Using antilenses to demonstrate tautochronism for a microwave lens

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(Received 5 August 1996; accepted 27 January 1997)

Tautochronism[1] [from the Greek: tauto≡same; chronos ≡time] is the requirement that every point on an electromagnetic wave front [a surface sharing a common phase] takes the same time to reach the corresponding point on a subsequent wave front. Tautochronism follows naturally from Huygens' principle.[2,3] For a converging lens, tautochronism means that the time (or number of cycles) between emission of a wave at a point S and its arrival at the geometric-optics image point S' is the same along all rays even though the distance traversed and the time spent in the lens varies from ray to ray. This implies that the object signal from S is distributed over the entire wave front impinging on the lens and the image amplitude at S' is built up from all over the wave front emerging from the lens.

In the lab, tautochronism may be shown by changing the phase of part of a wave front so as to create total destructive interference in the formation of the image. To implement this is not easy for optical wavelengths since it involves altering optical paths to a precision better than a wavelength. To get around this tolerance problem, we use centimeter microwaves and a paraffin lens which can be shaped to a precision of 1–2 mm.

The paraffin lens, whose refractive index "n" is 1.5, is made by pouring melted paraffin into a mold. The lens is mounted on a backing of thin Plexiglas™ or particle-board. Figure 1 shows a planar-convex lens of diameter $D=30$ cm and axial thickness 5 cm made using a paraboloid mirror antenna as the mold. The lens and backing have been painted black.

Microwaves of wavelength $\lambda=32$ mm are generated by a Gunn diode.[4] A diverging spherical wave front is produced by setting the transmitter horn directly behind a circular hole (of radius 2 cm) in a metal sheet. The microwave source is set to the left of the lens on the optical axis at an object distance [center of transmitter to center of lens] "s" of 100 cm. This makes the angle small between the marginal rays at the lens' edge and the optical axis,

$$\sin^{-1}(D/2s)\approx 0.15 \text{ rad}=8.6^\circ. \tag{1}$$

In this case, the impinging wave front almost matches the lens' plane face (radius of curvature $R_1=\infty$) and the lens' paraboloid face can be approximated as a spherical surface whose radius of curvature is $R_2=-25$ cm.

The lensmakers' formula for the focal length "f" of the lens,

$$1/f=[n-1][1/R_1-1/R_2], \tag{2}$$

gives $f\approx 50$ cm.

The image distance [center of lens to center of receiver horn] "s'" is fixed by

$$1/f=1/s+1/s', \tag{3}$$

so that $s'\approx 100$ cm to the right of the lens. It is possible to design an "ideal" lens shape for which the geometric-optics image of a point object is exactly a point with no aberrations; but the lens described here is "good enough" for our purposes.

The experimental setup is sketched in Fig. 2. An oscilloscope is connected to the microwave detector and displays the amplitude of the image as a dc voltage.

First one can check that removing the lens greatly reduces the detected voltage thus showing that the lens does serve to focus the impinging wave. Next, shift by 180° the phase of half the wave front entering the lens by covering the central half of the flat lens face with a Plexiglas disc [refractive index $n'=1.6$] of radius $D/2^{3/2}=10.6$ cm and thickness $\lambda/2[n'-1]=26.7$ mm. This disc thickness introduces a half-wavelength optical path difference between rays on the wave front traversing the disc and rays not going through the disc. The disc may also be made from paraffin, in which case the appropriate thickness is 32 mm.

The image at S' is made up by a superposition of two equal groups of amplitudes entirely out of phase with each other. Hence the resultant voltage at S' is zero. The disc serves as an "antilens" eliminating the geometric-optics image that would otherwise result from the lens.

Now we divide the wave front hitting the flat lens face into three co-circular regions of equal area. The antilens in this

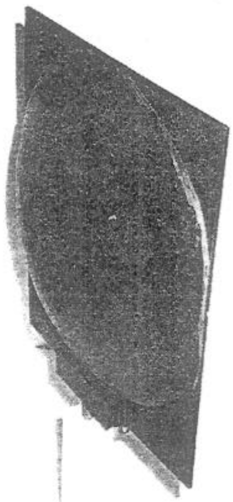

Fig. 1. Planar-convex paraffin lens of diameter 30 cm and axial thickness 5 cm made using a paraboloid mirror antenna mold. The lens and Plexiglas backing are painted black.

