# **Fuzzy Dynamics of Brain Activity.**

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#### Abstract

In this paper fuzzy dynamics are applied to the description of motivated behavior. We have shown that in an important case of instrumental reflex elaboration the theory gives a good agreement with the experiment and clears up an important biological and behavioral features of this phenomenon.

*Keywords:* fuzzy dynamics, fuzzy differential equations and inclusions, neural system, brain, conditional reflex.

# 1 Introduction

Motivation is a cause for actions leading to achievement of some goal. Main biologic motivations are: need of feeding, drinking, respiration, temperature regulation, sexual motivation, avoidance of danger. There are also artificial motivations as self-stimulation or drug-dependence. (see [1] and bibliography there for details). If an animal knows how to act, its behavior is almost deterministic and is governed by the long term memory patterns. If the animal does not know how to act, it tries to reach the goal by choosing an actions from the set of preferable actions in the given situation. This choice is uncertain enough. It is unlikely that such decision is determined by internal stochastic forces, because in this case the reaction would be looking as an averaging the activity of the individual neurons that has not been observed [2].

The motivated behavior acts as follows. Signals indicating the motivation reach the motivation centers and excite the one (or sometimes a few) of them. The neurons of the center send a reinforcement signal to the "behavior neurons" that generate an action. The action generation take place until the motivation cause disappear. Any motivation is connected with a reward (positive reinforcement) or with a punishment (negative reinforcement). There are experimental evidences [1] that the motivated activity of the behavior neurons is a function of the neuron "damage" caused by the reinforcement signal.

In non-deterministic motivated behavior the number of available actions must be as large as possible in order to find the solution, but the possibility of actions leading to dangerous situations must be minimized. In such a case the use of the *Fuzzy Dynamics* (FS-dynamic for short) [3] (see also [4] for details) is preferable since the *min* t-norm minimizes the maximal error. Also the use of this dynamic description does not require an averaging which, as mentioned above, was not found in the process of motivated action .

#### 2 Experimental results

An important particular case of the motivated behavior under which the animal learns how to avoid punishment is called an instrumental reflex elaboration. It was shown [2] that even a single neuron can produce an instrumental reflex reactions. Without training the neuron activity decreases after multiple repetition of the same stimulus. In the experiment, a single neuron of the snail Helix was trained to preserve the activity in response to multiply repeating conditional stimulus. The training procedure was as follows. The animal was punished (negative reinforcement) if the trained neuron did not generate an action potential in response to the conditional stimulus. So the activity of this neuron become an instrumental reaction. Typical behavior of the training and the surrounding (control) neurons are shown in Fig.1 and Fig.2.

At the last cycles of this procedure the activity of the trained neuron remain high, while the activity of the control (non trained) neurons become low. This show that the instrumental reflex was elaborated.

In these experiments forty sessions of neuron training were performed for each of the trained snail Helixes and several physiological parameters of the snail neurons were measured. It was found that the neuron *activity* and specific characteristic of the inner neuron state (which was called above as *damage*) play, apparently, main role in this situation.

Substantial for our consideration experimental results are shown in Fig.3 and Fig.4. These results could be summarized by the following qualitative claims:

- **a-b** The neuron activity increases if the neuron damage is large and decreases if the neuron damage is small.
- **c-d.** The damage increases with a negative reinforcement or high activity of the surrounding neurons and decreases with a positive reinforcement and a non-violent activity of the surrounding neurons.

We will describe the state of a neuron by the "exponents" of the activity and of the damage :

$$x_1 = \ln \frac{A_t}{A_0} \rightarrow \text{exponent of the activity},$$
  
 $x_2 = \ln \frac{D_t}{D_0} \rightarrow \text{exponent of the damage},$ 

for the training neuron and

$$y_1 = \ln \frac{A_c}{A_0},$$
  
$$y_2 = \ln \frac{D_c}{D_0},$$

for a control neuron, where  $A_t$ ,  $A_c$  and  $D_t$ ,  $D_c$  are current levels and  $A_0$ ,  $D_0$  are some typical levels of activity and damage of the neurons. The velocities of change of  $x_1$  and  $x_2$  will be denoted by  $u_1$  and  $u_2$  ( $v_1, v_2$  for  $y_1, y_2$  correspondingly). The reinforcement value is denoted by r. Then, the linguistic rules for the training neuron dynamics are:

