STABILITY OF SYSTEMS OF PLANE REFLECTING SURFACES

Naftali Schweitzer, Yaakov Friedman and Michael Skop

Department of Electro-Optics and Applied Mathematics

POB 16031, Jerusalem, Israel

Tel. 972-2-6751184

e-mail: friedman@math.jct.ac.il

A general rule for the stability of plane reflecting surface systems is derived using the features of the reflection matrix. It is proven that only two directions can be stable: the forward direction and the backward direction (retro-reflection). Examples for the application of this rule in the design of stable reflecting systems for optical communication are given.

Many indoor and outdoor optical systems incorporate reflecting devices for folding the optical axis, for scanning, or for aligning the transmitted beam in optical communication systems toward the receiver. It is essential for such reflecting devices to be optically stable in order to ensure that the entering beam of light exits from the system in the desired direction. The definition of an optically stable reflector is that for a given direction of an incoming beam of light, the reflecting device reflects the light in a fixed direction independent of reflector rotation (as long as the beam can enter the reflector aperture). Such reflectors are sometimes called constant-deviation systems [1].

The most familiar optically stable devices are corner cube reflectors, which reflect light at an angle of 180°. These retroreflectors are comprised of three mutually perpendicular reflecting surfaces - either plane mirrors or the surfaces of a tetrahedron cut from the corner of a glass cube [2,3], and they have been recognized for over half a century [4]. Several authors have studied the optical stability of corner cubes with dihedral angles a little different from 90° [5-7]. Skop et al. [8] found out that a combination of a roof prism and a right angle prism is nearly stable for reflection angles near 180°. Beggs [9] used the reflection matrix to study the stability of a system of mirrors under displacement.

Several questions are left open. Is it possible to obtain optically stable systems for angles not equal to 180°? If the answer is positive, what are the configurations of such systems? For example, is it possible to design an optically stable device for reflecting a laser beam at an angle of 40° in order to integrate it into an optical communication system, and what is the desired configuration? Are there correct and incorrect ways to construct a system of plane mirrors for folding the optical axis of a light ray? It generally seems that textbooks and publications have not dealt with these questions.

We will use the features of the reflection matrix in order to investigate the possible stable directions of reflection and to find the configurations of optically stable reflecting systems. First we choose a coordinate system in space. The direction of a ray is now expressed by a unit vector $\boldsymbol{r}$. The reflection matrix of a plane surface transforms the direction of an incoming ray into the direction of the outgoing ray. For a system of mirrors, an incoming ray undergoes several reflections. Thus, we may introduce a reflection matrix A of the system which is the product of the reflection matrices of each mirror from which this incoming ray was reflected in the same order as the ray was reflected. We will call here a ray "an incoming ray" if it undergoes the same sequence of reflections. The matrix will transform the direction $\boldsymbol{r}$ of an incoming ray into the direction $A\boldsymbol{r}$ of the exit ray.

Since the reflection matrix A is a 3x3 orthogonal matrix with real coefficients, at least one of the eigenvalues of this matrix is real and equal to 1 for an even number of reflections or -1 for an odd number of reflections [10]. This means that there is a direction $\boldsymbol{r}_0$ for which $A\boldsymbol{r}_0=\boldsymbol{r}_0$ so the exit ray has the same direction as the incoming one in the above first case, or $A\boldsymbol{r}_0=-\boldsymbol{r}_0$ and the exit ray will have the opposite direction in the above second case. Note that $\boldsymbol{r}_0$ may be or may not be a direction of an incoming ray.

Consider first the case $A\boldsymbol{r}_0=-\boldsymbol{r}_0$. In this case the following holds:

Proposition For any system of plane mirrors with an odd number of reflections there is always a plane P (containing the origin) such that the sum $\boldsymbol{r}+A\boldsymbol{r}$ of the direction $\boldsymbol{r}$ of any *incoming ray* and the direction $A\boldsymbol{r}$ of the corresponding *exit ray* is always parallel to P.

Proof

As mentioned above, for a given system there is a direction r_0 for which $Ar_0=-r_0$. Let $\boldsymbol{r}$ be an arbitrary incoming direction. From the orthogonality of A it follows that:

$$(r+Ar)\cdot r_0 = r\cdot r_0+(Ar)\cdot r_0=r\cdot r_0-(Ar)\cdot(Ar_0)=r\cdot r_0-r\cdot r_0=0,$$

where the dot sign denotes the scalar product. Thus, for any $\boldsymbol{r}$ the vector $r+Ar$ must be perpendicular to r_0. If P is the plane perpendicular to r_0, then $r+Ar$ is parallel to P. Q.E.D.

Note that even though the plane P of the Proposition was constructed with the help of the matrix A which is dependent on the choice of the coordinate system, this plane depends only on the optical system and is coordinate free. Moreover, this plane could always be determined without knowing the internal structure of an optical system (black box). To determine P, send a ray into the optical system and check the exit direction. Add the direction of the incoming ray to the direction of the outgoing ray. The direction of this sum is the bisector between the two directions and is coordinate free. This gives you one direction parallel to the plane. To obtain the second direction parallel to the plane repeat the process with another incoming ray that is close but not parallel to the first one. Two independent directions determine the plane P uniquely up till translations.

Let r and P be as above in the Proposition. If $r+Ar$ is a non-zero vector, then there is a rotation R such that after performing this rotation on the optical system, the direction r will still be an incoming direction and the vector $r+Ar$ will not be parallel to the rotated plane RP. For an unstable system the exit direction is changed in such a way that its sum with the incoming direction is parallel to RP. But if the system is stable, the direction Ar is still the exit direction for r and by the Proposition the vector $r+Ar$ *must* be parallel to RP. This contradiction shows that the assumption that $r+Ar$ is not zero was wrong and implies that $Ar=-r$. Thus, a stable system with an odd number of reflections is retroreflecting any incoming direction.

