Diffraction for Fresnel zones and subzones using microwaves

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I. INTRODUCTION

The Huygens–Fresnel approach to diffraction[1] is particularly straightforward for obstacles and/or apertures that are circularly symmetric about the transmitter–detector axis. The spiral "vibration curve"[2] is readily derived and serves as a powerful qualitative method for analyzing diffraction.

Microwaves provide an excellent means of experimentally examining Fresnel theory predictions in lecture hall demonstrations or in student labs. Microwaves allow easy examination of Fresnel diffraction, not only for an integral number of Fresnel zones, but also for *fractions of the first Fresnel zone*. Investigation of diffraction by part of the first zone is not practical for optical wavelengths; but, using microwaves, the surprising, nonintuitive results predicted from the vibration spiral are simple to realize in the lab.[3]

The general experimental setup is shown in Fig. 1. Microwaves of wavelength $\lambda=3.2$ cm are generated by a Gunn diode or klystron tube. The transmitter horn is set directly behind a circular (4 cm diam) hole in a metal sheet. The emerging spherical wavefront travels a distance "d" to the target. The detector horn sits on the axis at distance "s" from the target and is connected to an oscilloscope. The transmitter horn may need to be moved slightly to ensure that the wave emerges from the circular hole at a maximum amplitude. It is convenient to fix $d=s=1$ m. The targets—aluminum foil on 2-cm styrofoam backing and/or Plexiglas (or paraffin)—are mounted on a thin (0.5 cm) sheet of wood veneer. The metal foil completely reflects the portion of the wavefront impinging on it. The dielectric material alters the phase of the traversing wave. To achieve a phase delay of 180° in perspex with refractive index $n=1.6$ requires a thickness

$$t=\tfrac{1}{2}\lambda/(n-1)\simeq 27 \text{ mm}. \quad (1)$$

For paraffin, with refractive index 1.5, a half wavelength delay requires a thickness of 32 mm. Experimental determination of the refractive index of the dielectric may be carried out by the method outlined in Ref. 4 using Young's apparatus for microwaves.

For a spherical wavefront, the vibration spiral of Fig. 2 summarizes the contribution of all the Huygens wavelets to the wave amplitude at the detector, taking into account the phase variation and the gradual damping from the obliquity factor. The contribution of the first Fresnel zone is represented by the vector **AB**, the second zone by **BC**; the third zone by **CD** and so on. With no target the resultant contribution is:

$$\mathbf{AB}+\mathbf{BC}+\mathbf{CD}+\mathbf{DE}+\ldots=\mathbf{AO}. \quad (2)$$

Designate by "a" the amplitude AO which is viewed on the oscilloscope.

The radii of the first few Fresnel zones are well-approximated[5] by

$$R(k)=\sqrt{k\lambda ds/(d+s)}, \quad k=1,2,3,\ldots \quad (3)$$

For our setup, $R(1)=127$ mm; $R(2)=$ [illegible] mm; $R(3)=219$ mm; $R(4)=253$ mm; $R(5)=$ [illegible] mm; $R(6)=313$ mm; $R(7)=337$ mm.

II. INTEGRAL NUMBER OF FRESNEL ZONES

Some typical whole-zone targets are shown in Fig. 3.

Consider the following sequences of experiments and vibration spiral predictions involving an integral number of Fresnel zones. A blocked (≡b) zone is covered with metal foil. An open (≡o) zone is unobstructed. A retarded (≡r) zone is covered with a dielectric that alters the phase of the traversing wave by 180°.

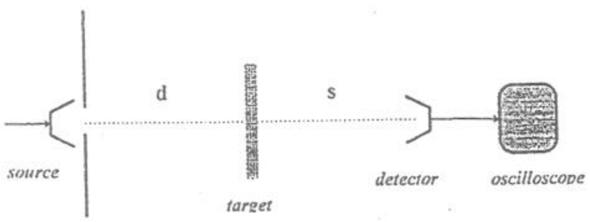

Fig. 1. General experimental setup.