- a)  $u_1$  is Positive for  $x_2$  Positive,
- **b)**  $u_1$  is Negative for  $x_2$  Negative,
- c)  $u_2$  is Positive for r Negative OR  $y_1$  Positive,
- d)  $u_2$  is Negative for r Positive AND  $y_1$  Negative,

while for the control neurons these rules are:

- **a')**  $v_1$  is Positive for  $y_2$  Positive,
- **b')**  $v_1$  is Negative for  $y_2$  Negative,
- c')  $v_2$  is Positive for r Negative OR  $y_1$  Positive,
- d')  $v_2$  is Negative for r Positive AND  $y_1$  Negative,

To these laws we add a natural restriction:

e)  $u_1, u_2$  and  $v_1, v_2$  are Limited.

These rules are logically related as

$$\begin{bmatrix} \mathbf{a} & or \mathbf{b} \end{bmatrix} \quad and \begin{bmatrix} \mathbf{c} & or \mathbf{d} \end{bmatrix} \quad and \mathbf{e} \end{bmatrix} \tag{1}$$

which mean that velocity  $u_1$  satisfy either rule **a**) or **b**) and velocity  $u_2$  satisfy either rule **c**) or **d**) and both of them satisfy **e**).

### 3 Theory

For the quantitative description of the fuzzy information about state of a neuron we introduce a fuzzy state-  $s(\mathbf{x}, t)$  as a grade of our belief or possibility that the system is in the physical state  $\mathbf{x} \in X$  at the time t. We will assume that this function has real values between 0 and 1 and the value  $s(\mathbf{x}, t) = 1$ corresponds to the most possible state, while  $s(\mathbf{x}, t) = 0$  to the less possible state  $\mathbf{x}$  at time t. Note that s is not a measure and in many cases it may not be integrable. For mathematical foundation of fuzzy logic and introduction to the main concepts see [6]. In accordance with FS approach we consider a fuzzy generator of the evolution -  $g(\mathbf{u}; \mathbf{x})$ , which is a possibility that system has the velocity  $\mathbf{u}$  at the state  $\mathbf{x}$ . We have to assume that [5]

$$\sup_{\mathbf{x}} s(\mathbf{x}, t) \le \sup_{\mathbf{u}} g(\mathbf{u}; \mathbf{x})$$
(2)

and (for definiteness) we normalize  $g(\mathbf{u}; \mathbf{x}, t)$  and  $s(\mathbf{x}, t)$  as

$$\sup_{\mathbf{u}} g(\mathbf{u}; \mathbf{x}, t) = \sup_{\mathbf{x}} s(\mathbf{x}, t) = 1.$$
(3)

Then, the dynamics law for fuzzy state can be represented in the form [3], [5]

$$s(\mathbf{x}, t+\delta) \approx \sup_{\mathbf{u}} \min \left\{ g(\mathbf{u}; \mathbf{x}, t); \ s(\mathbf{x} - \mathbf{u}\delta, t) \right\}.$$
(4)

where the time interval  $\delta$  is assumed to be small. From a general point of view such "pseudo-convolution" has been considered in [7].

It can be shown [3],[5] that in the limit  $\delta \to 0$  this equation is equivalent to:

$$\frac{\partial s\left(\mathbf{x},t\right)}{\partial t} + U^{j}(\mathbf{x},\nabla s\left(\mathbf{x},t\right),t,s\left(\mathbf{x},t\right))\nabla_{j}s\left(\mathbf{x},t\right) = 0$$
(5)

where  $\mathbf{U}(\mathbf{x}, \nabla s, s)$  is solution of the optimization problem:

$$(\mathbf{U} \cdot \nabla s) \to \text{is minimal with respect to } \mathbf{U},$$
 (6)

under the restriction

$$g(\mathbf{U}; \mathbf{x}, t) = s. \tag{7}$$

Equation (5) is the fuzzy analog of the Hamilton-Jacobi equation [8], with the Hamiltonian

$$H = U^{j}(\mathbf{x}, \nabla s(\mathbf{x}, t), t, s(\mathbf{x}, t)) \nabla_{j} s(\mathbf{x}, t)$$