case consists of: a Plexiglas disc of radius $D/2(3)^{1/2} = 8.7$ cm and thickness $2\lambda/3[n'-1] = 35.6$ mm covering the central region, a Plexiglas ring between the radii $D/2(3)^{1/2} = 8.7$ cm and $D/6^{1/2} = 12.2$ cm having a thickness $\lambda/3[n'-1] = 17.8$ mm and an open outer ring. See Fig. 3. At S' the amplitude contributions are the same from all three regions but the phase difference between adjacent regions is 120°. The resultant null amplitude at S' follows from the vector sum of the three amplitude vectors which combine to form an equilateral triangle.

Let us generalize the preceding to make a continuous antilens. Divide the wave front into N symmetric co-axial sections each of area $\pi[D/2]^2/N$. The outer radius of each section is given by

$$r_i = (i/N)^{1/2} D/2, \quad i = 1,2,3,\ldots,N. \tag{4}$$

The optical path difference between adjacent sections is λ/N so that the thickness of the "ith" section from the center ($i=1$) is

$$t_i^{\mathrm{CONVEX}} = [1 - i/N]\lambda/[n'-1] \tag{5a}$$

$$= [1 - r_i^2(2/D)^2]\lambda/[n'-1]. \tag{5b}$$

In the large N limit, the variables become continuous: $t_i \rightarrow t$, $r_i \rightarrow r$ so the thickness of the antilens as a function of radius is

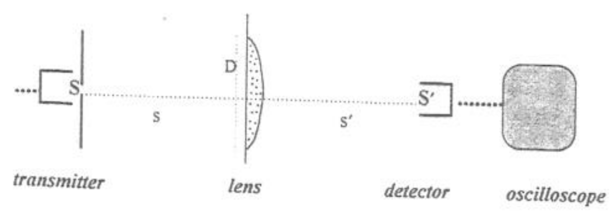

Fig. 2. Experimental setup consisting of microwave source, plano-convex lens, and a receiver connected to an oscilloscope. The receiver horn is placed at the geometric-optics image of the transmitter horn.

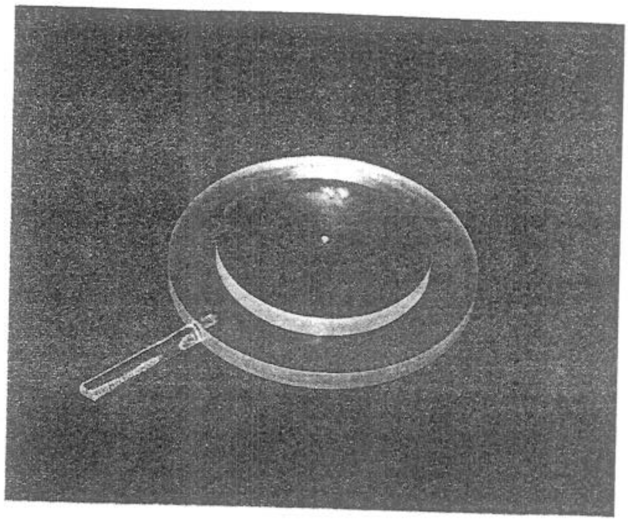

Fig. 3. Antilens made of three co-circular sections (the outer ring is open).

$$t^{\mathrm{CONVEX}} = [1 - 4r^2/D]\lambda/[n'-1]. \tag{6}$$

If the thickest section is taken not at the center but at the outer edge, then instead of Eqs. (5a) and (5b) one finds

$$t^{\mathrm{CONCAVE}} = [i/N]\lambda/[n'-1] \tag{7a}$$

$$= r_i^2[2/D]^2\lambda/[n'-1], \tag{7b}$$

which in the large N limit gives

$$t^{\mathrm{CONCAVE}} = [4r^2/D^2]\lambda/[n'-1]. \tag{8}$$

Both the convex-planar and the concave-planar antilenses can be made from paraffin.

Next, we cover a semi-circular half of the convergent lens by a Plexiglas slab of thickness $\lambda/2[n'-1] = 26.7$ mm. This produces a 180° phase change for half of the wave front traversing the lens. The resultant amplitude at S' is zero. This experiment can be performed even without the lens providing the slab is big enough to cover half the wave front propagating between the transmitter and receiver horn.

