Use of coordinates systems for 3D plot of discontinuous functions

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The study of functions of two real variables can be supported by visualization using a Computer Algebra System (CAS).Historically, contour plots were the first type of graphical representations. With the development of scientific computing, 3D plots were introduced and plotting the graph of a two-variable function has been made possible, including parametric plotting and implicit plotting. In most of the CAS such as MATLAB, Maple, Mathematica, the 3D plot may be builded using local coordinates systems and linear interpolation of the function using local parameters.

When the function is continuous, the uniform convergence of the approximated function to the function is proved by Bernstein Theorem [5].Then the 3D plot is independent of the type of local coordinate system. Therefore, the same 3D plot is generated by different local coordinates; see [2].

However, in a neighborhood of a discontinuity, Bernstein Theorem fails and the 3D plot is strongly dependant on the type of local coordinates chosen for the 3D plot.

In this present paper, we analyze the various aspects of the 3D plot created by different local coordinates. The study focuses on functions of the type $f(x,y) = \frac{P(x,y)}{Q(x,y)}$ where $P(x,y)$ and $Q(x,y)$ are polynomials of degree 2.

We distinguish different types of discontinuities:

1. $Q(x,y)$ is a linear function;
2. $Q(x,y)$ is a quadratic function;
3. $Q(x,y)$ contains linear and quadratic functions.

The choice of an adequate coordinate system is required before generating a 3D plot because of two main problems:

1. A non suitable choice of local coordinates may yield an inaccurate plot. In this case, the discretization of the function on the local coordinates miss the discontinuity points or lines.
2. Non accurate erratic behavior along the discontinuities appears.

3. Regular plots may be obtained when geodesics on the surface $Q(x,y) = k$ are used.

4. Multiple discontinuities are also analyzed. This analysis is based on image processing algorithm used for curve extractions.

Finally, a comparison of some different plotting software such as MATLAB, Maple, K3Dsurf around the choice of local coordinate systems, will be presented. We will focus on the application of image processing for the visualization of the discontinuities surfaces.

References

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