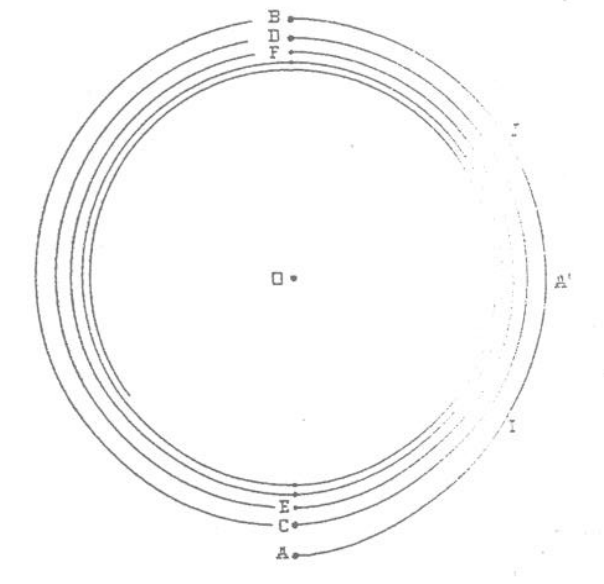

Fig. 2. Vibration curve for a spherical wavefront. The first Fresnel zone is associated with the arc from A to B, the second with the arc from B to C,... . The phase difference between A and I is 60°, between A and A' is 90°; between A and J is 120°; between A and B is 180°.

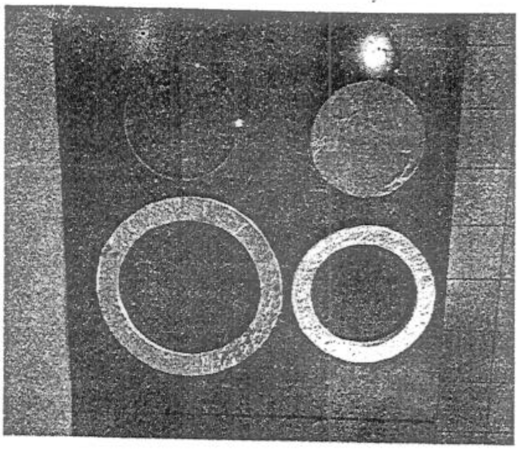

Fig. 3. Some typical whole-zone targets.

←---------------zones ----------------→

1	2	3	4	5	rest	amplitude	
b	o	o	o	o	o	$\|BO\| \simeq a$	(4a)
b	b	o	o	o	o	$\|CO\| \simeq a$	(4b)
b	b	b	o	o	o	$\|DO\| \simeq a$	(4c)
b	b	b	b	o	o	$\|EO\| \simeq a$	(4d)

Here we see Poisson's bright spot at the center of a circular shadow.

1	2	3	4	5	rest	amplitude	
o	b	b	b	b	b	$\|AB\| \simeq 2a$	(5a)
o	o	b	b	b	b	$\|AC\| \ll a$	(5b)
o	o	o	b	b	b	$\|AD\| \simeq 2a$	(5c)
o	o	o	o	b	b	$\|AE\| \ll a$	(5d)

The amplitude oscillates between maximum and minimum as more zones are unobstructed. The first zone, by itself, yields (on the symmetry axis) double the amplitude of the unobstructed wave. Opening two zones, instead of one, *reduces* the amplitude on the axis.

1	2	3	4	5	rest	amplitude	
b	o	b	b	b	b	$\|BC\| \simeq 2a$	(6a)
b	o	b	o	o	o	$\|BC+DO\| \simeq 3a$	(6b)
b	o	b	o	b	b	$\|BC+DE\| \simeq 4a$	(6c)
b	o	b	o	b	o	$\|BC+DE+FO\| \simeq 5a$	(6d)

As more even-numbered zones are opened, leaving lower-lying, odd-numbered zones blocked, the resultant amplitude on the axis grows. A "zone plate" over the first seven Fresnel zones is shown in Fig. 4.

1	2	3	4	5	rest	amplitude	
o	b	b	b	b	b	$\|AB\| \simeq 2a$	(7a)
o	b	o	o	o	o	$\|AB+CO\| \simeq 3a$	(7b)
o	b	o	b	b	b	$\|AB+CD\| \simeq 4a$	(7c)
o	b	o	b	o	o	$\|AB+CD+EO\| \simeq 5a$	(7d)
o	b	o	b	o	b	$\|AB+CD+EF\| \simeq 6a$	(7e)

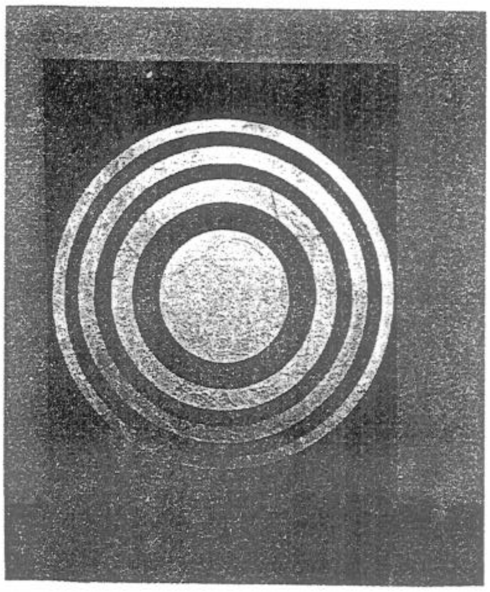

Fig. 4. Fresnel zone plate made from metallic foil over seven zones.