Figure 4 shows an antilens made of three sections, each subtending an angle of 120° at the lens center—one open, one introducing an optical path change of $\lambda/3$ and the other an optical path change of $2\lambda/3$. Here again one can carry out

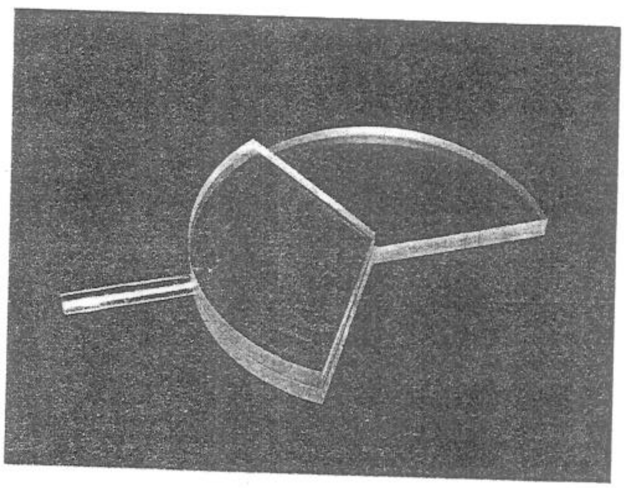

Fig. 4. Antilens made of three sections.

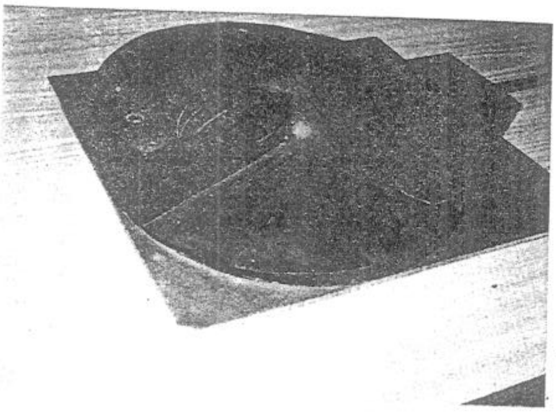

Fig. 5. Continuous "circular-ramp" antilens made from paraffin and painted black.

the experiment without the lens provided sufficiently large dielectric sections are positioned about the transmitter–receiver axis.

This construction can also be generalized to a "circular-ramp" antilens shown in Fig. 5 where the optical path changes continuously from 0 to λ over a counter-clockwise revolution. To construct this paraffin antilens, first make (by a mold) a paraffin cylinder of diameter $D=30$ cm and height 37 mm ($\lambda/2[n-1]=32$ mm plus a base of 5 mm). Now trace a right-angle paper triangle so that opposite the hypotenuse is a base of length $\pi D=94.25$ cm and a side of height 37 mm. Along the triangle base add a rectangle ($\pi D=94.25$ cm by 5 mm) then cut out the triangle plus rectangle combination. Wrap and attach the paper cut-out, hypotenuse edge up, around the paraffin cylinder so the two ends of the πD base meet. Using a sharp knife, *carefully* cut away (from the top, down) the exposed paraffin between the paper cut-out and the axis of the cylinder, making sure that on the remaining top surface each line between a point on the triangle hypotenuse and the axis is along a cylinder radius. This is best accomplished in small "steps" that are then themselves reduced and polished. Slip-ups are corrected with liquid paraffin. Cutting all the way around yields a continuous antilens on a 5-mm base.

ACKNOWLEDGMENTS

We thank E. Elnatan and I. Plaut for assistance in producing antilenses.

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[1]G. S. Landsberg, *Optica* (Nauka/Fizmatgiz, Moscow, 1976), p. 89 [in Russian].

[2]E. Hecht, *Optics* (Addison-Wesley, Reading, MA, 1987), p. 80.

[3]G. S. Gorelik, *Vibrations and Waves* (GITTL, Moscow, 1950), p. 361 [in Russian].

[4]"3-cm band" microwave transmitter/receiver systems specially designed for physics demonstrations are available from some major physics equipment suppliers. A much less expensive solution is to convert a 10–20 mw, 10-GHz microwave communications system for amateur applications (available from electronics suppliers) from frequency modulation to amplitude modulation. We adapted microwave communications components available from Advanced Receiver Research (Ar^2), P.O. Box 1242, Burlington, CT 06013.