the internal electrical bias field.

In the experiments we used monocrystalline specimens obtained by the Bridgman–Stockbarger method, and cut in the form of plates and parallelepipeds, the edges of which coincide with the axes of Cartesian coordinates. The X axis is superposed on the [100] direction, the Y axis coincides with the normal [010] to the plane of symmetry, and the Z axis lies near [001]; the monoclinic angle is $\beta = 91.2°$.

The domain structure and the processes of switching were observed by the method of nematic liquid crystals (NLC) with the aid of an optical polarizing microscope with 60-fold magnification.[2] Semitransparent electrodes of SnO were deposited on the planes of the polar cuts by the method of vacuum sputtering.

The formation kinetics of the internal electric field were investigated from the dielectric hysteresis loops and from visual observations of the polarization reversal process. Before the measurements the specimen was annealed for 1 h at 370 K, leading to complete disappearance of the "biographical" internal field. Monodomainization was effected by slow cooling to room temperature by application of a field of about 1.5 kV/cm along the P_s axis.

The thermostimulated currents (TSC) were measured by

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