Note that U depend only on the direction of  $\nabla s$  rather on  $\nabla s$ . If the minimum in (6) is obtained on a set of elements of  $\mathbf{u}$ , then  $\mathbf{U}$  is equal to the set of vectors, rather to a single one. Thus  $\mathbf{U}(\mathbf{x}, \nabla s, s)$  is a generalized vector field or a set-valued function. For points with s = 1 condition (6) does not make sense since in such point either  $\nabla s = 0$  or is not defined and  $\mathbf{U}$  must be taken equal to all  $\mathbf{u}$  satisfying (7). This can be shown by approximating  $g(\mathbf{u}; \mathbf{x}, t)$  monotonically with a function with unique maximum at any given  $\mathbf{u}$  satisfying (7).

The Hamilton-Jacobi equations with not-smooth  $\mathbf{U}(\mathbf{x}, \nabla s(\mathbf{x}, t), t, s(\mathbf{x}, t))$ , that appear in the control theory, have been studied in [9],[10]. In our case  $\mathbf{U}$  may be a set-valued map, so that a whole interval

$$\mathbf{U}^{(-)}\left(\mathbf{x},t\right) \leq \mathbf{U} \leq \mathbf{U}^{(+)}\left(\mathbf{x},t\right)$$

satisfy to (6)-(7) (for example, for s = 1, see above and in [3]). In this case the equation (5) is a differential inclusion rather then a differential equation. Such equations where used by several authors to describe "nonstochastic" uncertainty. See [11] and referenced literature therein for the definitions and the theorems of existence of solutions for such equations.

In according with the standard theory of partial differential equations of the first order [8], equation (5) can be solved by the method of *characteristics*  $\xi(t)$  and  $\zeta(t)$  of this differential equation:

$$s(\xi(t), t) = s(\mathbf{x}, t = 0) = s_0(\mathbf{x}).$$
 (8)

The characteristics satisfy the system of characteristic differential equations [8]:

$$\frac{d\xi}{dt} = \mathbf{U}(\xi, \zeta, t, s_0), \tag{9}$$

$$\frac{d\zeta}{dt} = -\frac{\partial U^i(\xi, \zeta, t, s_0)}{\partial \xi} \zeta_i - \beta \zeta, \qquad (10)$$

with the initial conditions:

$$\begin{aligned} \xi(0) &= \mathbf{x}, \\ \zeta(0) &= \nabla s_0(\mathbf{x}). \end{aligned}$$
(11)

The quantity  $\mathbf{U}(\xi, \zeta, t, s_0)$  should be found from the equations:

$$g(\mathbf{U};\xi,t) = s_0 \tag{12}$$

and

$$\zeta = \beta \frac{\partial g(\mathbf{U}; \xi, t)}{\partial \mathbf{U}},\tag{13}$$

that is equivalent to the optimization problem (7),(6) ( $\beta$  is a Lagrange multiplier).

Note, that s remains constant on the characteristics  $\xi(t)$ :

$$\frac{ds(\xi(t),t)}{dt} = \frac{\partial s}{\partial t} + (\zeta \cdot \mathbf{U}(\xi,\zeta,t,s)) \equiv 0.$$
(14)

