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Tensors in the lab—The thermal resistivity of wood

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Understanding tensors can be facilitated by the examination of tensor properties in the lab. In Refs. 1 and 2 we outlined some experiments for upper-division physics majors on electrical and optical properties related to second-order tensors. Here we show how to examine tensor behavior in a thermal physics context, heat flow in a plate. The observed isotherm boundary is compared to the requirements of the second-order thermal resistivity tensor.

We use a much-simplified version of Senarmont's method[3] to study heat flow from a ''point'' in a plane. A uniform (~0.5-mm) layer of paraffin is deposited on a plate (at least 5 mm thickness) with a small conical or cylindrical hole (diameter less than 1 cm). Heat is transferred to the plate from a metal cone or cylinder wedged into the hole and whose temperature (at least 100 °C) is maintained by a heat source. Paraffin melts and becomes transparent between 50 and 60 °C. Thus, depending on the temperature gradient that develops in the plate, a transparent region S grows about the hole. The boundary of S is an isotherm—circular, if the heat flow is isotropic and noncircular if the heat flow is anisotropic.

A soldering iron inserted snugly into the hole is a convenient heat source. The iron is kept in the hole until the expanding transparent region S has grown to centimeter dimensions. This takes less than a minute. Convection drives some of the liquid paraffin outward and there is also some leakage into the hole. As a result, the melted region and its boundary remain clearly distinguishable even after the heat source is removed.

Start with an isotropic material—a fiberglass or aluminum plate—for which circular isotherms are naturally expected. Next use straight-layer wood cut with the fiber direction perpendicular to the plate face. Here the heat flow in the plate face is also circularly symmetric.

To examine a thermally anisotropic system, it is convenient to use straight-layer wood with the fiber direction cut parallel to the plate face. The heat source should be in an area where the fibers are parallel and more-or-less evenly spaced. Heat flows much faster along the fiber direction than perpendicular to it. The characteristic elliptical isotherm boundary is shown in Fig. 1 for a 2-cm pine wood plate with a conical hole. Thinner (0.5- to 1-cm) plates with cylindrical holes are adequate. But, if the soldering iron is not kept vertical relative to the plate face or the iron-wood contact is not uniform, there may result some distortion from an elliptical boundary centered about the heat source.

To analyze the isotherm in tensor terms, consider first the general heat flow equations.[4] The rate of heat flow (energy per unit time per unit area perpendicular to the flow) $\mathbf{h}$ and the temperature T are related to the thermal resistivity tensor ρ_{ij} by

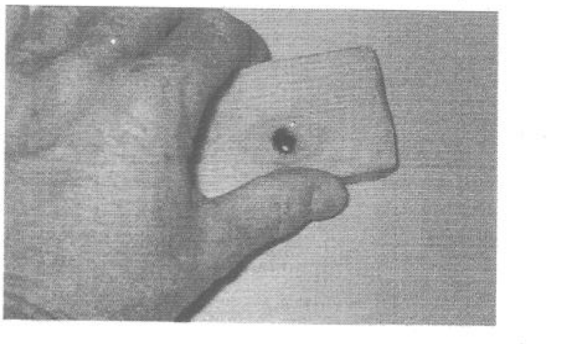

Fig. 1. Elliptical boundary of melted paraffin on a pine wood plate.

Table I. Radius (to the nearest half-mm) of the isotherm boundary at various angles with respect to the fiber direction. The measured values for $r^2(0°)/r^2(\theta)$ are shown along with the values calculated from Eq. (11).

θ	$\cos^2\theta$	$r(\theta)$ (mm)	Measured $r^2(0°)/r^2(\theta)$	Calculated $\cos^2\theta+\sin^2\theta r^2(0°)/r^2(90°)$
0°, 180°	1	14.0	1	1
30°, 210°	0.75	11.5	1.5	1.5
45°, 225°	0.50	10.0	2	2
60°, 240°	0.25	9.0	2.4	2.5
90°, 270°	0	8.0	3.1	input 3.1
120°, 300°	0.25	9.0	2.4	2.5
135°, 315°	0.50	10.0	2	2
150°, 330°	0.75	11.5	1.5	1.5

