PO Box 12037, 3950 Overdale Rd., Winston-Salem, NC 27117, (910) 784-4800.

[14] The cement used is a silica-based fireplace and furnace cement available from hardware stores.

[15] Valves used on natural gas lines have been found to work quite well with the vacuum system and can hold vacuum at pressures of 1 mTorr.

[16] When placing the heat pipe oven in storage, it is best to fill the oven with an inert gas at a pressure slightly above atmospheric pressure.

[17] S. Hansen, "Gauges 2—the thermocouple gauge," The Bell Jar 1 (4), 9–11 (1992). The Bell Jar is a source of information on low cost vacuum systems. The Bell Jar, 35 Windsor Dr., Amherst, NH 03031.

[18] A pressure transducer which used in conjunction with an amplifier and a meter provides a very adequate pressure gauge. The pressure transducer (part no. PX184-015V5V) is available from Omega Engineering Inc., PO Box 4047, Stamford, CT 06907, (800) 826-6342.

[19] An. N. Nesmeyanov, *Vapor Pressure of the Elements* (Academic, New York, 1963).

[20] An automotive quartz halogen lamp is typically used as a white-light source.

[21] J. Heinze, P. Kowalczyk, and F. Engelke, "Doppler-free polarization spectroscopy of the $B\ {}^1\Pi_u - X\ {}^1\Sigma_g^+$ band system," J. Chem. Phys. **89**, 45–53 (1987).

[22] A. J. Ross, P. Crozet, J. d'Incan, and C. Effantin, "The ground state, $X\ {}^1\Sigma_g^+$, of the potassium dimer," J. Phys. B: At. Mol. Phys. **19**, L145–L148 (1986).

[23] G. Herzberg, *Molecular Spectra and Molecular Structure*, Spectra of Diatomic Molecules (van Nostrand Reinhold Company, NY, 1950), 2nd ed.

[24] M. Terrell and M. F. Masters, "Laser spectroscopy of the cesium dimer as a physics laboratory experiment," Am. J. Phys. **64**, 1116–1120 (1996).

Extending Elihu Thomson's demonstration and Lenz's law

Benjamin S. Perkalskis and J. Reuben Freeman[a]
Jerusalem College of Technology-Machon Lev,-Jerusalem, Israel

(Received 7 October 1996; accepted 8 April 1997)

In 1889, Elihu Thomson (1853–1937), the founder of a precursor to the General Electric Company, introduced a dramatic demonstration[1] of the force on an induced current. An aluminum ring is placed on the end of an electromagnet—a solenoid around an iron core. When the electromagnet coil is connected to an ac generator, the ring flies into the air. The varying current in the coil has induced an opposing current in the ring. The repulsion between the opposing currents pushes the ring away.

We extend Elihu Thomson's experiment so that the secondary (i.e., of the induced current) circuit can be mostly capacitative or resistive and not only inductive as in the original experiment. The usual formulation of Lenz's law for induced currents is problematic in inductive or capacitative circuits.

The experimental setup is sketched in Fig. 1. A horizontal primary coil (inductance L', resistance R') may be connected to an ac generator [emf $\epsilon'(t)$, angular frequency ω]. A bundle of iron wires comprise the core. A secondary coil (inductance $L \ll L'$, resistance $R \ll R'$) is suspended about the iron core of the primary coil by strings. The connection of the capacitor to the secondary coil must be by lightweight wires that do not prevent the coil from swinging along the iron core.

The actual coils that we used are shown in Fig. 2. Our primary coil consists of two solenoids [inner diameter (i.d.) of 9 cm] connected in parallel, each solenoid being 3400 loops of 0.65-mm-diam copper wire and having a resistance of 62.5 Ω. The diameter of the iron core is 7 cm. Our secondary coil (i.d. of 8.5 cm) consists of 40 loops of 1.76-mm-diam copper wire having an overall resistance of about 0.06 Ω. The power mains provided 220 V at a frequency of 50 Hz. Our primary coil inductance is $L' = 0.87$ H and our secondary coil inductance is $L = 0.65$ mH.

With the capacitor out of the secondary circuit—switch 1 closed—the secondary circuit is of the LR type with the impedance dominated by the inductive reactance (ωL). The resistance R is small compared to (ωL). When the ac current flows in the primary coil—a second or so is enough—the secondary coil *swings away* from the primary coil.

With a capacitor in the secondary circuit—switches 2 and 3 closed, switch 1 open—the secondary circuit is of the LCR type. First, insert the values of ω and L into the resonance condition $\omega L = 1/\omega C$ to determine the initial choice of capacitance. At resonance, the secondary coil *just hangs* without swinging when the ac current is introduced into the primary coil.

Now decrease the capacitance so that the capacitative reactance $(-1/\omega C)$ dominates the inductive reactance (ωL) but without too great a difference between them (so as not to over-reduce the induced current). With the ac current in the primary coil, the secondary coil *swings toward* the primary coil. For our apparatus, the greatest effect was achieved at a capacitance of 7200 μF.

Let's analyze what is happening in these experiments. The instantaneous force between the primary and secondary coils is proportional to the product of the currents. If the product is positive [negative], the force is attractive [repulsive]. Designate the nontransient current in the primary [secondary] circuit by

$$i'(t) \equiv i_0' \sin(\omega t), \quad [i(t) \equiv i_0 \sin(\omega t + \delta)], \tag{1}$$

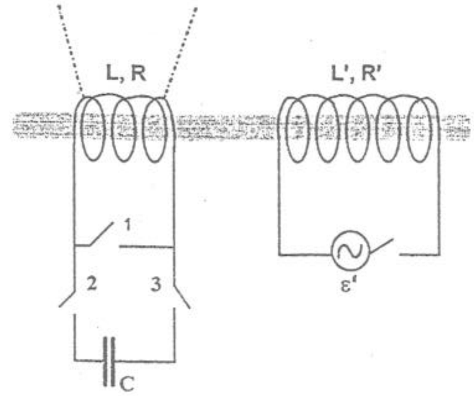


Fig. 1. A horizontal primary coil is connected to an ac generator. The secondary coil is suspended around the core of the primary coil. A capacitor may be kept in or out of the secondary circuit.

where i_0', i_0 are the current amplitudes,[2] t is the time, and δ is the relative phase.