The system (12)-(13) is covariant under the transformation  $\zeta \to f\zeta$  and  $\beta \to f\beta$ , where f is a positive scalar function of t and  $\xi$ . If we put  $\rho = f\zeta$  with

$$\frac{df^{-1}}{dt} = \beta, \quad f(0) = 1 \tag{15}$$

the system (9)-(10) becomes Hamiltonian:

$$\frac{d\xi}{dt} = \frac{\partial H}{\partial \rho},\tag{16}$$

$$\frac{d\rho}{dt} = -\frac{\partial H}{\partial\xi},\tag{17}$$

with the Hamiltonian

$$H(\xi, \rho, s_0) = \left(\rho \cdot \mathbf{U}(\xi, \frac{\rho}{|\rho|}, t, s_0)\right).$$
(18)

and with the initial conditions:

$$\begin{aligned} \xi(0) &= \mathbf{x}, \\ \rho(0) &= \nabla s_0(\mathbf{x}). \end{aligned}$$
(19)

For more information about FS-dynamics see [4]

# 4 Fuzzy dynamics of the reflex elaboration

Introduce a membership grade of the expression "*a is Positive*" as a smooth monotone increasing function Pos(a) such that:

$$0 \le Pos\left(a\right) \le 1,\tag{20}$$

In the mentioned above experiments the reinforcement r has been chosen as an increasing function of the training neuron activity, so we can put:

$$Pos(r) = Pos(x_1).$$
<sup>(21)</sup>

The truth value of the rule  $\mathbf{a}$ ) by use of the *min-max* version of fuzzy logic is

$$\mu_a = \min\left\{ Pos(\alpha u_1), Pos(\gamma x_2) \right\},\,$$

where  $\alpha$  and  $\gamma$  are some constants depending on the fuzziness of the statements. By use of (20) this gives:

$$\mu_a = Pos\left(\min\left\{\alpha u_1, \gamma x_2\right\}\right).$$

Similarly for the other rules we obtain:

$$\begin{split} \mu_b &= Pos\left(\min\left\{-\alpha u_1, -\gamma x_2\right\}\right), \\ \mu_c &= Pos\left(\min\left\{\alpha' u_1, \max\left\{-r, \gamma' y_1\right\}\right\}\right), \\ \mu_d &= Pos\left(\min\left\{\alpha' u_1, \min\left\{r, -\gamma' y_1\right\}\right\}\right). \end{split}$$

By the same way we have for surrounding neurons:

$$\begin{aligned} \mu_{a'} &= Pos\left(\min\left\{\alpha v_{1}, \gamma y_{2}\right\}\right), \\ \mu_{b'} &= Pos\left(\min\left\{-\alpha v_{1}, -\gamma y_{2}\right\}\right), \\ \mu_{c'} &= Pos\left(\min\left\{\alpha' v_{1}, \max\left\{-r, \gamma' y_{1}\right\}\right\}\right), \\ \mu_{d'} &= Pos\left(\min\left\{\alpha' v_{1}, \min\left\{r, -\gamma' y_{1}\right\}\right\}\right) \end{aligned}$$

and

$$\mu_{e}(u_{i}) = Pos\left(\min\left\{u_{mi} - u_{i}, u'_{mi} + u_{i}\right\}\right),\\ \mu_{e'}(v_{i}) = Pos\left(\min\left\{v_{mi} - v_{i}, v'_{mi} + v_{i}\right\}\right).$$

According to (1), the non-normalized possibility -  $\chi(u_1, u_2; x_1, x_2)$  that the velocity of change of the trained neuron state  $\mathbf{x} = (x_1, x_2)$  is  $\mathbf{u} = (u_1, u_2)$  can be expressed as

$$\chi_t = \min\{\max\{\mu_a, \mu_b\}, \max\{\mu_c, \mu_d\}, \mu_e\}.$$
 (22)

Correspondingly, for the control neuron we have:

$$\chi_c = \min\left\{\max\left\{\mu_{a'}, \mu_{b'}\right\}, \max\left\{\mu_{c'}, \mu_{d'}\right\}, \mu_{e'}\right\}.$$
(23)

The generator of evolution -  $g(\mathbf{u}, \mathbf{v}; \mathbf{x}, \mathbf{y})$  of the neuron system can be obtained by normalization with respect to  $\mathbf{u}$  and  $\mathbf{v}$  of min  $(\chi_t; \chi_c)$ . Thus:

$$g(\mathbf{u}, \mathbf{v}; \mathbf{x}, \mathbf{y}) = \frac{\min \left\{ \chi_t(\mathbf{u}; \mathbf{x}, \mathbf{y}); \chi_c(\mathbf{v}; \mathbf{x}, \mathbf{y}) \right\}}{\sup_{\mathbf{u}, \mathbf{v}} \min \left( \chi_t; \chi_c \right)}.$$
 (24)