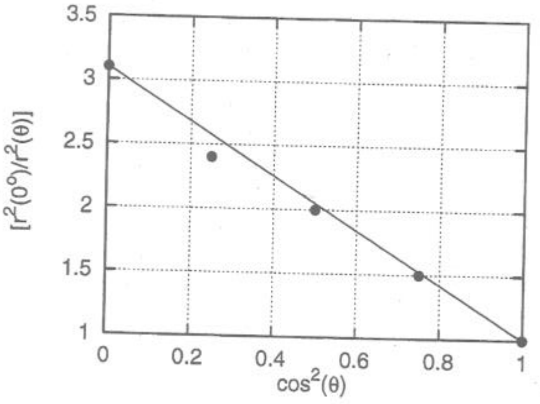

Fig. 2. The fiber-angle dependence of the isotherm boundary—$r^2(0°)/r^2(\theta)$ vs $\cos^2\theta$ according to Eq. (11) with $r^2(0°)/r^2(90°)=3.1$. Measured values are shown as dots.

$$-(\partial T/\partial r_i)=\sum_j \rho_{ij}h_j, \quad i,j=1,2,3, \tag{1}$$

where r_1, r_2, r_3 are the components of the position vector $\mathbf{r}$ along the orthogonal unit vectors **1**, **2**, **3** and the heat flow rate has components

$$\mathbf{h}=h_1\mathbf{1}+h_2\mathbf{2}+h_3\mathbf{3}. \tag{2}$$

The resistivity units are [m s °C/J].

When referred to its principal axes, the symmetrical, second-rank tensor ρ_{ij} becomes diagonal:

$$\rho_{ij}=\rho_i\delta_{ij}, \quad i,j=1,2,3, \tag{3}$$

where the ρ_i are the principal thermal resistivities.

For heat flow from a point source, the isothermal surfaces are similar in shape and orientation to the thermal resistivity ellipsoid:

$$\rho_1r_1^2+\rho_2r_2^2+\rho_3r_3^2=1. \tag{4}$$

In the case of heat flow from a point through a wood plate (in the **2**–**3** plane) whose fiber direction is parallel to the **3** axis, the isotherms are given by the resistivity ellipse

$$\rho_2r_2^2+\rho_3r_3^2=1. \tag{5}$$

The thermal resistivity $\rho(\theta)$ in a direction making an angle θ with respect to the wood fiber direction is fixed by the second-order tensor behavior under uniaxial symmetry:[1,5]

$$\rho(\theta)=\rho(0°)\cos^2\theta+\rho(90°)\sin^2\theta, \tag{6}$$

where

$$\rho(0°)=\rho_3, \quad \rho(90°)=\rho_1=\rho_2. \tag{7}$$

Introducing the polar coordinates

$$r_3=r\cos\theta, \quad r_2=r\sin\theta, \tag{8}$$

the isotherm equation (5) becomes

$$r^2[\rho_2\sin^2\theta+\rho_3\cos^2\theta]=1, \tag{9}$$

so that

$$r^2=1/\rho(\theta). \tag{10}$$

Hence, Eq. (6) becomes

$$r^2(0°)/r^2(\theta)=(1-r^2(0°)/r^2(90°))\cos^2\theta + r^2(0°)/r^2(90°). \tag{11}$$

The isotherm radii along the principal axes $r(0°)$, $r(90°)$ determine the radius $r(\theta)$ at any other angle θ, because the shape of the isotherm ellipse is determined by the semi-major and semi-minor radii.

Table I lists the measured isotherm radii for a series of angles together with the measured and calculated values of $r^2(0°)/r^2(\theta)$. Figure 2 shows the observed values of $r^2(0°)/r^2(\theta)$ as a function of $\cos^2\theta$ and the straight-line prediction of Eq. (11) with $r^2(0°)/r^2(90°)=3.1$.

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Three-dimensional display of light interference patterns

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I. INTRODUCTION

One of the challenging exercises in introductory physics courses is the concept of constructive and destructive interference of light rays and thus production of bright and dark fringes. These concepts are usually demonstrated in the laboratory environment by laser beam diffraction or a Michelson[1] interferometer. The results of these experiments are two-dimensional light intensity distributions in the plane transverse to the direction of propagation of the beams. Outside the laboratory the printed images of the fringes are usually accompanied by intensity distribution curves which are not sufficiently intuitive to most students.[2]

In this article we introduce a simple method to display