The average force ⟨F⟩ is then proportional to

$$\langle F\rangle \propto \frac{\omega}{2\pi}\int_0^{2\pi/\omega} i'(t)i(t)dt = \frac{1}{2}\, i_0' i_0 \cos\,\delta. \tag{2}$$

The phase δ determines whether the average force is repulsive, attractive, or zero.

From Faraday's law, we see that the emf, $\epsilon(t)$, induced in the secondary circuit is proportional to the negative of the rate of change of the primary current:

$$\epsilon(t) \propto -di'(t)/dt = \omega i_0' \sin(\omega t + 3\pi/2). \tag{3}$$

Thus the phase of the induced emf, $\epsilon(t)$, exceeds that of the primary current, $i'(t)$, by 270°. But the relative phase between the induced emf $\epsilon(t)$, and the induced current, $i(t)$, depends on the type of circuit.

If the secondary circuit is purely inductive, then $i(t)$ lags behind $\epsilon(t)$ by 90° so that the phase of $i(t)$ exceeds that of $i'(t)$ by

$$\delta_L = 270° - 90° = 180°. \tag{4}$$

By Eq. (2), the average force is repulsive.

If the capacitative reactance dominates the impedance in the secondary circuit, then $i(t)$ leads $\epsilon(t)$ by 90° so that the phase of the secondary current, $i(t)$, exceeds that of the primary current, $i'(t)$, by

$$\delta_C = 270° + 90° = 360°. \tag{5}$$

The average force is attractive.

If the secondary circuit is at resonance, the impedance is purely resistive with $\epsilon(t)$ and $i(t)$ in phase. Thus the phase of $i(t)$ leads that of $i'(t)$ by

$$\delta_R = 270° + 0° = 270° \tag{6}$$

and the average force between the coils vanishes.

It is very instructive to examine these experiments with regard to Lenz's rule for the direction of the induced current. A typical formulation of this law is: The induced current will appear in such a direction that it opposes the change of magnetic flux that produced it.[3] This may lead one to expect for the experiments described here that when the primary current $i'(t)$ is growing [falling] then the secondary current $i(t)$ should flow in the negative [positive] direction. The simple examples of induction where suddenly turning on or off the emf in the primary coil induces a current in the secondary coil reinforce this notion. But it is *not* always applicable for cases where the secondary circuit can store energy.

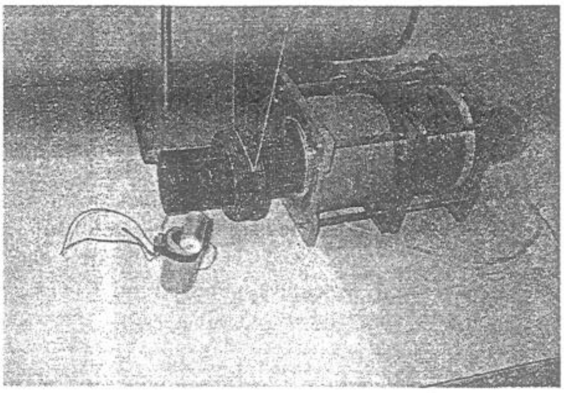
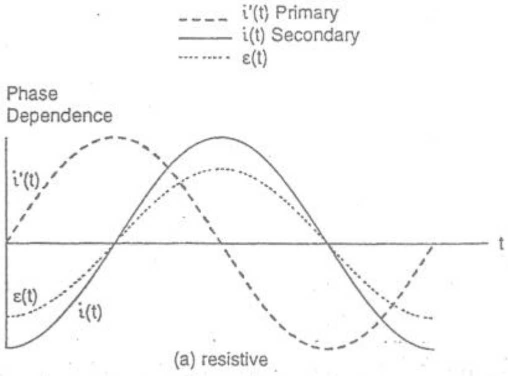
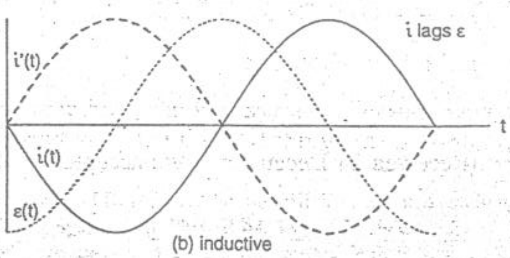


Fig. 2. Photograph of primary coil and secondary coil connected to a 7200-μF capacitor.

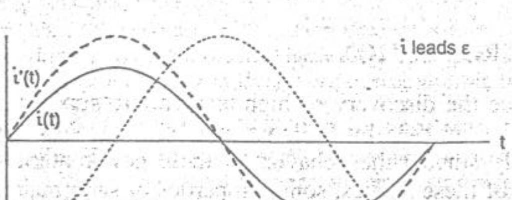
Consider the phase dependence of $i'(t)$, $i(t)$, and $\epsilon(t)$ over one cycle in each of the cases: purely resistive [Fig. 3(a)], purely inductive [Fig. 3(b)], and purely capacitative



(a) resistive



(b) inductive



(c) capacitative

Fig. 3. Phase dependence over one cycle for the primary current $i'(t)$, induced current $i(t)$, and induced emf $\epsilon(t)$ in the case when the secondary circuit is (a) resistive, (b) inductive, (c) capacitative.