For simplicity we put the value of our constants  $\alpha, \alpha', \gamma$  and  $\gamma'$  to be equal 1 and  $u_{mi} = v_{mi} = w$  (which is enough to obtain qualitative agreement with the experiment). Then

$$\sup_{\mathbf{u},\mathbf{v}} \min(\chi_t; \chi_c) = \\ = Pos\left(\min\left\{\frac{w}{2}, |x_2|, |y_2|, |\min\{-x_1, y_1\}|\right\}\right).$$
(25)

The dynamics of this system is described in the terms of "fuzzy trajectories" -  $\mathbf{X}(t; s_0)$ ,  $\mathbf{Y}(t; s_0)$ , where the constant  $0 \le s_0 \le 1$  is the preferability of the given trajectory. These fuzzy trajectories are obtained from the system:

$$\frac{d\mathbf{X}}{dt} = \frac{\partial (\mathbf{p}_t \cdot \mathbf{W})}{\partial \mathbf{p}_t},$$

$$\frac{d\mathbf{p}_t}{dt} = -\frac{\partial (\mathbf{p}_t \cdot \mathbf{W})}{\partial \mathbf{X}};$$

$$\frac{d\mathbf{Y}}{dt} = \frac{\partial (\mathbf{p}_c \cdot \mathbf{W})}{\partial \mathbf{p}_c},$$

$$\frac{d\mathbf{p}_c}{dt} = -\frac{\partial (\mathbf{p}_c \cdot \mathbf{W})}{\partial \mathbf{Y}}.$$
(26)

where  $\mathbf{p} = (\mathbf{p}_t, \mathbf{p}_c)$  and  $\mathbf{W} = (\mathbf{U}(\mathbf{X}, \mathbf{Y}; \mathbf{p}), \mathbf{V}(\mathbf{X}, \mathbf{Y}; \mathbf{p}))$  is obtained as:

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$$p_{ti} = \beta \frac{\partial g(\mathbf{U}, \mathbf{V}; \mathbf{X}, \mathbf{Y})}{\partial U_i},$$
  

$$p_{ci} = \beta \frac{\partial g(\mathbf{U}, \mathbf{V}; \mathbf{X}, \mathbf{Y})}{\partial V_i},$$
  

$$s_0 = g(\mathbf{U}, \mathbf{V}; \mathbf{X}, \mathbf{Y}),$$
(27)

where  $\beta$  is an auxiliary gauge factor. Substituting (22) into (??) we get:

$$\min \left[\Psi_t \left(\mathbf{U}; \mathbf{X}, Y_1\right); \Psi_c \left(\mathbf{V}; \mathbf{Y}, X_1\right)\right] = = Pos^{-1} \left(s_0 Pos \left(\min \left[\Psi_{tm}, \Psi_{cm}\right]\right)\right), \qquad (28)$$

where  $\Psi_{\ldots}=\{\psi_{\ldots 1},\psi_{\ldots 2}\}$  :

$$\psi_{t1} = \min \{ w - U_1, w + U_1; \\ \max [\min (X_2; U_1); \min (-X_2; -U_1)] \}, \\ \psi_{t2} = \min \{ w - U_2, w + U_2; \\ \max [\min (-\varphi; U_2); \min (\varphi; -U_2)] \}, \\ \varphi = -\min (-X_1, Y_2), \end{cases}$$

and the similar expressions for  $\psi_{ci}(\mathbf{V};\mathbf{Y},X_1)$ . Correspondingly we have:

$$\Psi_{tm} = \sup_{\mathbf{U}} \Psi_t \left(\mathbf{U}; \mathbf{X}, Y_1\right) = \\ = \min\left(\frac{w}{2}, |X_2|, |\varphi|\right), \\ \Psi_{cm} = \sup_{\mathbf{V}} \Psi_c \left(\mathbf{V}; \mathbf{Y}, X_1\right) = \\ = \min\left(\frac{w}{2}, |Y_2|, |\varphi|\right).$$

Here we consider only the case  $s_0 = 1, \mathbf{p}_{\dots}(0) \equiv 0$ , which is a "tube" of the most preferable trajectories. These trajectories does not depend on the explicit form of the function Pos(a).