As more odd-numbered zones are opened, leaving lower-lying, even numbered zones blocked, the resultant amplitude on the axis grows.

Adding 180° phase-delay targets allows a wide range of possibilities to be explored and tested against the Fresnel predictions. Some examples are listed below:

1	2	3	4	5	rest	amplitude	
r	b	b	b	b	b	$\|BA\| \simeq 2a$	(8a)
r	r	b	b	b	b	$\|CA\| \ll a$	(8b)
r	r	r	b	b	b	$\|DA\| \simeq 2a$	(8c)
r	o	o	o	o	o	$\|BA+BO\| \simeq 3a$	(8d)
r	r	o	o	o	o	$\|CA+CO\| \simeq a$	(8e)
r	r	r	o	o	o	$\|DA+DO\| \simeq 3a$	(8f)
r	b	r	b	o	o	$\|BA+DC+EO\| \simeq 3a$	(8g)
o	r	b	o	o	o	$\|AB+CB+DO\| \simeq 3a$	(8h)
r	o	b	b	b	b	$\|BA+BC\| \simeq 4a$	(8i)
o	r	b	b	b	b	$\|AB+CB\| \simeq 4a$	(8j)
o	r	o	o	o	o	$\|AB+CB+CO\| \simeq 5a$	(8k)
r	b	r	o	o	o	$\|BA+DC+DO\| \simeq 5a$	(8l)
o	r	o	b	b	b	$\|AB+CB+CD\| \simeq 6a$	(8m)
r	o	r	o	o	o	$\|BA+BC+DC+DO\| \simeq 7a$	(8n)
o	r	o	r	b	b	$\|AB+CB+CD+ED\| \simeq 8a$	(8o)
o	r	o	r	o	b	$\|AB+CB+CD+ED+EF\| \simeq 10a$	(8p)

A "phase zone plate" made from paraffin is shown in Fig. 5.

III. FRACTIONS OF THE FIRST FRESNEL ZONE

Some typical fractional-zone targets (with a whole zone in the foreground) are shown in Fig. 6.

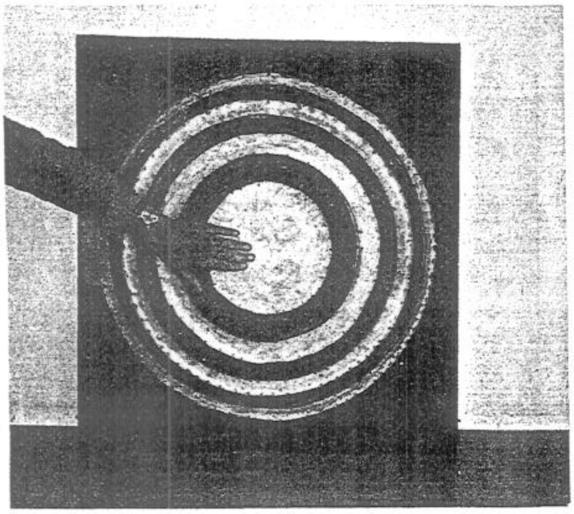

Fig. 5. Phase zone plate made from paraffin.

Block half of the first zone by a circular, foil target. "Half," here, means: up to the subzone corresponding to a 90° phase difference relative to the part of the wavefront traveling straight along the symmetry axis. The radius of the half-zone is

$$R(1/2)\simeq\sqrt{(1/2)\lambda ds/(d+s)}\simeq 89.4 \text{ mm}. \quad (9)$$

Since the wavelets between **A** and **A**′ on the vibration spiral are eliminated, the resultant amplitude is $|\mathbf{A'O}|\simeq a$. Again,