For a system with an even number of reflections, $Ar_0=r_0$, the proof is the same but $r+Ar$ has to be replaced by $r-Ar$. In this case the stable system will preserve all the incoming directions. Hence a system cannot be stable under rotation and simultaneously have incoming and exiting rays non-parallel.

Note that the Proposition can also be used to check whether the number of reflections in an optical system is odd or even. To do this determine first two planes P_1 and P_2 as described above, which are parallel to the sum or to the difference of the incoming and exit rays correspondingly. Now send a new incoming ray and check whether the sum of the incoming and the exit directions is parallel to P_1. If yes, the number of reflections is odd. If not, check if the difference of the incoming and the exit directions is parallel to P_2. If yes, the number of reflections is even. If not, it means that the ray sent does not hit the mirrors in the required order and is not an incoming ray according to our definition.

The above results show that there exist only two types of optically stable plane mirror systems - direction preserving or retroreflecting. Aside from the theoretical significance, these conclusions hold an important practical impact, as shown in the following. Any combination of plane mirrors which deviate light rays at an angle not equal to 180° or 0° can not be optically stable. Thus, such systems should be avoided if possible in optical communication systems. A laser beam sent from earth to a satellite (or to the moon) carrying retroreflecting devices will be retroreflected toward the transmitter with great precision, even after the retroreflector has suffered small mechanical deformations; but if the beam is to be reflected toward another point on earth, such deformation will destroy the communication system. There is no optical solution here only a mechanical one: the mechanical support of the reflector must be designed very carefully.

In optical devices designed to reflect light near the optimal angles - close to 0° or 180° - partial stability can be obtained [8]. By partial stability we mean that the incoming ray suffers a relatively small angular deviation (much smaller than 2α) when the optical device is rotated by an angle α. Recall that a plane reflecting surface deviates an incident light ray by an angle of 2α when it is rotated by an angle α around an axis which is perpendicular to the plane of incidence. This can be useful especially at short distances and for large aperture receivers.

While retroreflecting devices are widely known and used, direction-preserving reflectors have not been explicitly reported until now. Such reflectors are comprised of two parallel plane reflecting surfaces and are stable for an even number of reflections. In analog to the retroreflectors which can be embodied by three perpendicular mirrors or by a glass corner cube, the direction preserving reflectors can be embodied either by two parallel plane mirrors or by a pair of parallel surfaces of a rectangular glass prism.

The parallel-plane-mirror based periscope is commonly used and preferred over the perpendicular-plane mirror based periscope, because it produces an erect image, but no one seems to emphasize the optical stability feature of this configuration [1]. The following experiment can be easily carried out. Let a laser beam enter the aperture of a simple parallel-mirror periscope. Now rotate the whole system around any axis, looking at the projection of the exit beam on a distant screen. The point of light on the screen will remain fixed (within experimental errors)! This means that in order to construct an optically stable periscope for viewing, the two parallel mirrors must be integrated into the same mechanical system. In the same way, when two parallel mirrors are used for folding the optical axis of a transmitted collimated light beam, the two mirrors must be integrated into the same mechanical system in order to achieve optical stability.

Lets discuss a frequent example. Fig. 1 is a scheme of an optical bypass

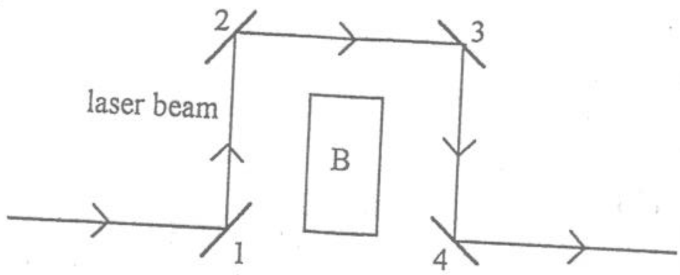

Fig. 1. An optical bypass. B - An obstacle. 1, 2, 3, 4 - plane mirrors.

The optical bypass is comprised of four plane mirrors 1, 2, 3, 4. B is an obstacle (such as a wall) in the path of a laser light beam used for optical communication. Suppose we have to design the mechanical supports of the optical system. If each mirror has a separate mechanical support, the system will not be optically stable. Any small rotation in angle α of any mirror will cause a deviation of 2α in the direction of the light beam. Nor can optical stability be achieved by integrating the mutually perpendicular mirrors 2 and 3 into a common mechanical support. We already know that an optically stable reflecting system either retroreflects or preserves the direction of an incoming ray. However, if we construct a common support for the parallel pair of mirrors 1 and 2 and another common support for the other parallel pair of mirrors 3 and 4, the four mirror system will be optically stable. Rotating and tilting the first pair will not change the direction of the beam, as long as it hits each of the two mirrors. The beam will (ideally) always hit mirror 3 at the same point. Similar considerations are true for the other pair of mirrors - 3 and 4. Thus the *whole* mirror system is optically stable for any angle between mirrors 2 and 3 and in spite of the fact that we designed separate optically stable direction preserving systems. This has much significance, especially when the distance between mirrors 2 and 3 is large. We see that by correctly using the rule, other optically stable multiple-mirror reflecting systems can be constructed, even where the components of the optical systems are separate and at a large distance from each other.

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