In this case a whole interval

$$\mathbf{U}^{(-)}(\mathbf{X}, \mathbf{Y}) \leq \mathbf{U} \leq \mathbf{U}^{(+)}(\mathbf{X}, \mathbf{Y}), 
\mathbf{V}^{(-)}(\mathbf{X}, \mathbf{Y}) \leq \mathbf{V} \leq \mathbf{V}^{(+)}(\mathbf{X}, \mathbf{Y}),$$
(29)

satisfy equation (??). Thus, the equations (??) become:

$$\mathbf{U}^{(-)}(\mathbf{X}, \mathbf{Y}) \leq \frac{d\mathbf{X}}{dt} \leq \mathbf{U}^{(+)}(\mathbf{X}, \mathbf{Y}), 
\mathbf{V}^{(-)}(\mathbf{X}, \mathbf{Y}) \leq \frac{d\mathbf{Y}}{dt} \leq \mathbf{V}^{(+)}(\mathbf{X}, \mathbf{Y}),$$
(30)

where  $\mathbf{U}^{(-)}(\mathbf{X}, \mathbf{Y})$ ,  $\mathbf{U}^{(+)}(\mathbf{X}, \mathbf{Y})$  and  $\mathbf{V}^{(-)}(\mathbf{X}, \mathbf{Y})$ ,  $\mathbf{V}^{(+)}(\mathbf{X}, \mathbf{Y})$  are lower and upper bonds of the solution of the equation (??) for  $s_0 = 1$ . The solution of (26) for initial condition:  $X_1(0) = Y_1(0) = a_0 \leq 0$  (moderate initial neuron activity) and  $X_2(0) = Y_2(0) \ll a_0$  (small initial damage) is shown in the Fig.5 and Fig.6. Qualitative agreement with the experiments is very good.

#### 5 Concluding remarks

The proposed theory has an obvious phenomenological form and, thus, does not need in assumptions about concrete physical and chemical mechanisms of neuron activity. Although this feature can be considered as an advantage of the approach, the problem of whether fuzzy dynamic is also realized on the "microscopic" physical and chemical level, is still unsolved.

The brain is a unique object. We usually examine brain behavior like that of any other object, but the brain itself also studies the environment. Which logic does the brain use for description? The choice of the means for the description of the environment is determined by the salient features of the brain, not only by the expedient. The huge non-stability of brain function [12], [13] at the every level of activity is well known. Animal behavior is not repeated from trial to trial. Neural centers may or may not participate in the same behavior. Reactions of individual neurons also are extremely non stable. The behaviors of postsynaptic potentials and potential-dependent channels of excitable membrane submit to quanta regularities. There is a factor desynchronizing the output of a single neuron with various postsynaptic neurons [14]. Increasing of chaos at the neuronal level has been demonstrated during elaboration of a local instrumental reflex in Helix neurons [1]. Increasing of the uncertainty during the middle stage of the reflex elaboration obtained here (see Fig.5 and Fig.6) is in good agreement with these experiments.

There are several reasons for the uncertainty of brain's decision-making. Firstly, there is a deficiency of knowledge about the environment. This component of uncertainty decreases after interaction with the environment. Secondly, there is the complexity of calculations executed by the brain. The error in successive processes grows. Although the brain's calculations are also in-parallel, we will demonstrate later that in the real brain this does not decrease the common error of calculations. However, the second component of uncertainty may also decrease after memorizing the results of the intermediate calculations during learning (e.g., the exactness of professional skill improves after training). The final level for decision-making is, apparently, molecular. There are data indicating that some elemental behavioral properties, especially simple learning, are inherent to the neuron itself ([15],[12], [16], [17]). The main arguments for learning localization in a neuron are that long-term memory is resistant to extensive nervous tissue ablation; that learning may be executed locally in a limited part of brain tissue or by means of local conditioned and unconditioned irritation of the neuron (see review [18]). Note that our theory do not assume a memorization process in the reflex elaboration. Thus, the considered mechanism can provide an adaptation even for creatures without long term memory.