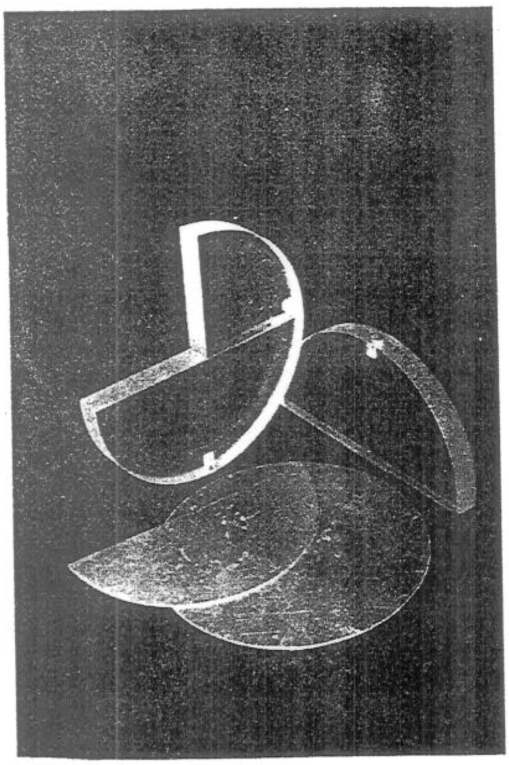

Fig. 6. Some typical fractional-zone targets (with a whole zone in the foreground).

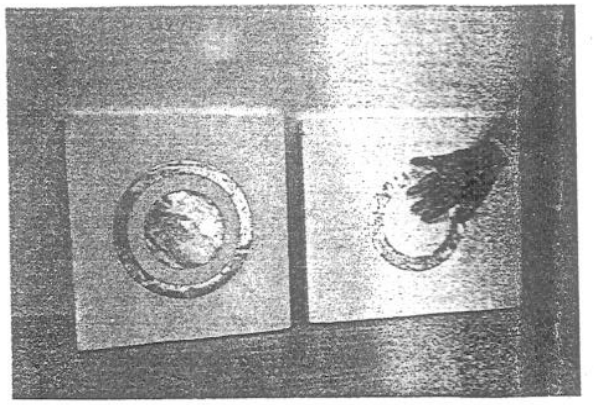

Fig. 7. On the right, only the middle third of the first zone is blocked; on the left, the inner third and outer third of the first zone are blocked.

we see Poisson's bright spot in the center of the shadow. A glance at the vibration spiral for the first zone shows that the amplitude for the Poisson spot is not sensitive to the radius of the foil target.

Now divide the first zone in half along a diameter. Block a semi-circular section, leaving open the other semi-circular section and the remaining zones. To understand how the vibration curve is affected, recall that, when obstructed, the first zone is comprised of a continuum of subzones each with approximately equal amplitudes and with phases varying from 0° to 180°. Blocking a semi-circular half of the first zone reduces by half the amplitude of each of the subzones. Thus instead of the arc **AA′B** with "diameter" $|\mathbf{AB}|\simeq 2a$, the vibration curve will be an arc between **A** and **O** of "diameter" $|\frac{1}{2}\mathbf{AB}|=|\mathbf{AO}|\simeq a$. The resultant amplitude for the system is then $|\frac{1}{2}\mathbf{AB}+\mathbf{BO}|\simeq 0$. A semi-circular blocking of the first Fresnel zone gives a dark spot on the axis!

If one semi-circular half of the first zone is covered by 180° phase-delaying Plexiglas and the other semi-circular half is left unobstructed and the remaining zones are blocked by metal foil, then the amplitude becomes

$$|\tfrac{1}{2}\mathbf{BA}+\tfrac{1}{2}\mathbf{AB}|=0, \quad (10)$$

again, a dark spot.

Next, divide the first zone into three circularly symmetric parts: The first part (the inner circle) corresponds to subzones with phases between 0° and 60° (i.e., between **A** and **I** on the vibration spiral of Fig. 2); the second part (the middle ring) has phases between 60° and 120° (i.e., between **I** and **J** on the spiral of Fig. 2); the third part (the outer ring) has phases between 120° and 180° (i.e., between **J** and **B** in Fig. 2). The outer radii of each part are

$$R(1/3)\simeq 73 \text{ mm}; \; R(2/3)\simeq 103 \text{ mm}; \; R(1)\simeq 127 \text{ mm}. \quad (11)$$

Blocking only the inner third yields the amplitude, $|\mathbf{IO}|\simeq a$. Blocking only the inner and middle thirds yields the amplitude $|\mathbf{JO}|\simeq a$—again, the Poisson bright spot. Now remove the foil target from the inner third so that *only* the middle third is blocked, as shown in the right side of Fig. 7. The amplitude becomes:

$$|\mathbf{AI}+\mathbf{JB}+\mathbf{BO}|=0. \quad (12)$$

Opening the circular hole has *decreased* the amplitude on the axis, extinguishing the Poisson bright spot.

Block only the entire first Fresnel zone. We have seen that the resultant amplitude is $|\mathbf{BO}| \simeq a$. Now open the middle third of the first zone, as shown in the left side of Fig. 7. The resultant amplitude becomes $|\mathbf{IJ}+\mathbf{BO}| \simeq 0$. The Poisson bright spot is also eliminated by opening the middle third of the first zone!

If only the outer third of the first zone is blocked, then the amplitude becomes $|\mathbf{AJ}+\mathbf{BO}| \approx a$. This result can be generalized: a metal ring target whose outer radius is $R(1)$ and whose inner radius lies between $R(1)$ and O, yields a resultant amplitude on the axis of about the same magnitude, "a," as when there is no target.

Next we examine "quarters" of the first Fresnel zone. If the entire first zone is covered by 180° phase-delaying perspex, the amplitude is $\sim 3a$ [see (8d)]. Now remove a quarter section, leaving a three-quarters section Plexiglas target. The contribution of the open quarter cancels that of a phase-delayed quarter leaving the contribution of the phase-delayed semi-circle half of the first zone as well as the remaining open zones.

$$|\tfrac{1}{4}\mathbf{AB}+\tfrac{3}{4}\mathbf{BA}+\mathbf{BO}| \simeq 2a. \tag{13}$$

Opening a quarter has reduced the amplitude from $3a$ to $2a$. Remove an additional quarter leaving only half the first zone covered by Plexiglas. The amplitude becomes

$$|\tfrac{1}{2}\mathbf{AB}+\tfrac{1}{2}\mathbf{BA}+\mathbf{BO}| \simeq a. \tag{14}$$

Opening two quarters has reduced the amplitude from $3a$ to a.

What happens if yet another quarter is removed leaving as the target only a quarter section of 180° phase-delaying perspex? *A dark spot results on the axis*! The Fresnel analysis, of course, predicts this surprising, nonintuitive result. The vibration curve of Fig. 2 gives

$$|\tfrac{1}{4}\mathbf{BA}+\tfrac{3}{4}\mathbf{AB}+\mathbf{BO}| \simeq 0. \tag{15}$$

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[1]See, for example, E. Hecht, *Optics* (Addison-Wesley, Reading, MA, 1990), Chap. 10.

[2]Reference 1, Sec. 10.3.2.

[3]B. S Perkalskis, *Wave Phenomena and Physical Demonstrations* (Tomsk U.P., Tomsk, 1984), p. 280, in Russian.

[4]B. S. Perkalskis and J. R. Freeman, "Demonstrating Crystal Optics Using Microwaves on Wood Targets," Am. J. Phys. **63**, 762–764 (1995).

[5]The approximation is good for $\lambda/2 \ll d$ or s.

On demonstrating transverse waves on a string

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Transverse waves are of great importance in physics, and a classroom demonstration is therefore extremely useful. However, the simple demonstration where the instructor moves one end of a horizontally stretched string up and down seldom works well. The frequency must be low, both for the waves to be clearly visible, and because the instructor's arm can only be made to oscillate slowly. If the frequency is low, however, the wavelengths produced tend to be too long for the demonstration to be effective. The reason for this is a simple constraint imposed by the presence of gravity.

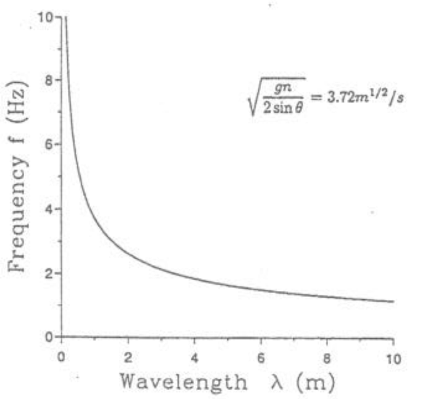

Fig. 1. Frequency as function of wavelength.

The velocity of transverse waves on a string with tension T and linear mass density ρ is $v = \sqrt{T/\rho}$. The mass density $\rho = M/L$, where M is the mass and L is the length of the string. Consider now a stretched string at rest in the presence of gravity, with its two ends held at the same height above ground. If at its ends the string makes an angle θ with the horizontal, $T = Mg/2 \sin \theta$, where g is the acceleration of gravity. Thus $v = \sqrt{gL/2 \sin \theta}$, independent of the mass of the string. To see n wavelengths on the string, one must have $L = n\lambda$, where λ is the wavelength, and since $v = f\lambda$, where f is the frequency,

$$f\sqrt{\lambda} = \sqrt{\frac{ng}{2 \sin \theta}}, \tag{1}$$

Eq. (1) gives the fundamental constraint. If the frequency is