It has been shown [19], [20], [21], [22] that a neuron becomes more excitable in its reaction to a stimulus as the biological significance of the stimulus increases, whereas it does not change its excitability to a neutral stimulus. The isolated neuron is also capable of primitive plasticity [23]. Associative learning in Drosophila is retained after metamorphosis, although the brain undergoes considerable degeneration, Any controlled entity which does not behave as a deterministic object may reorganization, and growth during metamorphosis [24].

It may be supposed, that the uncertainty of the brain's actions evoked by the various reasons must decrease during experience and as the result of the averaging. Nevertheless, this does not correspond to reality. The exceptions are the high degree the automation of actions, influence of a narcosis and other incidents of the unconsciousness, when brain's participation in the behavior is rather limited. It could be supposed that in large neuronal networks non-stability of activity of the individual neurons is averaged over the ensemble. However, this does not conform to experimental data. The result of the non-stability of the whole brain is not much smaller than the non-stability of individual neurons and smoothing is absent. This problem has been investigated on the example of elaboration of an instrumental conditioned reflex in mollusks. By deficiency of information for the decision making, when the brain uses trial-and-error methods [25], the condition of a neuron changes from trial to trial. In different neurons these alterations occurred coordinated. The actual choice is, evidently, simultaneously non stable, but not random, since the same choice takes place in the various brain neurons. Amalgamation of neurons in the ensembles is apparently the main factor for consciousness [26]. This returns us to the assumption that the absence of averaging by ensemble is determined by the fact that the decision in the brain is made by the network. Apart from the aforementioned objections, it should be pointed out that the time delay for decision-making in cortical neurons is only 20-50 ms [27] and therefore the time for a signal to travel around the neural network (as was supposed, e.g., in [28]) is absent. Faint averaging in the brain suggests that neurons receiving the same kind of information make similar decisions. The goal for brain function is survival of the organism and the "goal" for neuron function is its own survival. Since at every level of the neural system, from the organism to the macromolecule,

behavior is non-stable, the dependence of a reaction magnitude on a stimulus magnitude is not continued. At each level there is a threshold for activation of the given system, from disturbances in homeostasis, via action potential generation and opening of the channels of excitable membranes. During routine activity, e.g. during perception of signals, the brain's reactions are continuous. However, for the most essential cases it is important to choose between a maximum and a minimum. During threats on life the dangerous possibilities are rejected, but during the absence of an immediate threat on life the basis for behavior is the disturbance of neuronal homeostasis. This disturbance induces a transient damage of the determined brain neurons and their activities are directed to maximal protection from transient damage [1]. Therefore, for neurons playing a decisive role in actual behavior with an unclear result, the situation is generally critical and choice is simple: whether it is necessary to generate action potential, i.e. it is necessary to the choice is simple: whether or not it is necessary to generate an action potential, i.e., it is necessary to choose between a maximum and a minimum. We may conclude that activity of the active brain is extremely non stable in any level of its function. However, in spite of availability of enormous number of brain's elements  $(10^{10} - 10^{12} \text{ neurons}; 10^{12} - 10^{15} \text{ synapses and much more number}$ of the channels) and absence of the visible synchronizing influences (with exception of the reception of the essential biological information) activity of many different elements of the brain is coordinated and is not averaged by the ensemble. Besides, even normal brain functioning is based on the maintenance of the equilibrium between life and death in corresponding sensitive neurons. Therefore, the choice of uttermost decisions is the normal action of the brain. We may consider that at least in the important cases neural system have to make decisions by means of fuzzy logic. For a certain kind of neurons a mechanism that could carry out *min-max* operations was proposed in [29].

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